**Government 320: Public Opinion and Public Choice** 

Spring 2007

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Tuesday and Thursday 2:55–4:10 (MG 165)
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**Course web page:** 

http://macht.arts.cornell.edu/wrm1/gov320.html

- heresthetic and the critique of democracy: instability and meaninglessness
- based on social choice theory, in particular on Arrow's Theorem
- assumes individual rationality

• rationality: transitive and complete individual preference orders

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- transitivity:
  - transitivity of strict preferences: for every individual i and all alternatives a, b and c,  $aP_ib$  and  $bP_ic$  implies  $aP_ic$
  - transitivity of indifference:  $aI_ib$  and  $bI_ic$  implies  $aI_ic$
  - transitivity of weak preference:  $aR_ib$  means  $aP_ib$  or  $aI_ib$ ;  $aR_ib$  and  $bR_ic$  implies  $aR_ic$

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  - transitivity of strict preferences: for every individual i and all alternatives a, b and c,  $aP_ib$  and  $bP_ic$  implies  $aP_ic$
  - transitivity of indifference:  $aI_ib$  and  $bI_ic$  implies  $aI_ic$
  - transitivity of weak preference:  $aR_ib$  means  $aP_ib$  or  $aI_ib$ ;  $aR_ib$  and  $bR_ic$  implies  $aR_ic$
- completeness:
  - for every individual i and all alternatives a and b,  $aR_ib$  or  $bR_ia$
  - incompleteness, which implies inability to choose, is not the same as indifference (Buridan's ass example)

- axiomatic social choice theory
  - state as axioms general properties of any collective choice procedure (or any collective preference procedure)
  - derive characteristic properties of any procedure that the axioms describe
  - this may include showing that some desirable properties are mutually incompatible

- example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)
- consider three voters with the following preferences:
  - $-aP_ibP_ic$
  - $-cP_iaP_ib$
  - $-bP_icP_ia$

- example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)
- consider three voters with the following preferences:
  - $-aP_ibP_ic$
  - $cP_i aP_i b$
  - $bP_i cP_i a$
- consider majority rule as the collective choice procedure
- because majority rule considers only two alternatives at a time, the three alternatives must be considered in some order
- the order in which the votes occur is an agenda

- example: the paradox of voting
- consider three voters with the following preferences:
  - $-aP_ibP_ic$
  - $cP_i aP_i b$
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- consider three agendas with pairwise majority votes:
  - $-\ a$  versus b , then winner versus c
  - -a versus c, then winner versus b
  - b versus c, then winner versus a

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- the outcome depends on the agenda

- consider three voters with the following preferences:
  - $aP_ibP_ic$ ,  $cP_iaP_ib$ ,  $bP_icP_ia$
- consider three agendas with pairwise majority votes and sophisticated voting:
- with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently

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- consider three agendas with pairwise majority votes and sophisticated voting:
- with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently
- this is strategic voting only if each voter's vote depends on the voter's beliefs about everyone else's preferences
- implicitly there is usually a "common knowledge" assumption that implies (among other things) that those beliefs are not systematically wrong

- consider three voters with the following preferences:
  - $aP_ibP_ic$ ,  $cP_iaP_ib$ ,  $bP_icP_ia$
- pairwise majority votes and sophisticated voting:
- everyone knows everyone votes sincerely in the last stage
  - -a versus b, then winner versus c (tree diagrams help)



• consider three voters with the following preferences:

 $- aP_ibP_ic$ ,  $cP_iaP_ib$ ,  $bP_icP_ia$ 

- $\bullet$  with sincere voting, a beats b, then c beats a
- everyone knows everyone votes sincerely in the last stage
- so everyone knows cPa and bPc





• because everyone knows cPa and bPc, everyone knows the situation really is



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    - $\ast$  the first vote is really c versus b, because cPa
    - \* so the winner is b (voter 1 votes for b not a)

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    - \* so the winner is a (voter 2 votes for a not c)

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  - -a versus c, then winner versus b
    - $\ast$  the first vote is really a versus b , because bPc
    - \* so the winner is a (voter 2 votes for a not c)
  - $-\ b$  versus c, then winner versus a
    - $\ast$  the first vote is really c versus a, because aPb
    - \* so the winner is c (voter 3 votes for c not b)

- consider three voters with the following preferences:
  - $aP_ibP_ic$ ,  $cP_iaP_ib$ ,  $bP_icP_ia$
- pairwise majority votes and sophisticated voting:
- everyone knows everyone votes sincerely in the last stage
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    - $\ast$  the winner is a
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    - $\ast$  the winner is c

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- examples of voting rules
  - majority rule
  - plurality voting with one winner (winner take all, first past the post)
  - plurality voting with M>1
  - approval voting
  - Borda count
  - utilitarian systems

- examples of voting rules
  - majority rule
  - plurality voting with one winner (winner take all, first past the post)
  - plurality voting with M>1
  - approval voting
  - Borda count
  - utilitarian systems
- there is extensive theory for single districts, little for whole systems

