Congressional Campaign Contributions, District Service and Electoral Outcomes in the United States: Statistical Tests of a Formal Game Model with Nonlinear Dynamics*

by

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Abstract

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Using a two-stage game model, with the second stage being a system of ordinary differential equations, I argue that candidates, political parties and financial contributors interact strategically in American congressional elections in a way that is inherently nonlinear. The nonlinearity explains longstanding anomalies in the congressional elections literature regarding candidate finances, district service and votes for the incumbent. Congressional races in which the incumbent faces a challenge are generated by dynamical systems that have Hopf and saddle connection bifurcations. A small change in the challenger's quality or in the type of district service can change a stable incumbent advantage into a race with growing oscillations in which the incumbent's chances are uncertain. Normal form equations from local bifurcation theory, and topological considerations, motivate a statistical model that can recover qualitative features of the dynamics from cross-sectional data. I estimate and test the model using district-level data from the 1984 and 1986 U.S. House election periods for political action committee campaign contributions, intergovernmental transfers and general election vote shares.

Introduction

Ill-funded challengers are uncompetitive in U.S. congressional elections (Jacobson 1980; Jacobson 1985; Abramowitz 1991; Krasno 1994), but it is not clear how incumbents produce such challengers. Better challengers are more likely to enter a race when they are more likely to win (Bond, Covington and Fleisher 1985; Banks and Kiewiet 1989; Jacobson 1989; Jacobson 1990a). But why do potential challengers take their chances of winning as given? Large incumbent "war chests" of campaign funds in particular can deter quality challengers (Epstein and Zemsky 1995; Box-Steffensmeier 1996). But it is not clear why incumbents are able to accumulate such war chests: why would financial contributors want to make the election uncompetitive? Several interesting theoretical arguments suggest that district service such as casework and "pork barrel" spending ought to benefit incumbents (Austen-Smith 1987; Baron 1989a, 1989b, 1994; Hinich and Munger 1989; Snyder 1990; Morton and Cameron 1992), but the effects have been remarkably difficult to identify in empirical work. There is good evidence that local federal expenditure varies in response both to incumbents' involvement in "policy subsystems" (Stein and Bickers 1995) and to the proportion of Democratic voters in each district (Levitt and Snyder 1995), but evidence that district service affects votes has been hard to come by (Feldman and Jondrow 1984; Cain, Ferejohn and Fiorina 1987; Fiorina 1989; Stein and Bickers 1994; Levitt and Snyder 1997), and evidence regarding effects of district service on campaign contributions has been mixed (Kau and Rubin 1982; McAdams and Johannes 1987; Grier and Munger 1986; Snyder 1990; Endersby and Munger 1992; McCarty and Rothenberg 1996).

I argue that the lack of simple and reliable empirical relationships among campaign contributions, district service, challenger quality and election outcomes reflects a nonlinearity inherent in the strategic interactions of political parties, candidates and contributors. I use a two-stage game model in which the second stage is a realization of a system of ordinary differential equations.

The type of district service and the quality of challenger that, respectively, the incumbent and the opposing party are most likely to choose in the first stage of the game induce a particular kind of nonlinearity in the second stage dynamics. For service type and challenger quality values near the values that have the highest probability of occurring in the perfect Nash equilibrium solution of the game, the dynamics exhibit Hopf and saddle connection bifurcations (Guckenheimer and Holmes 1983, 150, 290). I show how the nonlinearity of the dynamics may be the reason for the complicated and contradictory empirical relationships that have been reported in the literature. I develop a nonlinear statistical model based on the normal form equations that local bifurcation theory specifies for Hopf bifurcation. I estimate the model using cross-sectional data from the 1984 and 1986 election periods. I use statistical tests to examine how well the dynamics the model recovers match predictions from the game.

Formal Theory: A Two-stage Campaign Game

In the first stage of the game, the incumbent chooses a type for service that will be provided after the election, while the opposing party simultaneously chooses the quality of a challenger to run against the incumbent. There is then a subgame (the game's second stage) during which the incumbent and challenger each produce a flow of proposals for rates at which each will provide the service after the election, if victorious, while a financial contributor simultaneously produces a flow of proposals for money it will give to each candidate's campaign. In the subgame the three players act in continuous time, their movements being restricted to the dynamics of a system of ordinary differential equations. That system starts from specified kinds of initial conditions and runs for a finite time that is common knowledge. At the end of that time the voters learn the then-current parameter values and the election occurs. The probability that the incumbent wins the election is a function of the current values of the service rates and the contributions. The incumbent, the opposing party, the challenger and the contributor have the function as common knowledge and all fully anticipate the probability. Payoffs to the incumbent, opposing party, challenger and contributor are functions of the probability, evaluated at the time of the election. The players have complete information about their own and one another's payoff functions. Overall, the solution concept for the game is perfect Nash equilibrium. In each subgame the solution concept is Cournot-Nash equilibrium, with some refinements to choose initial conditions and specify what occurs when no Cournot-Nash equilibrium exists.

The model draws on several ideas developed in other formal work. As in many other models, the parties, candidates and contributor interact before the election based on rational expectations of voters' behavior. In Baron's (1989a; 1989b; 1994) models of interactions between candidates and contributors, the probability that each candidate wins the election is an explicitly specified function. Baron (1989a) showed that solutions with incumbent advantages in both contributions and reelection chances can be produced using a variety of exogenous differences between the candidates, including differences in recognition, valuation of the office, service effectiveness and interest-group support. The present model treats the incumbent and the challenger asymmetrically, but the asymmetries do not necessarily lead to an incumbent advantage. Austen-Smith (1987; 1995) has also analyzed games with policy selection, campaign contributions and elections.

The present model is also informed by the concept that contributors view their contributions as investments. In connection with electoral outcomes, this idea has been explored in formal work especially by Welch (1980), Denzau and Munger (1986) and Baron (1989a), both formally and empirically by Snyder (1990), Stratmann (1992) and Grier, Munger and Roberts (1994), and empirically by McCarty and Rothenberg (1996). The present model and Snyder's model imply the same behavior for an "investor-contributor" in the case where, in the present model, the type of service being provided makes voters indifferent to the amount of contributions being given to the candidates. Snyder does not model the concept of different types of service. In this sense the present model can be viewed as a generalization of his approach.

The type of service concept is defined in terms of the reaction service provokes among voters (Denzau and Munger 1986). Voters in the model respond to the difference between the amount of service the incumbent will provide, if she wins the election, and the amount that would be provided by the challenger should he win. The type of the service determines whether a larger amount of service from a candidate attracts or repels voters. For types of service that repel voters, a candidate's chances of winning the election increase if he or she promises to provide less service.

The concept of different types of service is motivated by the observation that service can be distributed in a variety of ways and produce a variety of externalities in a district. The taxes that must ultimately be collected to pay for spending are a negative externality associated with each service increase. Service that distributes benefits widely may offset the costs this externality imposes on most constituents. Staff assigned to the district to support casework may be an example of this kind of service: many may choose not to use the staff, but any constituent who wishes to do so can. Service that targets benefits more narrowly will do little to offset the costs most voters face, unless the benefits create significant positive externalities. For example, highway construction contracts go to individual firms, and so provide highly concentrated benefits, but when completed the highway itself will be a local public good. Situations in the model where voters are hostile to service are supposed to represent situations where the benefits from service, including externalities, do not exceed the costs in the particular district.

I interpret challenger quality as referring to the way the challenger computes his payoffs during the campaign. I assume that the incumbent cares about the negative effect an increase in voters' hostility to service would have on her reelection chances. The incumbent would like to minimize those effects. If the challenger's quality is high then he also cares about the effect an increase in voters' hostility to service would have on his chances of winning. The highest quality challenger weighs these concerns as strongly as the incumbent does; the highest quality challenger thinks just like the incumbent. A low quality challenger ignores the effects of potential changes in voters' responses to service.[1]

Voters do not respond to the challenger's quality in any direct way in the model. This does not mean that the challenger's quality has no effect on the probability that the incumbent wins the election. Voters in the model respond to the rate at which each candidate turns contributions into service, and to the amount and type of service to be provided after the election. The amount of service is an increasing function of the contributions to each candidate and of each candidate's service rate. The service rates promised by the candidates, the contributions to the candidates, the service type and the quality of the challenger are all jointly determined, because of the strategic interactions among the parties, candidates and contributor. So the challenger's quality does affect voters, albeit indirectly.

In the system of differential equations that describes each subgame, the candidates adjust their intended service rates, while the contributor adjusts the contributions it will make to each candidate. The adjustment process is continuous time Cournot adjustment: the players all act noncooperatively, with each player making the adjustment in each parameter that would produce the largest improvement in its payoff if all the other parameters were to remain constant.[2]

Specification Details

In the first stage the incumbent chooses a real value for a service type parameter g while the opposition party chooses a challenger that has a quality value h in the range $0 \leq h \leq 1$. An increase in g represents an increase in the concentration of benefits, while a higher value of h represents a higher quality challenger. During the second stage the incumbent issues a flow of proposals for a post-election district *service rate* $r > 0$, the challenger issues a flow of proposals for an alternative service rate $q > 0$ and the contributor issues a flow of proposals for contributions to be made to the incumbent and challenger respectively in amounts $a > 0$ and $b > 0$. The combined flow of proposals evolves continuously according to a four-dimensional system of ordinary differential equations.

The incumbent seeks to maximize both the expected gain from her service rate and the extent to which an increase in the concentration of the benefits from service helps her reelection chances. The incumbent's payoff is

$$I = pr + \frac{\partial p}{\partial g}$$

where $r > 0$ is the incumbent's service rate and p is the probability that the incumbent wins reelection. Because the service rate r is received only if the incumbent wins, the incumbent acts to maximize the expected value pr. The term $\partial p/\partial g$ represents the effect an increase in the concentration of the benefits from the service would have on the incumbent's reelection chances. By the definition of p, below, voters are hostile to an increase in the value of service if $g > 0$, but respond favorably to more service if $g < 0$. One reason for the incumbent to care about $\partial p/\partial g$ would be if the incumbent believes that "policy subsystems" (Stein and Bickers 1995) or similar institutions tend to concentrate benefits beyond the degree expressed by the incumbent's own choice of g.

The challenger's payoff function is similar to the incumbent's, with one modification to represent the concept of challenger quality. The challenger's payoff is

$$J = (1 - p)q + h\frac{\partial(1 - p)}{\partial g}$$

where $q > 0$ is the challenger's service rate. Like the incumbent, the challenger seeks to maximize his expected service rate. But the degree to which the challenger is concerned about the effects of an increase in the concentration of benefits depends on the challenger's quality. The challenger is as concerned as the incumbent only if $h = 1$. For $h < 1$ the challenger is less sensitive than the incumbent; if $h = 0$, the challenger is completely insensitive. To avoid incentive compatibility complications I assume that the party opposing the incumbent has the same payoff function as the challenger.

The contributor wishes to maximize the return it gets in service, given the amount it is committing to pay in contributions. The contributor's payoff is

$$K = pr^2(1 + a)^2 + (1 - p)q^2(1 + b)^2 - a^2 - b^2$$

where $a > 0$ and $b > 0$ denote the contributions made respectively to the incumbent and the challenger. The contributor evaluates the cost of contributions quadratically. The costs of the contributions are therefore a^2 and b^2. The value of post-election service is $r^2(1+a)^2$ if the incumbent wins and $q^2(1 + b)^2$ if the challenger wins.[3] There is some service even if contributions become vanishingly small; service is at least either r^2 or q^2. But for fixed r and q there is always more service if contributions increase. Because service is provided only after the election, the contributor acts based on the expected value of the potential returns, given the reelection probability p.

Such a form for K entails the idea that the contributor views campaign contributions as investments (Snyder 1990). Given the restriction $\partial p/\partial a = 0$, it is easy to show that in equilibrium the contributor gives the incumbent a contribution equal to a permanent stream of income of one unit per period.[4] The specification for K therefore implicitly represents an expected long-term relationship between the contributor and the incumbent, whenever $\partial p/\partial a = 0$. Similar results can be obtained for the challenger. The solution to $\partial K/\partial a = 0$ does not simplify in such an appealing way when $\partial p/\partial a \neq 0$, but the result $\partial p/\partial a = 0$ nonetheless gives an intuitive interpretation of the service rates r and q. In trying to maximize r, the incumbent is trying to minimize the interest rate at which the contributor is willing to invest in the incumbent by making a contribution (compare Snyder 1990, 1198). For any given value of p, a higher value of r implies a lower interest rate, and therefore a higher "price" in terms of more contributions for the incumbent. The challenger's motives are analogous.

Voters treat the candidates asymmetrically in two respects. First, the incumbent enjoys a kind of recognition advantage. Voters respond to the service rate of the challenger only if the challenger succeeds in mounting a serious campaign. The classification of the challenge as serious or not occurs at the end of the game, based on the challenger's position at that time. The idea is that, through franked mail and other communications during her current term, the incumbent has already convinced the voters that she should get the benefit of the doubt in their decisions. Voters pay attention to the inherent merits of the challenger only if the media decide to cover the challenger as a worthy alternative. If this does not happen, voters take the challenger's service rate into account only when computing the value of the post-election service the challenger would provide. The media's decision is probabilistic, based on a horserace-type rule.

The second asymmetry is that the strength of voters' response to the candidates' service values depends on the level of contributions to the challenger's but not the incumbent's campaign. The idea here is that the burden is on the challenger to convince voters that they should compare the service levels the candidates are promising.

The two kinds of voter decision rules are indexed by $m \in \{0, 1\}$, where $m = 0$ represents the situation where the challenge is not serious. Given m, the probability that the incumbent wins is

$$\pi_m = [1 + \exp\{mq - r - gb[q^2(1+b)^2 - r^2(1+a)^2]\}]^{-1} .$$

The probability that the challenge is serious is $\Pr(m = 1) = \nu$, where

$$\nu = [1 + \exp\{\mu(\pi_0 - \tfrac{1}{2})\}]^{-1}$$

with $\mu \geq 0$ being an exogenously set constant. The challenger is likely to be taken seriously only if he already has significant support based solely on the comparison between his and the incumbent's service commitments. The horserace aspect of this formulation is clearest when μ is large, for then $\partial\nu/\partial\pi_0 = -\mu\nu(1-\nu) \leq 0$ is steep near $\pi_0 = \frac{1}{2}$, such that the value $\pi_0 = \frac{1}{2}$ becomes in effect the threshold below which the challenger must reduce the incumbent's support in order to be considered a serious threat. The unconditional probability that the incumbent wins, p, is the mixture of the serious-challenger and not-serious-challenger alternatives:

$$p = \nu\pi_1 + (1-\nu)\pi_0 \ .$$

Voters are attracted by higher amounts of post-election service if $g < 0$, but repelled if $g > 0$: $\partial\pi_m/\partial[r^2(1+a)^2] = -gb\pi_m(1-\pi_m)$ and $\partial\pi_m/\partial[q^2(1+b)^2] = gb\pi_m(1-\pi_m)$. The term $\partial p/\partial g$ in I can therefore give the incumbent an incentive to reduce her service rate. The incumbent's incentives regarding her service rate will depend on both the type of the service and campaign contributions.[5] Similar comments apply to the challenger, so long as $h > 0$.

During the second stage subgame, each player uses steepest ascent with respect to its payoff function to adjust its proposal values in continuous time. To keep the proposal values positive but always with smooth dynamics, I define the differential equations in terms of the natural logarithms of the proposal variables. Using t to denote time,

$$d\log r/dt = (\partial I/\partial r)/r \tag{1a}$$

$$d\log q/dt = (\partial J/\partial q)/q \tag{1b}$$

$$d\log a/dt = (\partial K/\partial a)/a \tag{1c}$$

$$d\log b/dt = (\partial K/\partial b)/b \tag{1d}$$

There is a *dynamic equilibrium* when the system (1) is at a fixed point, i.e., when $\partial I/\partial r = \partial J/\partial q = \partial K/\partial a = \partial K/\partial b = 0$. A dynamic equilibrium is a *Cournot-Nash equilibrium* only if the fixed point is a local maximum for each player, i.e., only if $\partial^2 I/\partial r^2 < 0$, $\partial^2 J/\partial q^2 < 0$ and the matrix

$$K_{ab} = \begin{bmatrix} \partial^2 K/\partial a^2 & \partial^2 K/\partial a \partial b \\ \partial^2 K/\partial a \partial b & \partial^2 K/\partial b^2 \end{bmatrix}$$

is negative definite.

The second-stage subgame that occurs for each (g, h) pair is a realization of system (1). I assume the following about the initial conditions for each realization. If for the (g, h) pair system (1) has a unique Cournot-Nash equilibrium point that is asymptotically stable (Hirsch and Smale 1974, 186), then that point is the subgame outcome. If system (1) has multiple Cournot-Nash equilibria for the (g, h) pair but only one equilibrium is asymptotically stable, then the players choose the stable point.[6] Cournot-Nash equilibria that are not asymptotically stable fixed points can be eliminated by a perfection argument.[7] If for the (g, h) pair no stable fixed points exist, I assume that the players begin at a point that is a Cournot-Nash equilibrium for some nearby (g, h) pair.

The ideal approach to solve the game would be to integrate system (1) for a fine grid of service type and challenger quality values, and then to use the resulting payoff values to find Nash equilibria for the first-stage choices of g and h. This would be backward induction. It is computationally infeasible to integrate system (1) for so many (g, h) pairs, so I use an approximation to the ideal method. I integrate system (1) near fixed points for a large number of (g, h) pairs, the goal being to find the set of (g, h) values at which the flows of the system change in a qualitatively significant manner. Such a set is called a *bifurcation set* (Guckenheimer and Holmes 1986, 119). If possible, each fixed point is a stable, Cournot-Nash equilibrium. Within each region of qualitatively similar behavior relatively few integrations of system (1) are needed to determine how the payoffs to the incumbent and to the opposition party (i.e., to the challenger) vary with (g, h). For each (g, h) pair I determine payoffs from the system (1) subgame as follows. When a stable fixed point exists, payoffs are evaluated at that point. When there is a stable limit cycle but no stable fixed point, payoffs are computed by averaging around the cycle. Where stable fixed points or cycles do not exist, I use the payoffs achieved after starting near a fixed point and integrating system (1) for about two time units.

Game Solution

Figure 1 shows a numerically approximated partial bifurcation set for system (1).[8] In region I there is always a stable Cournot-Nash equilibrium point at which the incumbent has a reelection advantage (i.e., $p > .5$). In region II, flows spiral away from the fixed point, approach a saddle point and then wander into a situation in which not only does the incumbent have an electoral advantage, but the challenger receives no contributions. In region III there is a stable limit cycle (Hirsch and Smale 1974, 250). Flows converge to an indefinitely repeated oscillation through most but not all of which the incumbent has an advantage. In region IV there are no stable fixed points. Flows wander rapidly to states in which $p = 1$. In region V, flows converge to the interior of a homoclinic cycle[9] and then to a stable fixed point at which $p = 0$. In region VI the incumbent usually ends up getting virtually no contributions but nonetheless runs at only a slight disadvantage ($.45 < p < .5$). For h very near zero, however, the flows become highly irregular and unpredictable. For $h = 0$, a frequent outcome is $p = 1$ and contributions to the incumbent increasing exponentially with time.

***** Figure 1 about here *****

Table 1 shows payoffs to the incumbent and challenger from the system (1) subgame for several *local maximum* and *effective boundary* pairs (g, h), arrayed so as to define the strategic form of the first-stage game. A (g, h) pair is a local maximum if small increases and small decreases in g both produce a worse payoff for the incumbent, or if small increases and small decreases in h both produce a worse payoff for the challenger. A (g, h) pair is an effective boundary if $h \in \{0, 1\}$ and the payoffs from system (1) do not materially change as g becomes more extreme. For instance, if g decreases below $g = -.08$ while $h = 0$, the incumbent's payoff from system (1) remains zero while the challenger receives the maximum possible payoff, $J_{\max}$. Table 1 also includes the payoffs from all the (g, h) pairs produced by crossing the g and h values from the local maximum and effective boundary pairs. The maximum payoffs, denoted $I_{\max}$ and $J_{\max}$, can each be arbitrarily large, depending on the length of time the system is imagined to run before the election. For the runs of about two time units used to construct Table 1, reasonable valuations are $I_{\max}, J_{\max} \geq 3$.

***** Table 1 about here *****

The game of Table 1 does not have a Nash equilibrium in pure strategies. The mixing probabilities for a mixed-strategy Nash equilibrium are shown in Table 2.[10] The mixing probabilities imply that the most likely outcome is the pair $(g, h) = (.0425, .487) \stackrel{\text{def}}{=} (g_0, h_0)$. Using $I_{\max} = J_{\max} = 3$ gives $\Pr(g = -.025) = .1$, $\Pr(g = .0425) = .9$, $\Pr(h = 0) = .45$ and $\Pr(h = .487) = .55$.

***** Tables 2 about here *****

Three of the four outcomes that have positive probability in the mixed-strategy equilibrium imply non-competitive elections. When $(g, h) = (-.025, 0)$, system (1) does not have a stable fixed point. Flows rapidly diverge in such a way that the probability that the incumbent wins the election falls to zero, resulting in a payoff to the incumbent of zero. I interpret this outcome as a case in which the incumbent drops out of the race: only if the incumbent retires is it *certain* that the incumbent will not win. When $(g, h) = (-.025, .487)$ or $(g, h) = (.0425, 0)$, system (1) also lacks a stable fixed point, but in these cases the probability that the *challenger* wins the election falls to zero. The natural interpretation of these cases is that the incumbent is running unopposed.