• heresthetic: changing outcomes without changing preferences

- heresthetic: changing outcomes without changing preferences
- heresthetic versus rhetoric (i.e., manipulation versus persuasion)
  - every individual's preference ordering remaining the same is the definition of no persuasion
  - changing people's beliefs about what the decision is is not considered persuasion
  - likewise, changing beliefs about what's at stake in a decision, or about the consequences of a decision, is not considered persuasion

- heresthetic: three basic kinds of manipulation
  - strategic voting
  - agenda control
  - dimension manipulation

- heresthetic: stories grouped by type manipulation (according to Riker)
  - strategic voting: Pliny (7); Massachusetts and SC
     delegates (8); Chicester (9); anti-school aid Republicans
     (11); Chrystal (5); Morris (4)
  - agenda control: Plott and Levine (3); Pliny (7); Reed and Norris (12)
  - dimension manipulation: Lincoln (1); DePew (2);
    Chrystal (5); city manager (6); Magnuson (10); Morris (4)

- dimension manipulation illustration: Warren Magnuson
- alternatives
  - *o*: leave shells of gas in Okinawa
  - -a: bring nerve gas to Alaska
  - G: detoxify gas "outside the U.S." (Gravel amendment)
  - *d*: support defense department
  - *n*: support Alaskan interests
  - -s: support constitutional authority of the Senate

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  - s: support constitutional authority of the Senate
- distinguish actions (for or against G) from implications of actions: preferences refer to implications
  - before Magnuson maneuver: for G means (o, n); against G means (a, d)
  - after Magnuson maneuver: for G means (o, n, s) (to some senators); against G means (a, d)

- dimension manipulation illustration: Warren Magnuson
- dimension manipulation is covert agenda control (sort of)
  - but what about Byrd and Thurmond, who "were unmoved by Magnuson's heresthetic"? likewise the "Church-Cooper supporters"?
  - Magnuson's implicit "amendment" via trying to redefine
     G only partially succeeded; not everyone saw it the same way
  - ("common knowledge" seems to be lacking)

- dimension manipulation illustration: Warren Magnuson
- different facts? (Mackie)
  - in Mackie's version the original Magnuson amendment is still on the table and is the actual reversion point if G fails
  - so Mackie says Riker has the legislative situation wrong

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  - hence according to Mackie there is
    - $\ast w$ : no nerve gas "to the United States"
    - \* before Magnuson maneuver: for G means (o, n); against G means (d, w)
    - \* after Magnuson maneuver: the same; s was irrelevant

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    - \* after Magnuson maneuver: the same; s was irrelevant
  - Mackie says Thurmond and Byrd cared about textiles, not s
  - moreover s was not about Senate prerogatives but just a tiff with the defense department

- axiomatic social choice theory
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- examples of properties of social choice rules
  - choice from all subsets
  - unrestricted preferences

- examples of properties of social choice rules
  - unanimity: if everyone has identical preferences, including  $aP_ib$ , then a is the sole choice from  $\{a, b\}$
  - weak Pareto: if for everyone  $aP_ib$ , then a is the sole choice from  $\{a, b\}$  regardless of what their other preferences are
  - strong Pareto: if one person has  $aP_ib$  and no one prefers b to a, then a is the sole choice from  $\{a, b\}$

- examples of properties of social choice rules
  - independence: the social choice from a set of alternatives depends only on the preferences individuals have about the alternatives in that set
  - pairwise independence: the social choice from  $\{a, b\}$ depends only on individuals' preferences between a and b
• majority voting satisfies pairwise independence

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- first-past-the-post (winner is the one with the most votes) does not (Craven's version)
- preferences showing first-past-the-post violates independence: choose from {b, c} while allowing votes to be cast for any of {a, b, c}

individuals I II

**2–10**  $aP_icP_ib$   $cP_ibP_ia$ 

- majority voting satisfies pairwise independence
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individuals I II

1	$bP_1cP_1a$	$aP_1bP_1c$
2–10	$aP_icP_ib$	$cP_ibP_ia$

- this may not be as crazy as it may appear: let *a* denote abstention
- (but, note, "voting" here is not behavioral)

- the Borda count does not satisfy independence
- preferences showing the Borda count violates independence: individuals
   I

1,2	$aP_ibP_ic$	$aP_icP_ib$
3,4	$bP_iaP_ic$	$bP_iaP_ic$
5	$cP_5bP_5a$	$cP_5bP_5a$

• the preferences regarding a and b are not changed, but b is chosen in I while a is chosen in II

- what properties describe majority rule?
- majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence

- what properties describe majority rule?
- majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence
- majority rule is the only social choice rule that satisfies independence, positive responsiveness, symmetry and anonymity
  - positive responsiveness: if there are two preference profiles  $R_i$  and  $R'_i$  such that  $aP'_ib$  whenever  $aP_ib$  and  $aR'_ib$ whenever  $aI_ib$ , then if  $C(a,b) = \{a\}$  with preferences  $R_i$ then  $C(a,b) = \{a\}$  with preferences  $R'_i$
  - anonymity (exchanging two voters' preferences does not change the social choice)
  - symmetry (exchanging a and b in everyone's preferences means a and b are exchanged in the social choice)

- an undesirable property: dictatorship
- dictatorship: if  $C_i(a,T) = a$ , then C(a,T) = a, and if  $C_i(U,T) = U$  then  $C(U,T) \subset U$