Of the four mixed-strategy equilibrium (g, h) pairs, only (g_0, h_0) induces dynamics that imply probabilities of election victory that are not either zero or one. A first-stage outcome of

$(g,h)=(g_0,h_0)$ leads to a competitive campaign in which the incumbent has a substantial advantage in terms of financial contributions and chances of reelection. With $(g,h)=(g_0,h_0)$, the point $(r,q,a,b)=(.84,1.02,.94,.52)\stackrel{\text{def}}{=}(r_0,q_0,a_0,b_0)$ is a fixed point that is a Cournot-Nash equilibrium: $\partial^2 I/\partial r^2 = -.138$, $\partial^2 J/\partial q^2 = -.762$ and $K_{ab} = \begin{bmatrix} -1.75 & .076 \\ .076 & -1.77 \end{bmatrix}$. In qualitative dynamic terms, (r_0,q_0,a_0,b_0) is a center (Hirsch and Smale 1974, 95): flows in a neighborhood of (r_0,q_0,a_0,b_0) are attracted to a surface of closed periodic orbits that surround (r_0,q_0,a_0,b_0). Figure 2 illustrates the pattern of convergence to the attracting surface and the magnitude of the variations around the periodic orbits. The figure shows a flow in system (1) for $(g,h)=(g_0,h_0)$, beginning with (r,q,a,b) near (r_0,q_0,a_0,b_0). The flow rapidly converges to a closed orbit.[11] Around the closed orbit the probability that the incumbent wins the election varies between .655 and .68. Contributions to the incumbent range from .8 to 1.1, while contributions to the challenger range from .51 to .55.

***** Figure 2 about here *****

The (g,h) pair (g_0,h_0) is a bifurcation point for system (1): small changes from those values induce qualitative changes in the system's flows (Guckenheimer and Holmes 1986, 117). Figure 3 magnifies the bifurcation set diagram of Figure 1 near (g_0,h_0), which is marked as point O. For (g,h) values in region III the fixed point is unstable but there is a stable limit cycle. For (g,h) values in region II the fixed point is a spiral source and there is at least one saddle point; flows that start near the source in general approach the saddle point before wandering permanently away from both fixed points. For (g,h) values in region I the fixed point is a spiral sink.[12] Crossing the open segments O-B and O-A, *saddle connection* bifurcations occur (Guckenheimer and Holmes 1986, 290–294). Crossing the open segment O-C, there are *Hopf* bifurcations (ibid. 1986, 150–152).

***** Figure 3 about here *****

Small variations in the incumbent's choice of a service type g or in the opposition party's choice of quality h for the challenger may therefore lead to qualitatively different subgame dynamics. Because the dynamics occur during a finite time period, the consequences of the qualitative differences among the dynamics may be in one sense quantitatively small. For (g, h) values near (g_0, h_0), flows that start near the dynamic equilibrium point (r_0, q_0, a_0, b_0) approach or leave the point so slowly—after having been quickly attracted to an invariant surface—that in general at the end of the game the service rate and contributions variables have values near one of the closed orbits that exist when the equilibrium values for g and h are chosen exactly. So given similar initial values for (r, q, a, b), realizations of system (1) that have different (g, h) values near (g_0, h_0) may all leave the candidates and the contributor in similar quantitative configurations at the end of the game.

Implications for Previous Empirical Data Analysis

The solution of the game can explain several important features of the empirical relationships among votes, contributions and district service that have been reported in the literature. To see how this is so, consider those races in which the incumbent runs in the general election against an opponent. Suppose that the population of such competitive campaigns is generated by random sampling from the set of points where the flows of realizations of system (1) that have (g, h) pairs near (g_0, h_0) and initial values for (r, q, a, b) near (r_0, q_0, a_0, b_0) can be at the end of the time period of the second stage subgame. This would be a set of campaigns generated by the players' actions being subject to small random departures from equilibrium behavior. Such a population will be a random sample of points from a convex region shaped roughly like the one shown in Figure 2.

It is easy to see how a linear regression analysis would suggest that the challenger's financial resources are very strongly related to the election outcome, while the incumbent's resources are at best weakly related. Such is the finding repeatedly found by Jacobson (1980; 1985; 1990a; cf.

Green and Krasno 1988; 1990; Thomas 1989; Levitt 1994). Simulating random data as indicated above and then in the simulated data regressing the probability that the incumbent wins on the contributions to the candidates reproduces the familiar regression results. A characteristic example, using ordinary least squares regression with 150 simulated observations, is

$$\begin{array}{ccccccccccc} p & = & .948 & + & .0008 & a & - & .540 & b & + & u \\ & & (.008) & & (.0020) & & & (.014) & & & \end{array}$$

(standard errors in parentheses). As is usually observed in real data, the linear effect of the challenger's campaign finances is statistically significant and large, while the effect of the incumbent's campaign money is statistically insignificant and tiny. Such a result is the correct outcome for linear analysis of the data, but from the perspective of the current game it is causally spurious.[13] Increases in challenger finances do not cause the incumbent's vote share to fall. Rather candidate finances, district service and votes are jointly determined by strategic interactions among candidates, parties and financial contributors.

A linear analysis also produces spurious results for the effect district service from the incumbent has on the election outcome. Adding the incumbent's post-election service to the preceding regression gives

$$\begin{array}{cccccccccccccc} p & = & .986 & + & .014 & a & - & .612 & b & - & .0045 & r^2(1+a)^2 & + & u \\ & & (.016) & & (.005) & & & (.030) & & & (.0017) & & & \end{array} .$$

The estimate for the effect of the incumbent's campaign money is now statistically significant, but still tiny in comparison to the estimated effect of contributions to the challenger. The estimated effect of service is statistically significant, but with what some accounts (Cain, Ferejohn and Fiorina 1987; Fiorina 1989) would say is the wrong sign. Such an estimate echoes the difficulty Feldman and Jondrow (1984) and others (Stein and Bickers 1994) have encountered trying to find significant effects of local federal spending.

Statistical Estimation and Testing: Hopf Models

Any effort to test the empirical reality of the dynamics predicted by the solution of the game faces two difficulties. First, system (1) is certainly not an exactly correct description of the dynamics of a real campaign. At best, system (1) captures important features of the true dynamics. Second, the variables in terms of which system (1) is defined cannot be measured in a truly dynamic fashion during a campaign. Time series can be constructed to record the times at which candidates received particular monetary contributions. But it is not clear how well such data would measure the evolving intentions of individual contributors. And in any case the succession of service commitments that the candidates may be making—offers and counter-offers—would be extremely difficult if not impossible to observe. It will be necessary to use cross-sectional data to assess the game's predictions regarding the dynamics.

A solution to both problems is to use a statistical model that represents key features of system (1) in a way that is robust even to gross errors of functional form and measurement. To develop such a model I focus on the fact that system (1) exhibits Hopf bifurcations for (g, h) values near (g_0, h_0). For empirical analysis, $(g, h) = (g_0, h_0)$ is the most important of the equilibrium outcomes, because it is the only outcome that leads to a race in which the incumbent runs and faces opposition. By the theory of the game, all actual campaigns in which the incumbent runs for reelection and faces at least minimally funded opposition ought to have dynamics like those of system (1) for $(g, h) \approx (g_0, h_0)$ with $(r, q, a, b) \approx (r_0, q_0, a_0, b_0)$. All such campaigns ought to have contribution levels, vote totals and post-election service amounts that are generated by processes that are qualitatively similar to system (1) near $(g, h) = (g_0, h_0)$ and $(r, q, a, b) = (r_0, q_0, a_0, b_0)$.

Using local bifurcation theory's method of normal forms, it can be shown that any bivariate system of differential equations that exhibits Hopf bifurcation has a Taylor series expansion of degree 3 that can be brought into the following form by smooth changes of coordinates,

$$dx/dt = (\delta\mu + \alpha(x^2 + y^2))x - (\omega + \gamma\mu + \beta(x^2 + y^2))y \tag{2a}$$

$$dy/dt = (\omega + \gamma\mu + \beta(x^2 + y^2))x + (\delta\mu + \alpha(x^2 + y^2))y \tag{2b}$$

where $\delta \neq 0$ and $\omega > 0$ (Guckenheimer and Holmes 1983, 150–151). When $\alpha\delta\mu < 0$, system (2) has a circular limit cycle. The theorems of local bifurcation theory that justify system (2) as a generic representation of Hopf bifurcation imply the crucial point about its robustness: the qualitative property of exhibiting Hopf bifurcation is not affected by terms of degree higher than 3 in the Taylor series expansion (Arnold 1988, 270–275; Guckenheimer and Holmes 1983, 151–152). As a representation of Hopf bifurcation, system (2) is perfectly robust. In practical terms, the robustness of system (2) means that no matter how complicated the system may be that actually generates the data we can observe from a competitive campaign, if that generating system is near a fixed point and exhibits Hopf bifurcation at that point, then almost any set of measurements of the system can be smoothly transformed into a set of coordinates such that the equations of (2) are adequate to characterize the qualitative properties of interest for testing the predictions of the game. If the theory of the game is qualitatively correct in predicting that all competitive campaigns occur near a continuum of Hopf bifurcations, then empirical models built on the formulation of system (2) should accurately and reliably approximate the qualitative features of the true dynamics.

In the Appendix I show that the theoretically crucial qualitative properties of system (2) can be recovered from cross-sectional data by using a simultaneous statistical model for four observed variables, denoted v^*, w^*, x^* and y^*. Using $v = v^* - \tilde{v}$, $w = w^* - \tilde{w}$, $x = x^* - \tilde{x}$, $y = y^* - \tilde{y}$, the functional form of the model is

$$\begin{aligned} u_v = v + [&- \gamma_{vy}y + \gamma_{vw}w + \gamma_{vx}x + (\alpha_{vy}v - \beta_{vy}y)(v^2 + y^2 + \epsilon_{vy}vy) \\ &+ (\alpha_{vw}v + \beta_{vw}w)(v^2 + w^2 + \epsilon_{vw}vw) + (\alpha_{vx}v + \beta_{vx}x)(v^2 + x^2 + \epsilon_{vx}vx)]/3 \end{aligned} \tag{3a}$$

$$u_w = w + [- \gamma_{wy} y - \gamma_{wx} x - \gamma_{vw} v + (\alpha_{wy} w - \beta_{wy} y)(w^2 + y^2 + \epsilon_{wy} wy)$$
$$+ (\alpha_{wx} w - \beta_{wx} x)(w^2 + x^2 + \epsilon_{wx} wx) + (\alpha_{vw} w - \beta_{vw} v)(w^2 + v^2 + \epsilon_{vw} wv)]/3 \quad (3b)$$
$$u_x = x + [- \gamma_{xy} y + \gamma_{wx} w - \gamma_{vx} v + (\alpha_{xy} x - \beta_{xy} y)(x^2 + y^2 + \epsilon_{xy} xy)$$
$$+ (\alpha_{wx} x + \beta_{wx} w)(x^2 + w^2 + \epsilon_{wx} xw) + (\alpha_{vx} x - \beta_{vx} v)(x^2 + v^2 + \epsilon_{vx} xv)]/3 \quad (3c)$$
$$u_y = y + [\gamma_{vy} v + \gamma_{wy} w + \gamma_{xy} x + (\alpha_{vy} y + \beta_{vy} v)(v^2 + y^2 + \epsilon_{vy} vy)$$
$$+ (\alpha_{wy} y + \beta_{wy} w)(w^2 + y^2 + \epsilon_{wy} wy) + (\alpha_{xy} y + \beta_{xy} x)(x^2 + y^2 + \epsilon_{xy} xy)]/3 \quad (3d)$$

The random vector $\mathbf{u} = (u_v, u_w, u_x, u_y)^\top$ is assumed to be normally distributed with mean $E\mathbf{u} = 0$ and covariance matrix $E\mathbf{u}\mathbf{u}^\top = \mathbf{\Sigma}$. Unknown parameters, to be estimated, are $\tilde{v}$, $\tilde{w}$, $\tilde{x}$, $\tilde{y}$ and α_{ij}, β_{ij}, γ_{ij} and ϵ_{ij} for $ij \in \{vy, xy, wy, wx, vw, vx\}$, with $-2 < \epsilon_{ij} < 2$. I refer to this model as the *four-dimensional Hopf* (4DH) model.