- other desirable properties
  - collective rationality
    - \* RC1: if  $C(a,b) = \{a\}$  and  $C(b,c) = \{b\}$  then  $C(a,c) = \{a\}$
    - \* **RC2:** if  $C(a, b) = \{a, b\}$  and  $C(b, c) = \{b, c\}$  then  $C(a, c) = \{a, c\}$
    - \* **RC3**: if a is in C(T) and b is in T, then a is in C(a, b)
    - \* RC4: if a is in C(a, b) and a is in C(T), then a is in  $C(T \cup b)$

- other desirable properties
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    - \* **RC3**: if a is in C(T) and b is in T, then a is in C(a, b)
    - \* RC4: if a is in C(a, b) and a is in C(T), then a is in  $C(T \cup b)$
- majority rule fails collective rationality, since it produces voting cycles

- recall that we presume
  - individual rationality
    - \* **RC1:** if  $C_i(a, b) = \{a\}$  and  $C_i(b, c) = \{b\}$  then  $C_i(a, c) = \{a\}$
    - \* **RC2:** if  $C_i(a, b) = \{a, b\}$  and  $C_i(b, c) = \{b, c\}$  then  $C_i(a, c) = \{a, c\}$
    - \* **RC3**: if a is in  $C_i(T)$  and b is in T, then a is in  $C_i(a, b)$
    - \* **RC4:** if *a* is in  $C_i(a, b)$  and *a* is in  $C_i(T)$ , then *a* is in  $C_i(T \cup b)$

- Arrow's theorem
- generalizes the conclusions about majority rule, almost universally

- Arrow's theorem (Vickrey variant)
- Theorem 3.3: There is a dictator if there are at least three alternatives and the social choice rule satisfies

choice from all subsets

unrestricted preferences

pairwise independence

weak Pareto

and if the social choices satisfy rationality conditions RC1 to RC4

- Arrow's theorem (Vickrey variant)
- proof in two steps
  - 1. epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives
  - 2. the dictator: show that the fully decisive set can contain only one individual

- epidemic
- set D of individuals is semidecisive for a over b if
   C(a, b) = {a} when aP<sub>i</sub>b for everyone in D but bP<sub>i</sub>a for
   everyone else
- assume D is semidecisive for a over b

- epidemic
- set D of individuals is semidecisive for a over b if
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- set D of individuals is semidecisive for a over b if
   C(a,b) = {a} when aP<sub>i</sub>b for everyone in D but bP<sub>i</sub>a for
   everyone else
- assume D is semidecisive for a over b
- unrestricted preferences make all these profiles relevant
- pairwise independence means these profiles can be considered in isolation from the rest of the preference orders, which may vary within each set of individuals

- assume D is semidecisive for a over b

- assume D is semidecisive for a over b
- preferences demonstrating semidecisiveness epidemic:
   individuals
   I
   II
   III
   III
   III
   III
   III
  - **rest**  $bP_icP_ia$   $bP_idP_ia$   $cP_iaP_ib$
- I: C(a, b) = {a} (semidecisive assumption); C(b, c) = {b} (weak Pareto); C(a, c) = {a} (RC1); D is semidecisive for a over c (definition)

- assume D is semidecisive for a over b
- preferences demonstrating semidecisiveness epidemic: individuals I II III in D aP<sub>i</sub>bP<sub>i</sub>c dP<sub>i</sub>aP<sub>i</sub>b bP<sub>i</sub>cP<sub>i</sub>a
  - **rest**  $bP_icP_ia$   $bP_idP_ia$   $cP_iaP_ib$
- I: C(a,b) = {a} (semidecisive assumption); C(b,c) = {b} (weak Pareto); C(a,c) = {a} (RC1); D is semidecisive for a over c (definition)
- II: C(a, b) = {a} (semidecisive assumption); C(d, a) = {d} (weak Pareto); C(d, b) = {d} (RC1); D is semidecisive for d over b (definition)

- combining and generalizing examples I and II gives
- SD1: if *D* is semidecisive for *a* over an alternative *y*, then *D* is semidecisive for any other alternative *x* over *y*
- SD2: if *D* is semidecisive for an alternative *x* over *b*, then *D* is semidecisive for *x* over any other alternative *y*

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- combining and generalizing examples I and II gives
- SD3: if *D* is semidecisive for *a* over *b*, then SD1 implies that *D* is semidecisive for any alternative *x* over *b*

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- SD4: if *D* is semidecisive for *a* over *b*, then SD2 implies that *D* is semidecisive for *a* over any alternative *y*

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- combining and generalizing examples I and II gives
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- SD4: if *D* is semidecisive for *a* over *b*, then SD2 implies that *D* is semidecisive for *a* over any alternative *y*
- SD5: if *D* is semidecisive for *a* over *b*, then SD3 and SD4 imply that *D* is semidecisive for any *x* over any *y* (except for *b* over *a*)

- assume D is semidecisive for a over b
- III: C(b,c) = {b} (SD5); C(a,c) = {c} (weak Pareto); C(a,b) = {b} (RC1); D is semidecisive for b over a (definition)

- assume D is semidecisive for a over b
- III: C(b,c) = {b} (SD5); C(a,c) = {c} (weak Pareto); C(a,b) = {b} (RC1); D is semidecisive for b over a (definition)
- if *D* is semidecisive for *a* over *b*, then *D* is semidecisive for any *x* over any *y*

- semidecisiveness implies decisiveness
- *D* is a decisive set of individuals if for any alternatives *x* and *y*, *C*(*x*, *y*) = {*x*} whenever *xP*<sub>*i*</sub>*y* for everyone in *D* whatever preferences others have

- semidecisiveness implies decisiveness
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- semidecisiveness implies decisiveness
- assume D is semidecisive (i.e., for any x over any y)
- preferences demonstrating decisiveness: individuals
  - in D  $dP_i eP_i c$
  - in E  $eP_idP_ic$
  - in F  $eP_icP_id$
  - rest  $eP_icI_id$
- C(d, e) = {d} (semidecisiveness); C(c, e) = {e} (weak Pareto); C(c, d) = {d} (RC1); D is decisive for d over c (definition)

- extension to decisiveness over choices from sets larger than pairs
- $\bullet$  assume D is a decisive set
- assume everyone in D has  $aP_ib$  for all other  $b \in T$
- then  $C(a,b) = \{a\}$  for all  $b \in T$

- extension to decisiveness over choices from sets larger than pairs
- assume *D* is a decisive set
- assume everyone in D has  $aP_ib$  for all other  $b \in T$
- then  $C(a,b) = \{a\}$  for all  $b \in T$
- let  $b \neq a$  and  $b \in T$  and suppose  $b \in C(T)$ ; then  $b \in C(a, b)$ (RC3), which contradicts  $C(a, b) = \{a\}$ ; therefore  $b \notin C(T)$ ; therefore  $C(T) = \{a\}$

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- let  $b \neq a$  and  $b \in T$  and suppose  $b \in C(T)$ ; then  $b \in C(a, b)$ (RC3), which contradicts  $C(a, b) = \{a\}$ ; therefore  $b \notin C(T)$ ; therefore  $C(T) = \{a\}$
- $C(T) = \{a\}$  matches the preference of the members of D

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- let  $b \neq a$  and  $b \in T$  and suppose  $b \in C(T)$ ; then  $b \in C(a, b)$ (RC3), which contradicts  $C(a, b) = \{a\}$ ; therefore  $b \notin C(T)$ ; therefore  $C(T) = \{a\}$
- $C(T) = \{a\}$  matches the preference of the members of D
- a set *D* is fully decisive if *D* can ensure that the most preferred alternative of all individuals in *D* is the social choice

- Arrow's theorem (Vickrey variant)
- proof in two steps
  - 1. epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives
  - 2. the dictator: show that the fully decisive set can contain only one individual

- the dictator
- the collective
  - i. a collective is the smallest fully decisive set
  - ii. all sets that include all members of the collective are fully decisive
  - iii. no set that does not include all members of the collective is fully decisive
  - iv. there is only one collective

- the collective
- intersecting fully decisive sets: D and E are fully decisive sets that have members in common

in D and in  $E = bP_i cP_i a$ 

in D, not in  $E = aP_ibP_ic$ 

in E, not in  $D = cP_i aP_i b$ 

rest  $aP_icP_ib$ 

- the collective
- intersecting fully decisive sets: D and E are fully decisive sets that have members in common

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in E, not in  $D = cP_i aP_i b$ 

rest  $aP_icP_ib$ 

C(b,c) = {b} (D decisive); C(a,c) = {c} (E decisive);
C(a,b) = {b} (RC1); D ∩ E is semidecisive for b over a;
therefore D ∩ E is a fully decisive set
- the dictator
- suppose the collective contains two or more members
- $\bullet\,$  let some but not all collective members be in E
- these assumptions lead to a contradiction

in  $E \quad aP_icP_ib$ 

rest  $cP_ibP_ia$ 

C(a,b) = {a,b} (neither subset is semidecisive);
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  C(a,c) = {a,c} (neither subset is semidecisive);
  C(b,c) = {b,c} (RC2); but C(b,c) = {b,c} violates weak
  Pareto
- if only one subset contains the collective, there is no contradiction
- the collective is impossible to divide only if has only one member

- Arrow's theorem (as proven)
- Theorem 3.7: There is a dictator if there are at least three alternatives and the social choice rule satisfies

choice from all subsets

unrestricted preferences

pairwise independence

weak Pareto

and if the social choices satisfy rationality conditions RC1, RC2 and RC3

• manipulation

## • manipulation

 a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences

## • manipulation

- a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences
- a rule is non-manipulable if it is not open to manipulation by any individual on any set of alternatives

- Gibbard-Satterthwaite theorem (variant)
- Theorem 5.4: There is a dictator if there are at least three alternatives and the social choice rule satisfies

choice from all subsets

unrestricted preferences

weak Pareto

non-manipulability

and if the social choices satisfy rationality conditions RC1 to RC4

- Riker and Arrow (heresthetic and the general impossibility theorem)
- Arrow's Theorem (variant): There is a dictator if there are at least three alternatives and the social choice rule satisfies: choice from all subsets; unrestricted preferences; pairwise independence; weak Pareto; and if the social choices satisfy rationality conditions RC1, RC2 and RC3
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- heresthetic is motivated by the basic principles: avoid dictatorship; ensure that collective choices reflect everyone's preferences; have collective choices be effective
- agenda control: follows from giving up transitivity
- dimension manipulation: follows from giving up independence
- strategic voting: follows from giving up independence, via nonmanipulability