The bifurcation set shown in Figure 3 exhibits Hopf bifurcations in system (1) only crossing the open segment O-C. The other bifurcations shown in the figure are saddle connection bifurcations rather than Hopf bifurcations. There is no guarantee that the equations of (2) will provide good approximations to the system's qualitative properties as (g, h) varies in the direction of the saddle connection bifurcations. Unfortunately for the goal of empirical testing, a saddle connection bifurcation is a global rather than a local phenomenon (Guckenheimer and Holmes 1983, 295). No normal form such as (2) exists that can generically represent the qualitative properties of such a bifurcation. But in light of the quantitative similarities across all three regions of Figure 3 in the flows that begin with $(r, q, a, b) \approx (r_0, q_0, a_0, b_0)$, at least not too far from $(g, h) = (g_0, h_0)$, it is plausible that for actual campaigns the different kinds of dynamics will be quantitatively sufficiently similar that model (3) will nonetheless provide a good approximation.

Statistical Tests of Qualitative Hypotheses about the Dynamics

Differences between kinds of political action committees (PACs) allow us indirectly to observe variations between presidential and midterm election periods in one of the two variables that are chosen in the first stage of the game model and that determine the type of dynamics that occur in system (1). The variable is challenger quality (h). We can therefore formulate predictions about differences that ought to be observed in the qualitative character of the dynamics in moving from one election period to the other. I use these predictions to motivate statistical tests of the key qualitative properties of the formal theory, using data from the 1984 and 1986 election periods.

Challenger quality ought to vary systematically between election years. In terms of vote share, the President's party invariably lost support in midterm congressional elections from 1918 through 1990 (Alesina and Rosenthal 1995, 84). But if the chances that an opposition party candidate will win the general election are reliably higher in the midterm year, then Banks and Kiewiet's (1989) analysis that shows that challengers ought to run when their chances of winning are greatest implies that challengers *not* of the president's party ought to be of higher quality at midterm than during the presidential election year. Banks and Kiewiet's argument further suggests that higher quality opposition party challengers ought to have an easier time of it during the primary season, as the expectation that the higher quality challengers will enter ought to deter lower quality opposition party challengers from entering. A similar argument suggests that challengers of the same party as the President ought to be of higher quality during the presidential election year than at midterm.[14]

Challenger quality also ought to vary systematically across kinds of PACs. Due to the respective parties' policy positions, labor PACs during the 1980s tended to favor Democratic incumbents (Grier and Munger 1986; Endersby and Munger 1992) and challengers (McCarty and Poole 1998). Corporate PACs tended to have been biased in favor of Republican incumbents (Grier and Munger 1986)[15] and challengers (McCarty and Poole 1998). Due to the conservative ideological bent of many high-profile non-connected PACs throughout the 1980s (Latus 1984), non-connected PACs tended to be biased in favor of Republican candidates during the 1984 and 1986 election cycles (McCarty and Poole 1998). Given the Republican presidential victory in 1984 and challengers' likely reactions to the midterm loss phenomenon, it follows that the Democratic challengers that labor PACs typically preferred to support ought to have increased in quality from 1984 to 1986, while the more Republican mix of challengers that corporate and non-connected PACs favored ought to have decreased in quality.

According to the theory that leads to Figure 3, such systematic variations in challenger quality across years and across PACs ought to imply corresponding variations in the stability of campaign dynamics. For labor PACs, from the 1984 to the 1986 campaign periods there ought to be movement up the challenger quality axis of the bifurcation diagram of Figure 3. The dynamics for labor PACs ought therefore appear more stable during the 1984 period than during the 1986 period. For corporate PACs and non-connected PACs, from 1984 to 1986 there ought to be movement down the challenger quality axis. The dynamics for corporate and non-connected PACs ought therefore appear less stable during the 1984 period than during the 1986 period.

I use two tests to evaluate whether the stability of the dynamics changes as predicted between election periods. The *distance* test assesses whether the 4DH model's estimated origin for the dynamics, $\hat{\hat{\mathbf{z}}} = (\hat{\hat{v}}, \hat{\hat{w}}, \hat{\hat{x}}, \hat{\hat{y}})^\top$, is farther from the sample mean ($\bar{\mathbf{z}}^*$) of the observed data $\mathbf{z}^* = (v^*, w^*, x^*, y^*)^\top$ during the election period for which more unstable dynamics are predicted than it is during the period for which more stable dynamics are expected. The greater the distance between $\hat{\hat{\mathbf{z}}}$ and $\bar{\mathbf{z}}^*$, the more likely it is that the dynamics are occurring in region II of Figure 3, where the fixed points are a spiral source and a saddle point, rather than in Figure 3's regions I (spiral sink) or III (limit cycle). As I explain in the Appendix, under the null hypothesis of no difference in stability between election periods, the test statistic for the distance test has a doubly noncentral F-distribution.

The second test, the *divergence* test, checks for a geometric feature that distinguishes stable from unstable dynamics. Stable dynamics push flows closer together, while unstable dynamics spread flows farther apart. The rate at which flows in a vector field ξ tend in this way either to increase or to reduce the volume of a bounded set can be measured by integrating the divergence of the vector field, denoted $\operatorname{div} \xi$, over the interior of the set. To estimate the divergence for the observed data I use vectors $-\hat{\mathbf{u}}$ to estimate the vector field. The set of vectors $\hat{\mathbf{u}}$ is computed by plugging the parameter estimates into model (3) and then evaluating the resulting equations for each observed data point. The divergence estimate is $\operatorname{div}(-\hat{\mathbf{u}}) = -(\partial \hat{u}_v/\partial v + \partial \hat{u}_w/\partial w + \partial \hat{u}_x/\partial x + \partial \hat{u}_y/\partial y)$. The divergence test is a one-tailed t-test for the equality of the sample means of $\operatorname{div}(-\hat{\mathbf{u}})$ between election periods. The mean divergence should be greater for the period predicted to be more unstable. The Appendix gives a more complete explanation of the divergence test.

I estimate the 4DH model by maximum likelihood with district-level data for the U.S. House elections of years 1984 and 1986.[16] The observed variables correspond to the formal variables of system (1) that were used in Figure 2 to illustrate the system's flows.[17] Variable v^* represents post-election district service, measured by intergovernmental transfers from the federal government to local governments in each congressional district during the year *following* each election: I use 1985 transfers for the 1984 election period and 1987 transfers for the 1986 election period. Intergovernmental transfers are a kind of district service that Members of Congress are well-known to affect (Arnold 1979; Haider 1974; Stein and Bickers 1995). I consider separately four types of transfers: education transfers; highways transfers; social welfare transfers; and other transfers. v^* is the natural logarithm of the amount originally measured in units of \$1000 per person.[18] Variable w^* represents incumbent contributions, measured by the total amount of PAC campaign contributions to each incumbent during each two-year campaign period. Variable x^* measures the total of all such contributions given to any challengers in each district. I consider separately contributions from corporate PACs, labor PACs and non-connected PACs (Federal Election Commission 1984–88). w^* and x^* are the natural logarithms of amounts originally measured in units of \$1 per person, based on district population (Bureau of the Census 1983; 1986). Variable $y^* = \log\{P/(1-P)\}$, where P is the proportion of all general election votes cast for the incumbent (Scammon and McGillivray 1983; 1985), represents the probability p that the incumbent wins. While $P \neq p$, p ought to be stochastically increasing in P.[19]

The test results, in Table 3, give extremely strong support to the qualitative hypotheses. In every instance, both the distance test and the divergence test indicate that for corporate and non-connected PACs the dynamics are less stable during the 1984 election period than during the 1986 election period. For labor PACs, the distance test does not indicate any significant increases between 1984 to 1986 in the separation between $\hat{\bar{\mathbf{z}}}$ and $\bar{\mathbf{z}}^*$. But for all four types of spending the divergence test indicates that the 1986 dynamics for labor PACs are significantly less stable than the dynamics of the 1984 period.

The estimated vector fields plotted in Figures 4 and 5 illustrate the kinds of changes in the dynamics that the distance test is measuring.[20] For each district i that has observed data $\mathbf{z}_i^*$, a vector is represented by an arrow that has base at $\mathbf{z}_i^*$ and head at $\mathbf{z}_i^* - .75\hat{\mathbf{u}}_i$.[21] Each Figure shows the vector field for four observed variables—post-election intergovernmental transfers for highways (v^*), corporate PAC contributions to the incumbent (w^*) and to challengers (x^*), and incumbent vote share (y^*)—projected into one subfigure for each pairing of the variables. In each subfigure a circle marks the estimated origin, i.e., the appropriate pair of the estimates $\hat{\bar{v}}$, $\hat{\bar{w}}$, $\hat{\bar{x}}$ and $\hat{\bar{y}}$. Figure 4 shows estimates for the 1984 election period and Figure 5 shows estimates for the 1986 election period. It is easy to see that the dynamics are much more unstable in 1984 than in 1986. In the three subfigures of Figure 4 that project the vector field into the planes defined by the post-election transfers and each of the other three variables, the estimated origin $\hat{\mathbf{z}}$ is clearly at a remove from the bulk of the data. There is no such pattern in Figure 5. In Figure 5 most of the vectors seem to be pointing inward, toward the centrally located origin.

***** Figures 4 and 5 about here *****

For corporate and non-connected PACs, the formal test results and estimated vector fields such as those shown in Figures 4 and 5 strongly suggest that during the 1984 election period there are unstable dynamics like those in region II of Figure 3's bifurcation diagram, but that during the 1986 period there are stable dynamics like the spiral sinks of Figure 3's region I. Vector field estimates for labor PACs (not shown) do not suggest a qualitative change between 1984 and 1986, but the divergence test strongly indicates that some kind of change does occur. The dynamics involving labor PACs during the 1986 election period are more unstable than the dynamics during the 1984 election period, but it is unlikely that the dynamics are as unstable as those in Figure 3's region II.

According to the theory that leads to Figure 3, the simplest explanations for the differing patterns of change are two. One possibility is that all three types of PACs are interested in the same type of service from the winner of the election, but somehow the challengers that corporate and non-connected PACs support during the 1984 election period are of higher quality than the challengers that labor PACs support during the 1986 election period. In this case, in Figure 3, the service type (g) would have roughly the same value for all three types of PACs, but the greater quality (h) of the challengers that the corporate and non-connected PACs support in 1984 would place them further up the h axis. The greater h values would induce dynamics that are substantially more unstable than those induced by the challengers that labor PACs supported in 1986.

The other simple possibility is that there is not much difference across types of PACs in the range of challengers' quality. Rather labor PACs may be interested in types of service that provide benefits that are more concentrated than the benefits that motivate corporate and non-connected

PACs. The idea is that such a difference in the type of service may place the dynamics for labor PACs to the right of the Cournot-Nash equilibrium point in Figure 3 (i.e., $g > g_0$), while putting the dynamics for corporate and non-connected PACs to the left of that point ($g < g_0$). Then an increase in challenger quality capable of shifting the dynamics for corporate and non-connected PACs from region I into region II may not be sufficient to move the dynamics for labor PACs out of region I. The divergence of the vector field for labor PACs would increase, but the qualitative character of the dynamic equilibrium point would not change. It would remain a sink, with flows tending to spiral in on it.