- restrictions on preferences
  - preference restrictions can avoid majority rule voting cycles (see Craven chapter 6)
  - recall that majority voting is not manipulable if there are no cycles
  - but falsely stating preferences can create a cycle even when no cycle exists in true preferences

• potentially manipulable preferences (Table 6.6):

ndividuals	number	true	false
E	$n_1$	$aP_ibP_ic$	$aP_i'bP_i'c$
F	$n_2$	$bP_iaP_ic$	$bP_i'aP_i'c$
G	$n_3$	$bP_icP_ia$	$bP_i'cP_i'a$
H	$n_4$	$cP_ibP_ia$	$cP_i'aP_i'b$

- if  $n_1 > n_2 + n_3 + n_4$ , then aPb and aPc (no manipulation)
- if  $n_2 + n_3 + n_4 > n_1$ ,  $n_1 + n_4 > n_2 + n_3$  and  $n_3 + n_4 > n_1 + n_2$ , then bPc, bPc and cPa but aP'b, bP'c and cP'a

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- the (one-dimensional) spatial model

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  - are preferences single peaked?

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  - example: YES
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    - \* person D:  $xP_iyP_iz$
    - \* person E:  $zP_iyP_ix$
    - \* person F:  $yP_izP_ix$

- restrictions on preferences
- the (one-dimensional) spatial model
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  - preferences:
    - \* person D:  $xP_iyP_iz$
    - \* person E:  $zP_iyP_ix$
    - \* person F:  $yP_izP_ix$
  - one alternative is never ranked last
  - each person has an IDEAL POINT



alternative

- the spatial model
- single-peaked preferences

- the spatial model
- single-peaked preferences
- which alternative wins in a series of pairwise votes?



- the spatial model
- single-peaked preferences
- which alternative wins in a series of pairwise votes?
- with single-peaked preferences, the median is the Condorcet winner

- the spatial model
- single-peaked preferences and measures of distance
  - example with symmetry and same distances for different people
  - tent metrics:  $u_{ik} = -b|x_k x_i|$



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$$u_{ik} = -b|y_k - y_i|$$
  
=  $-b|a + cx_k - (a + cx_i)$   
=  $-bc|x_k - x_i|$ 

 the transformation changes the distance measure but not the shape of the preference curves: intensity doesn't matter



- the spatial model
- single-peaked preferences and measures of distance
  - euclidean metrics:  $u_{ik} = -b[(x_k x_i)^2]^{1/2}$
  - symmetry
- the median remains the Condorcet winner



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$$u_{ik} = -b[(y_k - y_i)^2]^{1/2}$$
  
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=  $-bc[(x_k - x_i)^2]^{1/2}$ 

 the transformation changes the distance measure but not the shape of the indifference curves

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- with symmetric single-peaked preferences
- which is to say, in a pure one-dimensional spatial model...

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- with symmetric single-peaked preferences
- which is to say, in a pure one-dimensional spatial model...
- all information about the chooser's preferences that is relevant for choice behavior is summarized by the chooser's ideal point







- a two-dimensional spatial model
- two-dimensional separable euclidean preferences:

$$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$$

• an example with three voters

Three Voters with Separable Euclidean Preferences


- a two-dimensional spatial model
- two-dimensional separable euclidean preferences:

 $u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$ 

- with two-dimensional spatial preferences, in general the winset of any point x is not empty
- with separable preferences, the median on one dimension can be defeated by alternatives that shift along both dimensions
- with nonseparable preferences, a one-dimensional median is even more unstable

## • two-dimensional nonseparable euclidean preferences:

$$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$$

Three Voters with Nonseparable Euclidean Preferences



• two-dimensional nonseparable euclidean preferences:

 $u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$ 

- with two-dimensional spatial preferences, whether separable or nonseparable, in general the winset of any point x is not empty
- with nonseparable preferences, the median on one dimension can be defeated even by alternatives that shift along the same dimension

- instability with multidimensional spatial preferences
- the chaos theorems

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  - there is a finite path of pairwise majority votes from any alternative to any other alternative, and back
  - power of the agenda setter given sincere voters

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Three Voters



policy X

- Pareto set:
  - the set of all points that are not unanimously inferior to any other point
- each point in the Pareto set
  - 1. is not unanimously inferior to any other point
  - 2. is unanimously superior to at least one other point

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  - the set of all points that are not unanimously inferior to any other point
- each point in the Pareto set
  - 1. is not unanimously inferior to any other point
  - 2. is unanimously superior to at least one other point
- with three voters, the boundaries of the Pareto set coincide with the median lines
- a median line connects two voters such that half (or half plus one for an odd number of voters) of the remaining voters are on either side of the line

Three Voters with Pareto Set



policy X

Three Voters Facing an Agenda



Indifference Curves At Status Quo





policy X



policy X



policy X

At Status Quo

At First Alternative

3

С

2

С

2

4

4



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- chaos is not possible only if there are Condorcet winners, in other words, only if the core (the set of Condorcet winners) is not empty

- instability with multidimensional spatial preferences
- the chaos theorems
  - there is a finite path of pairwise majority votes from any alternative to any other alternative, and back
  - power of the agenda setter given sincere voters
- chaos is not possible only if there are Condorcet winners, in other words, only if the core (the set of Condorcet winners) is not empty
- under certain conditions on the type of agenda, strategic voters can keep outcomes inside the Pareto set



Indifference Curves At First Alternative, with Pareto Set



- instability with multidimensional spatial preferences
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  - technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"

- instability with multidimensional spatial preferences
- strategic voters can keep outcomes inside the Pareto set
  - technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"
- if the agenda is endogenous, which means the voters propose alternatives, then strategic voting can keep outcomes inside the uncovered set

- the uncovered set
- McKelvey, Richard D. 1986. "Covering, Dominance, and Institution-Free Properties of Social Choice." American Journal of Political Science 30: 283–314.
- x covers y, denoted xCy, if xPy and for all z such that zPx, zPy
- the uncovered set is the set of all alternatives w such x C w is false for all x

- the uncovered set and the yolk (generalized median set)
- the generalized median set (or the yolk) is the smallest ball that intersects all median lines

- the uncovered set and the yolk (generalized median set)
- the generalized median set (or the yolk) is the smallest ball that intersects all median lines
- the generalized median is the center point of the yolk
- the uncovered set is contained in the a region four times the yolk's radius
- in general, the yolk shrinks as the number of voters increases (assuming sufficient dispersion)

Seven Voters



Seven Voters with Median Lines



Seven Voters with the Yolk



Seven Voters



Nine Voters



Nine Voters with Median Lines



Nine Voters with the Yolk



Seven Voters with the Yolk



- the Pareto set and four times the yolk
- the uncovered set is contained in the intersection of these two sets




- the yolk is small when a Condorcet winner almost exists
- in this case, strategic, open-agenda outcomes are near where the Condorcet winner would be (if it existed)

Seven Voters



Seven Voters with Median Lines



#### Seven Voters with Yolk



#### Seven Voters with Yolk



### Seven Voters with Uncovered Set Range



• the generalized median and indifference curves

#### Voters with Generalized Median



dimension A



dimension A



dimension A

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- an agenda setter can produce any result with sincere voters (short of victory for an alternative that loses to every other possible alternative; with spatial preferences there is no such alternative)

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- strategic voters can keep outcomes inside the Pareto set—specifically, inside the uncovered set—if the agenda is an amendment agenda
  - an agenda is an amendment agenda iff it is symmetric and continuous
  - a symmetric agenda is nonrepetitive, complete and uniform

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- strategic voters can keep outcomes inside the Pareto set—specifically, inside the uncovered set—if the agenda is an amendment agenda
  - an agenda is an amendment agenda iff it is symmetric and continuous
  - a symmetric agenda is nonrepetitive, complete and uniform
- for other agendas, strategic voting can confine outcomes to alternatives that are dominant or are in the dominant cycle

 "But agenda power has two sources: the ability to choose a procedure for selecting from fixed set of alternatives and the ability to choose that set." (Ordeshook and Schwartz 1987, 194) • political parties: solutions to coordination or collective action problems?





## Coordination

	Melissa	Mariah
Melissa	2,2	0,0
Mariah	0,0	1,1

# PD

Don't Confess Confess

	$Don't \ Confess$	Confess
efess	3,3	0,4
fess	4,0	1, 1

### PD

Don't Confess Confess

	$Don't \ Confess$	Confess
fess	3,3	0,4
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• collective action problem (Aldrich Table 2.1)

	Bill		
Legislator	Χ	Y	Ζ
Α	4	3	-9
В	3	-9	4
С	-9	4	3

• collective action problem (Aldrich Table 2.3)

	Bill		
Legislator	Χ	Υ	Ζ
Α	3	-1	-1
В	-1	3	-1
С	-1	-1	3

- why parties?
- parties may help solve coordination dilemmas
  - electoral (party-in-elections)
  - legislative (party-in-government)
  - one role for activists: high intensity or high resource people who work or spend to promote focal points
  - party labels

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  - party labels
- parties may help deploy resources efficiently
  - information about policies and candidates (cheap for voters and candidates)

- strategic voting in electoral systems
- examples of electoral system features
  - districts and district magnitude (M)
  - presidents versus parliaments
  - proportional representation (PR)
    - \* with quotas (e.g., Hare: Q = V/M)
    - \* with averages (divisor methods) (e.g., d'Hondt:  $a_i(t) = v_i/(s_i(t) + 1)$ )
  - party lists
  - runoffs (dual ballots)

- examples of voting rules
  - majority rule
  - plurality voting with one winner (winner take all, first past the post)
  - plurality voting with M>1
  - Borda count
  - instant runoff voting (Hare systems)
  - approval voting
  - utilitarian systems
  - range voting

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  - approval voting
  - utilitarian systems
  - range voting
- there is extensive theory for single districts, little for whole systems