Are the benefits that labor PACs seek more concentrated than the subsidies, tax exemptions, regulatory changes and special legislation that most often interest the sponsors of corporate or non-connected PACs? Presumably labor PACs focus on the interests of their sponsoring union memberships. The current data are not sufficient to pursue this question, but the possibility is an intriguing and surprising suggestion from the analysis. Whatever the correct answer to the question may be, the way that it comes to the forefront here helps demonstrate the depth of substantive insight that can be supported by the 4DH model's ability to recover information about dynamics from cross-sectional data.

Discussion

The analysis using the game and statistical models has uncovered a hitherto unknown, powerful phenomenon at the heart of what happens before, during and after a congressional election. The phenomenon is a bifurcation pattern, comparable to that in Figure 3, that is qualitatively well modeled using the normal form equations for a dynamical system subject to Hopf bifurcation. The success of the hypotheses predicting particular changes between 1984 and 1986 in the stability of the dynamics for different types of PACs provides strong evidence that the dynamic patterns recovered by the 4DH model are substantively real. Because they strongly support the hypotheses, the recovered dynamics tend to verify a central result that the game model implies about the effect of variation in challenger quality. And through the argument used to motivate the hypotheses, the recovered dynamics connect to core facts about the American political process, in particular the midterm loss phenomenon and the partisan biases of different types of PACs' allocations of financial contributions. That the recovered dynamics are in these profound and surprising ways substantively meaningful is the best kind of evidence of the reality of the mathematical phenomenon—the bifurcation—that is the primary connection between the game and statistical models. The evidence that a bifurcation that includes features of both the Hopf and the saddle connection bifurcations is a nonlinear phenomenon inherent in the politics of congressional elections is therefore strong.

The apparent existence of the bifurcation has many substantive and methodological implications. Here I consider a few that seem to me to be among the most important.

Stability and Oscillation in Campaigns

The existence of the bifurcation implies that the kind of race an incumbent may face can vary greatly in response to relatively small changes in circumstances. A small change in the challenger's quality or in the type of service going to the district can change the race qualitatively, from one in which the incumbent has a stable advantage to one in which not only the incumbent's chances of winning and the amount of money dumped into the campaign but also the commitments the candidates make regarding post-election service may undergo substantial, growing oscillations.

Even when the incumbent's advantage is dynamically stable, the path of the campaign is a spiral. The incumbent's expected share of the votes, both the incumbent's and the challenger's financial support, the candidates' respective service commitments and much else will all inevitably oscillate during the campaign. These oscillations are not random wobbles, but rather reflect mutual, multiway strategic adjustments chosen by many of those who are interested in the election outcome. In many cases the variations will not be large enough to make a substantive difference, but a candidate or contributor observing a change of direction as it is occurring cannot be confident that the flow of the campaign is not about to wander dramatically from whatever the current situation may be. Usually the incumbent will win, but every incumbent has reason to be nervous.

The bifurcation pattern of oscillations and qualitative sensitivity to small changes will tend to dominate other patterns that may reflect alternative strategies, additional participants or other deviations from the specification of the current formal model. The theorems from local bifurcation theory that prove the robustness of the normal form equations (2) imply that, near the dynamic equilibrium point, motion in accord with the qualitative features that the equations describe will tend to be orders of magnitude larger than other features of the flow of the campaign.

Methodological Implications

The existence of the bifurcation makes it useless to estimate linear or generalized linear models of the relationships among most of the variables of interest in studies of congressional elections. The inherently nonlinear dynamics associated with the bifurcation imply that each congressional campaign evolves in a manner that resembles motion around a loop. Around the loop the relationship between a pair of variables will cycle among all possible orientations, sometimes increasing, sometimes decreasing. It is impossible to map such covariation faithfully into a linear model. A line segment cannot model a circle. The existence of the bifurcation means that linear models of congressional election phenomena such as campaign contributions, district service and votes are necessarily misspecified. If the bifurcation exists, such models must necessarily fail to produce correct results.

Both the current theoretical models and the claim that the bifurcation is inherent in congressional politics have many more testable implications than I have discussed. To get a sense of the range of such implications, consider that the "campaign" of the game model is not restricted to any particular time period within a legislator's term in office, and that the actions of the model such as the incumbent's choosing a service type may reach deep into the legislative process. Arguably, for instance, choosing a service type includes securing one's portfolio of committee assignments. On the most capacious interpretation, the game model should be read as predicting that the bifurcation pattern occurs in some form in the committee assignment process. Because the provision of district service often involves coordinated action across the federal, state and local levels of government, traces of the bifurcation pattern ought to be found as well in actions taken by nonlegislators at many levels of the federal system. The 4DH model can be extended in various ways to support many of the forms of data and particular hypotheses that would be most appropriate for empirical tests in these and other areas. Statistical models quite different from the 4DH model may also prove useful.

Implications for Representation

The current analysis suggests that voter preferences only partly determine election outcomes and subsequent policy choices. In the game model, voters do not behave strategically, but parties, candidates and contributors are all constrained by their correct anticipations of what voters are likely to do. To the extent that voters' behavior in the model reflects their preferences over policy outcomes, one can say that the game solution is an example of "the rule of anticipated reactions," a relation between outcomes and preferences that some would say indicates that voters are powerful (Nagel 1975; Arnold 1990). In the game, the best measure of voters' preferences is the concentration parameter, g: when $g < 0$, the benefits from service are widely dispersed and voters prefer to have post-election service increase, and when $g > 0$, any benefits from service are concentrated on a few and voters prefer to have service decrease.

The ideal for voters, then, is to have dispersed benefits and service as high as possible. In two of the four types of campaign in the game solution, that is what they get. The choices of service concentration and challenger quality that place the campaign in region IV or region V of Figure 1 result in service provision levels that strictly increase for as long as the campaign is considered to run after either the challenger (in region IV) or the incumbent (in region V) drops out. In the former case, voters are getting the best policy outcomes but are also being given the least choice—i.e., *no* choice—in the election. When there is a competitive race with the incumbent running the voters have already lost out, because in that case $g > 0$ but the amount of post-election service will be positive.

It does not follow, however, that voters are best off when the campaign is not competitive. In the other kind of uncompetitive race, with $(g, h) = (.0425, 0)$, the unpredictability of the dynamics in region VI of Figure 1 muddies the picture a bit, but as long as $h = 0$ the outcome is quite unfavorable for voters. For in this case, once the challenger disappears (i.e., $p = 1$), contributions to the incumbent and therefore the amount of post-election service from the incumbent increase very rapidly. Voters dislike this service, but there is nothing they can do to stop it. The game model suggests that the reason for the incumbent to be running unopposed is much more likely to be such an unfavorable dynamic than the one that gives voters their ideal outcome. Given that the incumbent is unopposed, the probabilities for the mixed-strategy equilibrium of the first-stage game suggest that the unfavorable dynamic is roughly seven times more likely than the ideal one to be the reason.[22] The dynamics of anticipated reaction are most likely to have driven the candidates and the financial contributor into a *de facto* conspiracy against the voters.

Appendix

Derivation of the Hopf Model: To apply system (2) to cross-sectional data, I assume that the realization of system (1) in each district begins at small, random displacements from $(g, h) = (g_0, h_0)$ and $(r, q, a, b) = (r_0, q_0, a_0, b_0)$. Each realization then runs for a random time period with positive mean and finite variance. The precise form of the random elements is not important, but it is convenient to assume that the random elements are independently and identically distributed across districts. The result is a data set consisting of the points at which the realizations terminated.

Given only cross-sectional data, the equations of (2) have to be simplified to eliminate parameters that are not uniquely identifiable with such data. Parameters δ, μ and γ are not distinctly defined: replacing δ with $\delta^* \neq 0$, μ with $\mu^* = \mu\delta/\delta^*$ and γ with $\gamma^* = \gamma\delta^*/\delta$ leaves (2) unchanged. Further, for any choice of $\omega > 0$ there is a value of γ that leaves $\omega + \gamma\mu$ invariant. I normalize by setting $\delta\mu = 1$ and collapsing $\omega + \gamma\mu$ into a single parameter γ.

The next step is to build in transformations to adjust for the coordinate system used to measure the observations x^* and y^*. I translate the observations to the origin using $x = x^* - \tilde{x}$ and $y = y^* - \tilde{y}$ where $\tilde{x}$ and $\tilde{y}$ are parameters with values to be determined. I also allow the form of the periodic orbits (the limit cycles of system (2)) to be elliptical rather than exactly circular. As Figure 2 illustrates, ellipses ought to be better approximations to the form of the orbits than circles would be. In system (2) the appropriate change is to replace $(x^2 + y^2)$ with $(x^2 + y^2 + \epsilon xy)$ where ϵ is a parameter valued in the range $-2 < \epsilon < 2$.

To produce a statistical model I replace the time derivatives dx/dt and dy/dt of system (2) with random variables u_x and u_y, which I assume to be bivariate normal with expectations $Eu_x = Eu_y = 0$ and covariance matrix $\mathrm{cov}(u_x, u_y) = \mathbf{\Sigma} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$. I assume that u_x and u_y are independently and identically distributed across observations. The distributional assumption is justified by the fact that the elliptical orbits are symmetric, and we expect observations to be randomly distributed on orbits near the equilibrium. For observed data x^* and y^* we now have the bivariate model

$$u_x = x - \gamma y + (\alpha x - \beta y)(x^2 + y^2 + \epsilon xy) \tag{4a}$$

$$u_y = y + \gamma x + (\alpha y + \beta x)(x^2 + y^2 + \epsilon xy) \tag{4b}$$

where $x = x^* - \tilde{x}$, $y = y^* - \tilde{y}$. Unknown parameters are α, β, γ, ϵ, $\tilde{x}$, $\tilde{y}$ and $\mathbf{\Sigma}$. Model (4) may be construed as specifying a nonlinear covariance structure.

The bivariate model (4) illustrates the basic method of changing time derivatives into random variables, but we should not expect a bivariate specification to be sufficient to represent the dynamics of system (1) near $(g, h) = (g_0, h_0)$ and the Cournot-Nash equilibrium point $(r, q, a, b) = (r_0, q_0, a_0, b_0)$. By the robustness of system (2), we can reasonably treat the dynamics as two-dimensional. But to represent system (2) in terms of the variables of system (1) it is necessary to embed system (2) in a space of dimension greater than two. Because system (2) is valid only in a neighborhood of the equilibrium point, it is reasonable to consider the data for which system (2) is relevant as confined to a compact set that includes (r_0, q_0, a_0, b_0). Treating that set as a compact manifold in which the flows of system (1) are occurring, we can then use the Whitney embedding theorem (Hirsch 1976, 23–27) to conclude that a four-dimensional model is necessary and sufficient to represent system (2).

Indeed, we have direct evidence from system (1) that two-dimensional models will fail to recover the dynamics. In Figure 2, in the projection into the plane "contributions to the challenger" by "probability that the incumbent wins," the flow of system (1) appears to cross itself. The resulting figure '8' pattern is fundamentally unlike the circular pattern of system (2)—a figure '8' cannot be smoothly mapped into a circle. Moreover, by the fundamental uniqueness theorems for differential equations, a flow that crosses itself is impossible (Hirsch and Smale 1974, 161–176). Therefore, even if system (1) were correct, a two-dimensional analysis of the relationship between challenger finances and votes would fail to confirm the dynamics of system (2), leading to the rejection of system (1). But the apparent failure of the model would be an artifact of the chosen pair of variables.

To do a four-dimensional analysis I extend model (4) to define model (3), which in four equations combines the six models of form (4) that can be constructed between pairs of the four observed variables. The log-likelihood, for n observations, is $l = \sum_{i=1}^{n} \log[(2\pi)^{-2}|\boldsymbol{\Sigma}|^{-1/2} \exp\{-\frac{1}{2}\mathbf{u}_i{}^{\top}\boldsymbol{\Sigma}^{-1}\mathbf{u}_i\}]$. To identify the model I set $\boldsymbol{\Sigma} = (n - k/4)^{-1} \sum_{i=1}^{n} \mathbf{u}_i\mathbf{u}_i{}^{\top}$, where $k = 28$ is the number of parameters in model (3). Doing so reduces the log-likelihood to $l = -n[2\log(2\pi) + \frac{1}{2}\log|\boldsymbol{\Sigma}|] - 2(n - k/4)$.

Derivation of the Tests of the Qualitative Dynamic Hypotheses: The distance test statistic is motivated by the key feature of the dynamics in region II of Figure 3, which is the existence of at least two fixed points, one being a spiral source and one a saddle point. As noted in the text, flows of system (1) that start near the source in general approach the saddle point before wandering unboundedly. An empirical implication is that the observed data should be concentrated near a point distinct from the estimated origin of the dynamics. The sample mean $\bar{\mathbf{z}}^*$ of the observed data should be distinct from the estimated origin, $\hat{\bar{\mathbf{z}}}$. Of course, the origin and the sample mean should be distinct even if the dynamics are not in Figure 3's region II, in part because flows in the four-dimensional data cannot be expected to be confined to a plane,[23] and in part because the orbits in regions I and III of Figure 3 cannot be expected to be circular. But if the distance between the origin and the mean is greater for one election period than another, it is reasonable to conclude that the dynamics are more unstable during the former period than during the latter. If the distance is dramatically different between periods, then the most likely explanation would be that during the less stable period the dynamics are occurring in region II while during the more stable period they are occurring in region I or region III.

Conditioning on the MLE $\hat{\bar{\mathbf{z}}}$ and treating the sample mean $\bar{\mathbf{z}}^*$ as random, a measure of the distance between the origin and the mean for election period j is

$$D_j = (\bar{\mathbf{z}}^*_j - \hat{\bar{\mathbf{z}}}_j)^\top \left(\frac{\mathbf{T}_j}{n_j}\right)^{-1} (\bar{\mathbf{z}}^*_j - \hat{\bar{\mathbf{z}}}_j)$$

where n_j is the number of observations and $\mathbf{T}_j = (n_j - k)^{-1} \sum_{i=1}^{n_j} (\mathbf{z}^*_{ij} - \hat{\bar{\mathbf{z}}}_j)(\mathbf{z}^*_{ij} - \hat{\bar{\mathbf{z}}}_j)^\top$, with $k = 28$ being the number of parameters in model (3). Under the hypothesis that $\hat{\bar{\mathbf{z}}} = \bar{\mathbf{z}}^*$, D_j has the χ^2_4 distribution. As noted above, however, such a hypothesis of equality is not reasonable for model (3). The distribution for D_j should therefore be taken as noncentral $\chi^2_{4;\lambda_j}$ with noncentrality parameter λ_j. The degree of instability between election periods can be compared by comparing the magnitudes of D_j for the respective periods, via the ratio $\rho = D_U/D_S$. D_U is the value of D_j for the period t_U that is predicted to be more unstable and D_S is the value for the period t_S that is predicted to be more stable. In general, ρ has the doubly noncentral F distribution, $F_{4,4;\lambda_U,\lambda_S}$ (Johnson, Kotz and Balakrishnan 1995, 480). The hypothesis $D_U = D_S$, which asserts that the t_U election period is neither more nor less unstable than the t_S period, implies $\lambda_U = \lambda_S$. Under the hypothesis, ρ therefore has the distribution $F_{4,4;\lambda_S,\lambda_S}$. Values of ρ significantly greater than 1.0—i.e., $\Pr(F_{4,4;\lambda_S,\lambda_S} > \rho) < \alpha$ for test level α—indicate departures from equality in the theoretically predicted direction.

The divergence test statistic is motivated by the contrasting effects flows in regions I and III of Figure 3 have on the volumes of bounded sets near the fixed point. In region I, flows decrease the volume of such a set, while in region III flows cause the volume of such sets to increase.[24] By Liouville's theorem (Arnold 1978, 69–70), the rate of change that system (1) induces in the volume of a bounded set is equal to the integral of the *divergence* of system (1)'s vector field over that set.[25] The divergence of a vector field at each point is the trace of its Jacobian matrix evaluated at that point (Weibull 1995, 251). Writing the vector field for system (1) as

$\xi(r,q,a,b) = ((\partial I/\partial r)/r, (\partial J/\partial q)/q, (\partial K/\partial a)/a, (\partial K/\partial b)/b)^\top$, the divergence is

$$\operatorname{div} \xi(r,q,a,b) = \frac{\partial[(\partial I/\partial r)/r]}{\partial r} + \frac{\partial[(\partial J/\partial q)/q]}{\partial q} + \frac{\partial[(\partial K/\partial a)/a]}{\partial a} + \frac{\partial[(\partial K/\partial b)/b]}{\partial b} .$$

To estimate the divergence for each observed data point $\mathbf{z}_i^*$, I reverse the approach used to derive the statistical model (4) from system (2) and treat each value $-\hat{\mathbf{u}}_i$ as an estimate of the value of the vector field at $\mathbf{z}_i^*$.[26] I estimate the divergence at $\mathbf{z}_i^*$ by using finite differences to compute $\operatorname{div}(-\hat{\mathbf{u}}_i) = -(\partial \hat{u}_{vi}/\partial v + \partial \hat{u}_{wi}/\partial w + \partial \hat{u}_{xi}/\partial x + \partial \hat{u}_{yi}/\partial y)$. The test statistic τ is the t-statistic for the difference of means between the set of values $\operatorname{div}(-\hat{\mathbf{u}}_i)$ for the election period that is predicted to be more unstable and the set of values for the period that is predicted to be more stable. The theory predicts that the differences will be significantly positive.

Notes

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1. The relationship between my concept of challenger quality and Jacobson's (1990b) challenger quality variable that measures past experience in office is not clear.

2. Fudenberg and Tirole (1991, 23–26) discuss the basic Cournot adjustment process.

3. As is well known, quadratic losses in the form of euclidean preferences are often used in spatial models. Berger (1985) discusses pros and cons of various loss functions.

4. If $\partial p/\partial a = 0$, then the solution to the first-order necessary condition for equilibrium, $\partial K/\partial a = 0$, is $a^* = pr^2/(1-pr^2)$. For $pr^2 < 1$, this solution is a maximum: $\partial^2 K/\partial a^2 = 2(pr^2-1) < 0$. Evaluated at $a = a^*$, the contributor's expected rate of return $\kappa = pr^2(1+a)^2/a^2 - 1$ is $\kappa^* = 1/pr^2 - 1$. Solving for pr^2 gives $pr^2 = 1/(1+\kappa^*)$, so that pr^2 is the one-period-ahead discount rate at interest rate $\kappa^* > 0$. But $a^* = 1/\kappa^* = \sum_{i=1}^{\infty} 1/(1+\kappa^*)^i$ is the present discounted value of a permanent future income of one unit per period. Notice that the rate of return in Snyder's (1990,

1197 eq. 1) contingent-claim formulation corresponds to $\kappa^* + 1$. I.e., Snyder's concept of the value of favor sold by a candidate (here the incumbent) corresponds to $r^2(1+a)^2$, while his concept of the investor contribution to the candidate corresponds to a^2.

5. The sign of r in $\partial p/\partial g$ is negative: $\partial p/\partial g = b[q^2(1+b)^2 - r^2(1+a)^2]f_1$ where $f_1 = \nu\pi_1(1-\pi_1) + (1-\nu)[1+\mu(\pi_0-\pi_1)\nu]\pi_0(1-\pi_0)$ is nonnegative because $\pi_0 \geq \pi_1$. This negative effect works against the incentive to increase r suggested by the term pr in I. But even here it is important to notice that $\partial pr/\partial r = p + [r - 2gbr^2(1+a)^2]f_1$ can be negative if $g > 0$.

6. If there are multiple, stable Cournot-Nash equilibria then the players can be assumed to choose one at random, but so far as I have been able to determine, there is never more than one stable fixed point.

7. If the Cournot-Nash equilibrium point is not stable, then the flows of system (1) do not converge to the point as the initial errors go to zero. Given any initial deviation from the Cournot-Nash equilibrium point, the flows wander away from the point. Compare Fudenberg and Tirole (1991, 351–352).

8. *MACSYMA* (Symbolics 1991) was used to do 4th-order Runge-Kutta method numerical integration. All simulations were computed using $\mu = 10$.

9. A homoclinic cycle occurs when flows connect one or more saddle points so as to create a circuit (Guckenheimer and Holmes 1986, 45). The cycles in region V include three saddle points.

10. One may verify by direct calculation that neither the incumbent nor the challenger can gain by unilaterally switching to one of the pure strategies of Table 1, as long as $I_{\max} > .41$ and $J_{\max} > 1.54$. As noted in the text, reasonable valuations are $I_{\max}, J_{\max} \geq 3$. Because I have not computed payoffs for all (g,h) pairs on a fine, bounded lattice (the computing demands are prohibitive), I cannot definitively assert that the equilibrium of Table 2 is unique (compare Fudenberg and Tirole 1991, 34–36), but nothing about the dynamics for the (g,h) pairs I have

simulated would suggest otherwise. Except for (g, h) pairs with small positive values of h in region VI of Figure 1, for which the dynamics of system (1) are highly irregular, there would appear to be no barrier in principle to demonstrating uniqueness by applying the Dasgupta-Maskin theorem that Fudenberg and Tirole (1991, 487-489) review. The computing requirements would be immense, however.

11. The orbit is not exactly closed because the values $g = .0425$ and $h = .487$ are approximate.

12. For examples and pictures of spirals, sources, sinks and saddle points, see Hirsch and Smale (1974, 90–96).

13. Results produced using a generalized linear model (McCullagh and Nelder 1989) would also be spurious.

14. It may be that special efforts by the opposition party to recruit strong challengers during the presidential election year are sufficient to overcome the tendency to have lower quality opposition party challengers then. Opposition party politicians may think it would be especially valuable, for instance, to have a robust legislative majority to support what they hope will be their new President's early policy initiatives. But one would expect such recruitment efforts to succeed only when the opposition party is widely believed to be highly likely to capture the White House, which was not the case in 1984.

15. The key regression coefficient in Grier and Munger's (1986) analysis has the correct sign for a conclusion that corporate PACs favored Republican incumbents but is statistically insignificant.