- current "election reform" debates about voting rules
- Borda count
  - favored by Saari (perhaps with different weights)
  - problem is positional methods are manipulable

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- Borda count
  - favored by Saari (perhaps with different weights)
  - problem is positional methods are manipulable
- approval voting
  - claimed to have many virtues (see Brams et al.)
  - among the flaws is it need not select a Condorcet winner (SC&W election example)
  - outcomes depend almost entirely on strategies, in particular on how many alternatives each voter decides to approve

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- instant runoff voting (Hare systems)
  - claimed to have many virtues (see Richie et al.)
  - major flaw is it violates monotonicity
- range voting
  - has strong advocates (see RangeVoting.html)
  - is approval voting with more degrees of freedom

- Duverger's Law: electoral systems cause party systems
  - plurality voting (with M = 1) favors a two-party system
  - PR favors a multiparty system
- motivated by strategic voting (voters) and coalition formation (elites)

- Duverger's Law: electoral systems cause party systems
  - plurality voting (with M = 1) favors a two-party system
  - PR favors a multiparty system
- motivated by strategic voting (voters) and coalition formation (elites)
- generalizing Duverger's Law for M > 1
  - the M+1 rule
  - non-Duvergerian outcomes: parties finishing below the M + 1-th place get equal numbers of votes greater than zero
  - leads to the bimodality hypothesis, which Cox tests using the SF ratio:  $v_{M+2}/v_{M+1} = 0$  or ,  $v_{M+2}/v_{M+1} = 1$

- strategic voting under plurality voting (Cox)
- preferences: von Neumann-Morgenstern utilities  $u_i = (u_{i1}, \ldots, u_{iK})'$ ,  $\max(u_i) = 1$ ,  $\min(u_i) = 0$
- beliefs:  $F_i = F$  (common knowledge)
- expectations: proportions  $\pi_i = (\pi_{i1}, \ldots, \pi_{iK})'$
- publicly generated expectations:  $\pi_i = \pi$
- vote to maximize expected utility  $\sum_{k=1}^{K} \pi_{ik} u_{ik}$

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- vote to maximize expected utility  $\sum_{k=1}^{K} \pi_{ik} u_{ik}$
- rational expectations condition: if given beliefs F everyone votes optimally in light of  $\pi$ , the result is expected vote shares that equal  $\pi$
- Theorem 1: If  $0 < \pi_j < \pi_{M+1}$  for j > M+1, then  $\pi$  is not a limit of rational expectations
- Corollary: If  $\pi$  is a limit of rational expectations, then  $\pi_j \in \{0, \pi_{M+1}\}$  for all j > M+1

 Duverger's Law, the M + 1 rule and other strategic voting: examples (using > to denote P<sub>i</sub>), find the rational expectations Nash equilibrium vote distributions

$$1/8 \quad A > B > C$$

- $4/8 \quad B > C > A$
- $3/8 \quad A > C > B$
- simple plurality
  - (A, 1/2; B, 1/2; C, 0)
  - (A, 1/2; B, 0; C, 1/2)
- plurality with M=2
- two-stage with plurality M = 2 in first stage
- Borda count

- consider coalition-proof refinement to Nash equilibrium

- Duverger's Law, the M + 1 rule and other strategic voting: examples (using > to denote P<sub>i</sub>)
  - $1/8 \quad A > B > C > D$
  - $2/8 \quad B > C > D > A$
  - $2/8 \quad D > B > C > A$
  - $1/8 \quad A > C > D > B$
  - $2/8 \quad D > A > C > B$
- simple plurality
  - (A, 1/2; B, 1/2; C, 0; D, 0)
  - (A, 0; B, 0; C, 1/2; D, 1/2)
- plurality with M = 2

- why parties: an informational rationale (Snyder and Ting)
  - candidates: one-dimensional policy plus benefits from holding office
  - benefit to candidate from holding office: w > 0
  - candidate ideal point:  $z \in [-1, 1]$
  - party:  $i \in \{L, R\}$
  - party position:  $x_i \in [-1, 1]$
  - cost of party affiliation:  $c \in [0, w)$
  - policy loss scale factor:  $\alpha>0$
  - affiliated candidate utility:  $w \alpha (x_i z)^2 c$
  - relative benefit of holding office as a party member:  $\theta = \sqrt{(w-c)/\alpha}$ ; assume  $\theta < 1$

- why parties: an informational rationale (Snyder and Ting)
  - voters: one-dimensional policy given uncertainty about candidates
  - ideal point of the median voter in a district:  $\boldsymbol{y}$
  - mean of ideal points of candidates affiliated with party i:  $\mu_i$
  - variance of ideal points of candidates affiliated with party *i*:  $\sigma_i^2$
  - expected utility of the median voter in a district for a candidate affiliated with party *i*:  $-(y \mu_i)^2 \sigma_i^2$
  - expected utility of the median voter in a district for an unaffiliated candidate :  $-y^2 1/3$  (i.e.,  $\mu_U = 0$  and  $\sigma_U^2 = 1/3$ )

- why parties: an informational rationale (Snyder and Ting)
  - sequence of game play (page 96)
  - platform and affiliation choices when parties maximize the share of offices won (page 100)
  - platforms when parties maximize total net benefits of their members (page 102)