16. The 4DH model log-likelihood exhibits severe global nonconcavity. To find global optima, I use *GENOUD* (Mebane and Sekhon 1996; Sekhon and Mebane 1998), an improved version of Michalewicz's (1992) evolution program.

17. Data from Ohio are excluded from the analysis due to difficulties encountered in trying to match parts of counties to parts of congressional districts after 1984.

18. Transfers in each category are district totals estimated from raw data in Bureau of the Census (1984; 1986–91; 1991) using the procedure of Mebane (1993). "Social welfare transfers" includes transfers for public welfare, employment security, health and hospitals and housing. Per capita amounts are computed first for each county using population values from Bureau of Economic Analysis (1990) and then allocated to districts in proportion to each county's share of the district population, as derived from Bureau of the Census (1985; 1986).

19. Observation counts for each type of PAC in 1984/85 (1986/87) are: corporate 162 (150), except highways spending 92 (101); labor 114 (94), except highways spending 63 (67); non-connected 186 (167), except highways spending 103 (119).

20. Vector field plots give no information about the divergence, because the divergence is a function of partial derivatives of the vectors rather than of the vectors themselves.

21. Using $\mathbf{z}^* - .75\hat{\mathbf{u}}$ rather than $\mathbf{z}^* - \hat{\mathbf{u}}$ makes the plots easier to interpret by reducing clutter.

22. If $I_{\max} = J_{\max} = 3$, then $\Pr(g = .0425, h = 0) / \Pr(g = -.025, h = .487) = 7.1$.

23. See the preceding discussion of embedding.

24. Let $\mathbf{B}(0)$ be a bounded set of positive volume in the four-dimensional space of system (1) at time $t = 0$. Let $\mathbf{B}(t)$ be the set of points produced by starting a flow of system (1) at each point of $\mathbf{B}(0)$ and running the system for time period t. The flows are said to have decreased (resp. increased) the volume of $\mathbf{B}(0)$ if the volume of $\mathbf{B}(t)$ is less (resp. greater) than that of $\mathbf{B}(0)$.

25. Weibull (1995) reviews applications of Liouville's theorem to assess stability in multipopulation evolutionary game models.

26. The order of time is ambiguous in the recovered dynamics, so that it is not clear a priori whether $\hat{\mathbf{u}}$ or $-\hat{\mathbf{u}}$ should be used to estimate the vector field. For the results discussed in the text I am using $-\hat{\mathbf{u}}$. Using $\hat{\mathbf{u}}$ reverses all the results for the divergence test in Table 3 and all the arrows in Figures 4 and 5.

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Table 1: First-stage Game, Based on the Candidates' Payoffs in the Second-stage Subgames

	$h = 0$	$h = .20$	$h = .325$	$h = .487$	$h = .68$	$h = 1$
$g = -.08$	$0, J_{\max}$	$0, J_{\max}$	$.9, .23$	$I_{\max}, 0$	$I_{\max}, 0$	$I_{\max}, 0$
$g = -.025$	$0, J_{\max}$	$2.0, .22$	$.94, .23$	$I_{\max}, 0$	$I_{\max}, 0$	$I_{\max}, 0$
$g = -.015$	$0, J_{\max}$	$1.7, .23$	$.92, .235$	$.41, .38$	$.24, .471$	$.06, .69$
$g = 0$	I_1, J_1	$1.5, .228$	$.91, .24$	$.42, .37$	$.29, .466$	$.17, .63$
$g = .0425$	$I_{\max}, 0$	$1.3, .30$	$.6, .29$	$.53, .35$	$I_{\max}, J_0$	$I_{\max}, J_0$
$g = .10$	$I_{\max}, 0$	$.95, .26$	$.74, .31$	$.52, .36$	$I_{\max}, J_0$	$I_{\max}, J_0$
$g = .24$	$.89, .43$	$.67, .35$	$.58, .33$	$.48, .36$	$.43, .42$	$I_{\max}, J_0$

Legend: g, service concentration; h, challenger quality.

Candidates' payoffs: $I_{\max} > \ldots > I_1 > 0$; $J_{\max} > \ldots > J_1 > J_0 > 0$.

Table 2: Mixed Strategy Equilibrium Probabilities in the First-stage Game

strategy	probability
$g = -.025$	$.35/(J_{\max} + .35)$
$g = .0425$	$J_{\max}/(J_{\max} + .35)$
$h = 0$	$(I_{\max} - .53)/(2I_{\max} - .53)$
$h = .487$	$I_{\max}/(2I_{\max} - .53)$

Legend: g, service concentration; h, challenger quality.

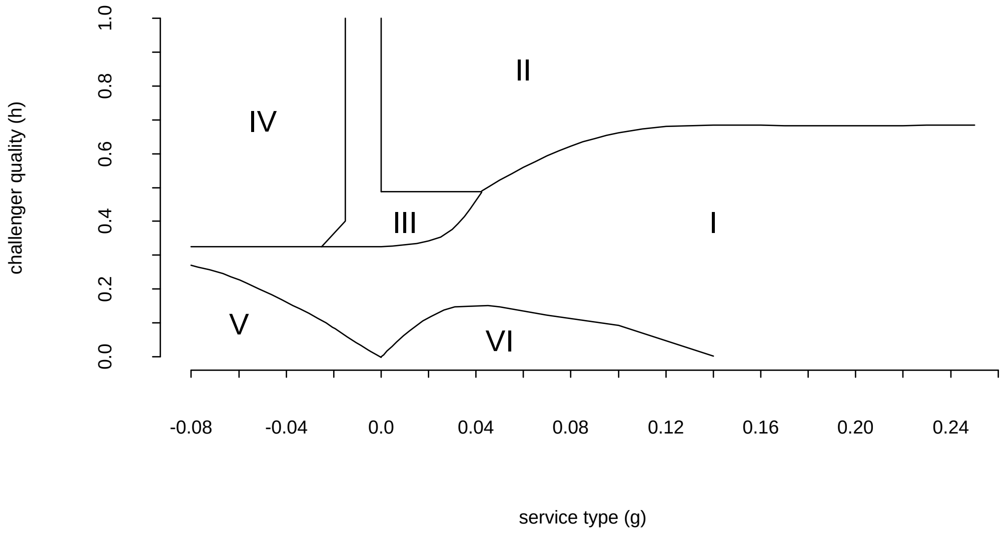
Candidates' payoffs: $I_{\max} > 2$; $J_{\max} > 2$.

Table 3: Tests of Hypothesized Changes in Stability

type of post-election spending	type of PACs	Distance Test ρ	Pr	Divergence Test τ	Pr
education	corporate	4.65	.00	13.87	.00
highways	corporate	2.50	.01	5.67	.00
social welfare	corporate	3.47	.00	11.07	.00
other	corporate	3.11	.01	12.08	.00
education	labor	1.10	.41	4.23	.00
highways	labor	1.06	.46	1.71	.04
social welfare	labor	.66	.82	2.84	.00
other	labor	.44	.96	1.51	.06
education	non-connected	7.45	.00	13.66	.00
highways	non-connected	4.33	.02	6.81	.00
social welfare	non-connected	2.34	.01	9.42	.00
other	non-connected	12.32	.00	14.73	.00

Notes: Probabilities show the upper-tail cumulative distribution function value. For ρ the distribution is $F_{4,4;\lambda_S,\lambda_S}$. For τ the distribution is $t_{n_U+n_S-2}$.

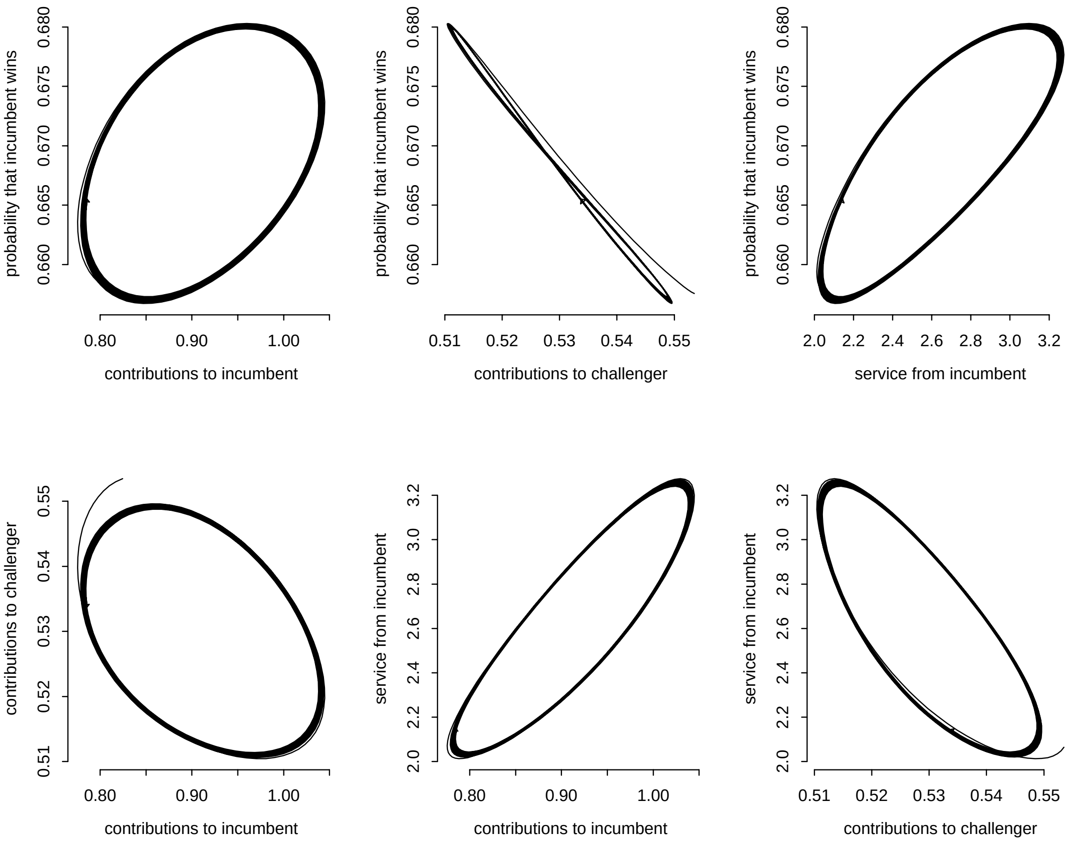
Figure 1: Campaign Dynamics Partial Bifurcation Set



Legend: I, stable incumbent advantage; II, no challenger contributions; III, limit cycle; IV, challenger drops out; V, incumbent drops out; VI, no contributions to one candidate (unpredictable).

Source: Numerical computations.

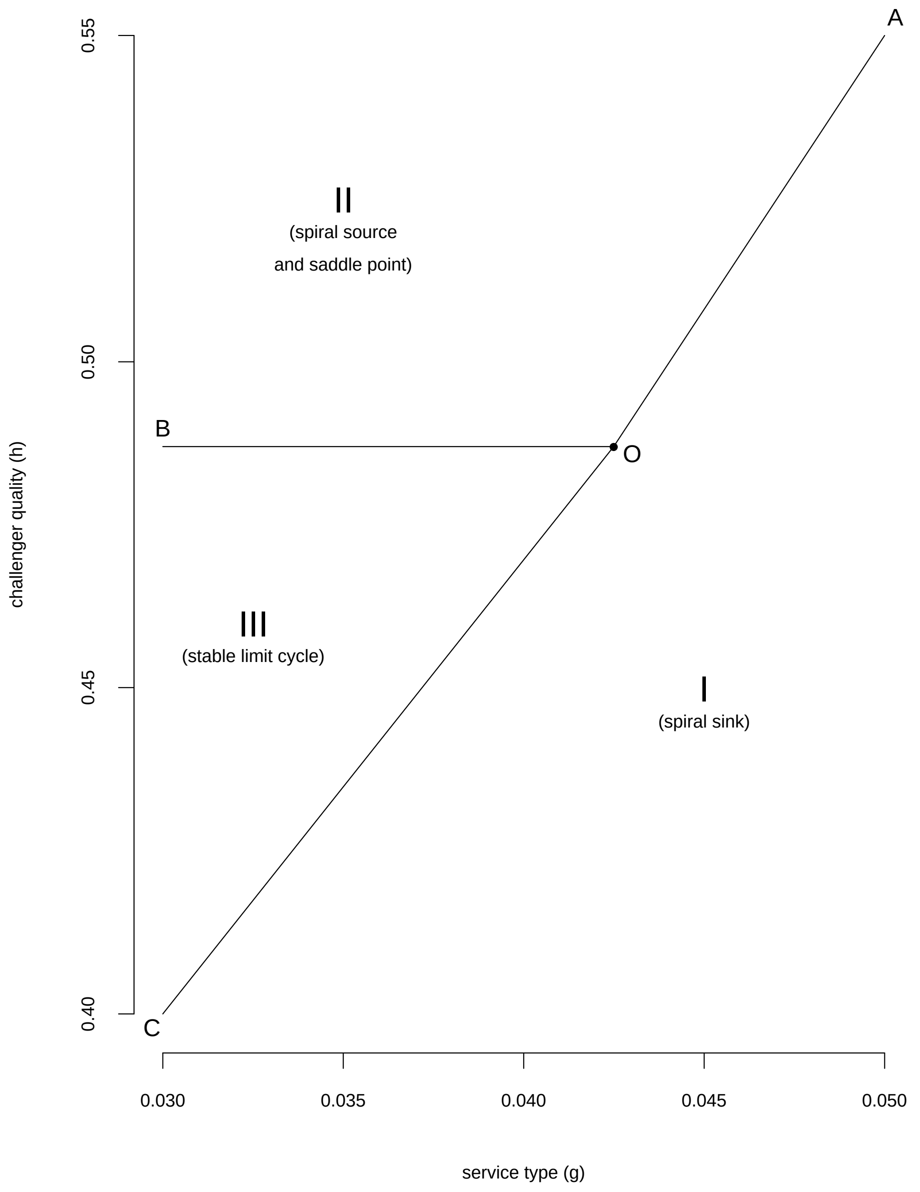
Figure 2: A Flow Near a Closed Orbit at the Most Likely[a] Campaign



Source: Runge-Kutta numerical integration. 2D orbit projections, $g = .0425$, $h = .487$.

Notes: [a] According to the mixed strategy equilibrium solution of Table 2.

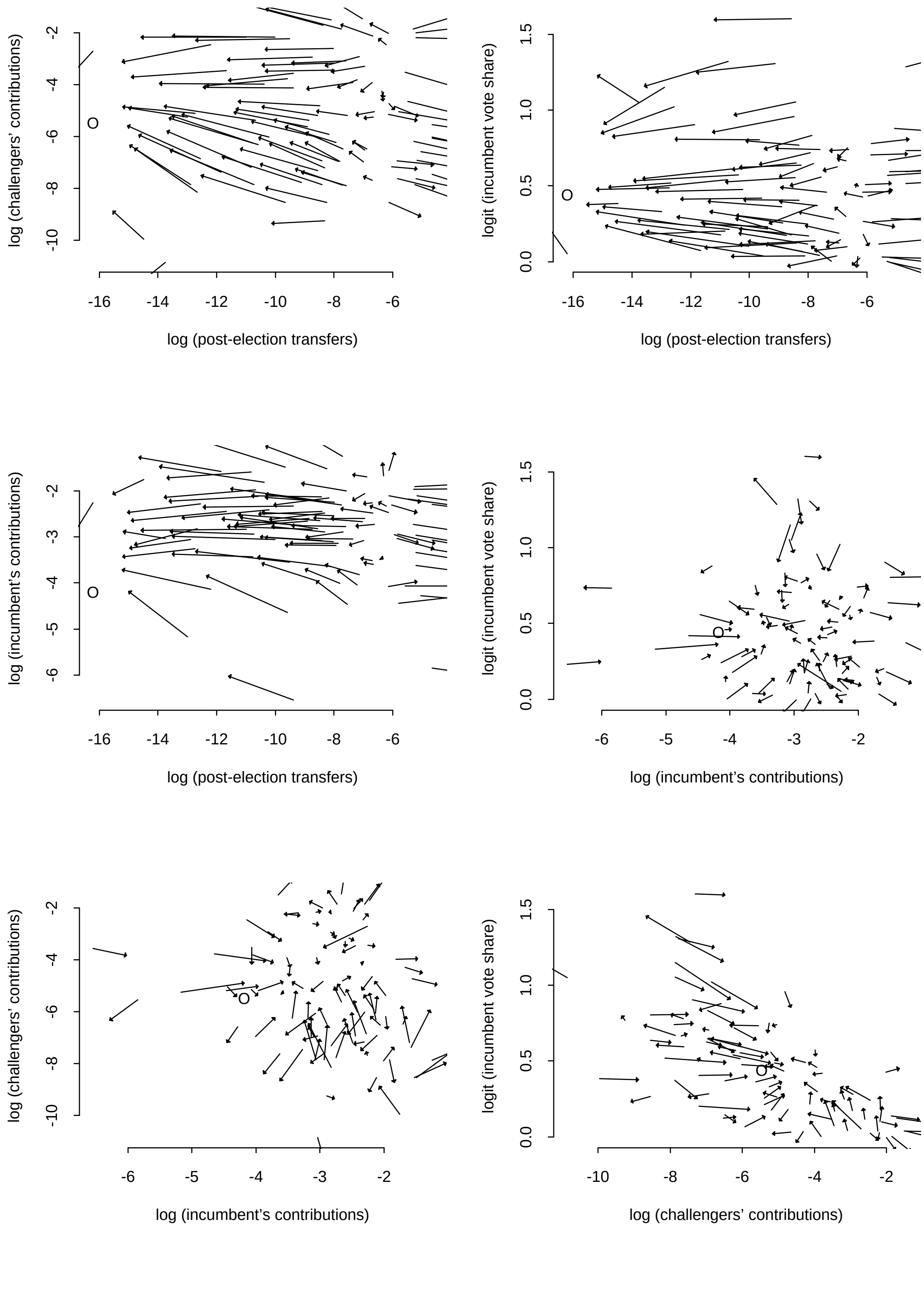
Figure 3: Partial Bifurcation Set Near the Most Likely[a] Type and Quality Values



Source: Runge-Kutta numerical integration.

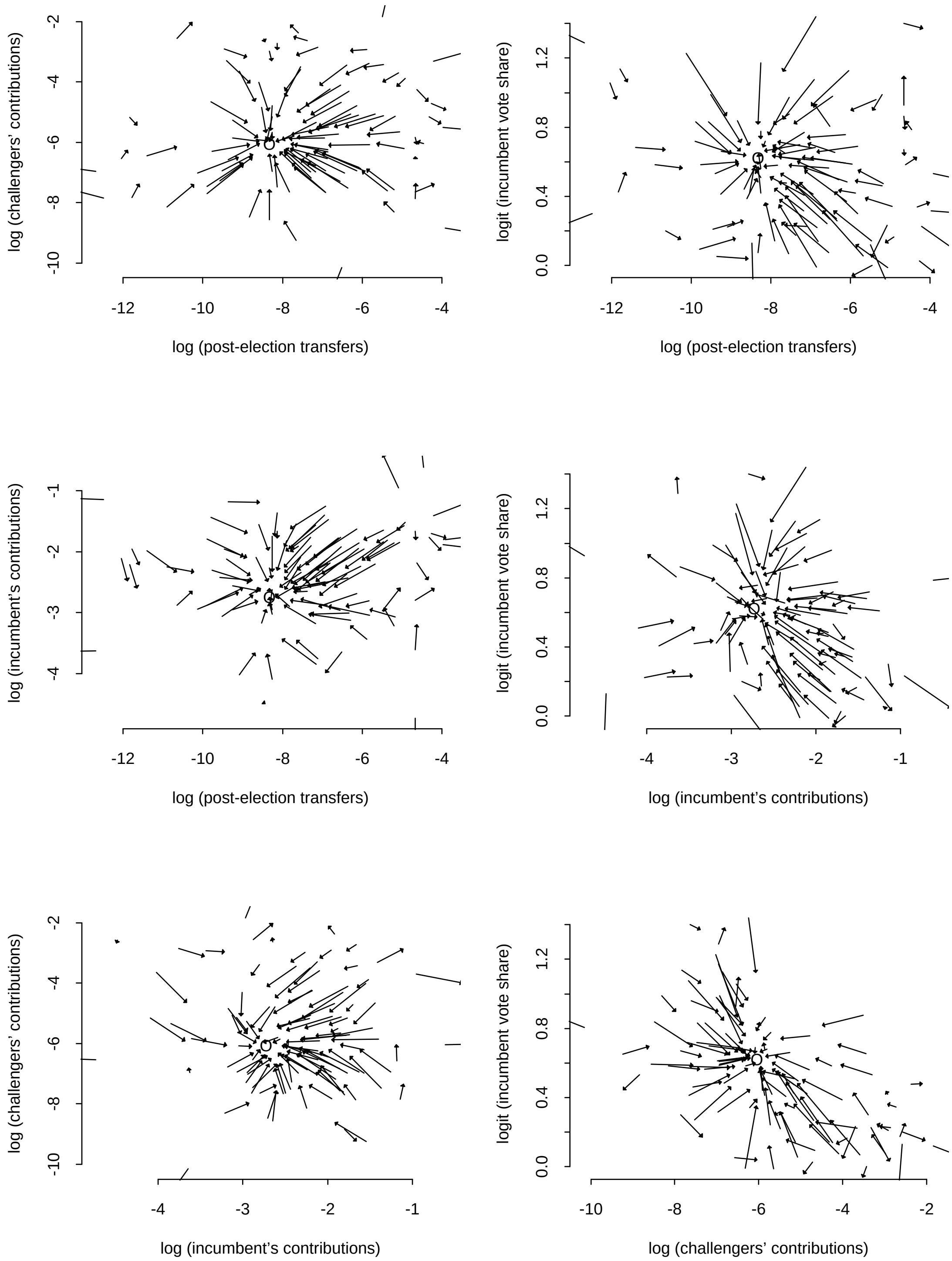
Notes: [a] According to the mixed strategy equilibrium solution of Table 2.

Figure 4: Vector Field for Highway Transfers, Corporate PAC Contributions and Votes, 1984



Source: $\hat{\mathbf{u}}$ computed at each observed data point using the model (3) parameter MLEs.

Figure 5: Vector Field for Highway Transfers, Corporate PAC Contributions and Votes, 1986



Source: $\hat{\mathbf{u}}$ computed at each observed data point using the model (3) parameter MLEs.