- simple agenda setting: the setter model (Rosenthal)
  - one-dimensional spatial setting
  - median voter
  - reversion or status quo point
  - game is: proposal followed by accept or reject

- gerrymanders and partisan bias
- a formal model (Cox and Katz 1999)
  - $-\nu$ : average vote share,  $0 < \nu < 1$
  - $\rho$ : responsiveness,  $\rho\in(-\infty,\infty)$
  - $-\lambda$ : partisan bias parameter,  $\lambda \in (-\infty,\infty)$
  - -s: seat share; as a function of average vote share

$$s(\nu;\rho,\lambda) = \frac{\exp\left[\rho\log\left(\frac{\nu}{1-\nu}\right)\right]}{\exp\left[-\lambda\right] + \exp\left[\rho\log\left(\frac{\nu}{1-\nu}\right)\right]}$$

- partisan bias measure:  $\exp[\lambda]/(\exp[\lambda] + 1) 0.5$
- responsiveness: if  $\rho < 1$ , then the smaller party is overrepresented ( $\nu < s$ )

- a formal model (Cox and Katz 1999)
  - $\nu$ : average vote share,  $0 < \nu < 1$
  - $\rho$ : responsiveness,  $\rho\in(-\infty,\infty)$
  - $\lambda$ : partisan bias parameter,  $\lambda \in (-\infty, \infty)$
  - $s(\nu; \rho, \lambda)$ : seat share, a function of average vote share
- feasible plans: a smooth, convex subset of  $(\rho, \lambda)$  space
- hence the strong party must tradeoff responsiveness and bias

## • cube law

$$\frac{S}{1-S} = \left(\frac{V}{1-V}\right)^3$$

- some Voting Right Act concepts
- racially polarized voting
- minority vote dilution
  - gerrymander
  - at-large election plans
  - majority-runoff requirements
  - anti-single-shot devices (defeats "bullet votes")

- some Voting Right Act concepts
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  - anti-single-shot devices (defeats "bullet votes")
- Section 5 preclearance
- Section 2, Mobile v. Bolden, and the 1982 VRA extension
- 1975 VRA extension and language minority provisions

- electoral systems (Grofman and Davidson, 7)
  - at-large
  - single-member districts
  - mixed systems

- electoral systems (Grofman and Davidson, 7)
- at-large
  - "In an at-large system, all the contested seats on a governmental body, such as a city council, county commission, or school board, are filled by voters in the jurisdiction at large. If there are eight seats to be filled, all voters have eight votes and theoretically have a chance to influence who gets elected to all eight seats."

- electoral systems (Grofman and Davidson, 7)
- single-member districts
  - "the city is divided into geographical districts, and voters in each district, like voters in congressional elections, are limited to a vote for a single candidate running to represent their district"
- mixed systems
  - "some of the seats are voted on at large, and some by district"

- districting in House elections under the Voting Rights Act
  - majority minority districts: changes over time
  - "bleaching" districts: partisan gerrymander disguised as nondilution

- districting in House elections under the Voting Rights Act
  - majority minority districts: changes over time
  - "bleaching" districts: partisan gerrymander disguised as nondilution
- substantive representation versus(?) symbolic representation
  - is the 65% majority-minority "rule" appropriate?
  - Cameron, Epstein, O'Halloran suggest no for U.S. House districts
  - Mapps suggests no for state legislative districts

- election fraud: is fraud (legitimate) political manipulation?
- detecting anomalies
- distinguishing anomalies from fraud
- diagnosing fraud

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- history of fraudulent elections in the United States
- elsewhere (and election monitoring: observers, PVT)

- detecting anomalies
- Florida 2000: wrong outcome, but why?
  - ex-felon lists
  - butterfly ballot
  - other machines and ballots

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- Florida 2000: wrong outcome, but why?
  - ex-felon lists
  - butterfly ballot
  - other machines and ballots
- Florida 2004: fraud alleged
  - conservative Democrats
  - hacked machines?

## • Election Forensics

- statistically analyzing recorded vote counts to detect anomalies and try to diagnose fraud
- regularities and departures from regularities
  - using relationships with covariates to detect outliers
  - checking whether vote counts match expected distributions

- election forensics and recounts
  - two kinds of errors (or frauds) in vote counts
    - $\ast\,$  miscounting the ballots that were cast
    - \* counting falsified ballots

- election forensics and recounts
  - two kinds of errors (or frauds) in vote counts
    - \* miscounting the ballots that were cast
    - \* counting falsified ballots
- recounts can detect the first kind but not the second kind
  - exception: physically inspecting ballots may spot signs that some or all are fake
  - this depends on there being physical ballots to inspect
- statistical analysis may be able to detect both kinds of distortions

- an example from the 2006 Mexican presidential election
  - relationship between presidential votos nulos and senate votos nulos
  - use casilla (ballot box) counts
  - the linear predictor is

 $Z_i = d_0 + d_1 \text{logitz}(\text{SenateVN}_i)$ 

SenateVN represents the proportion of votos nulos for senate votes at casilla  $i \operatorname{logitz}(p)$  denotes the log-odds function adjusted to handle zero counts (add 1/2 to each count before computing p)

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- estimate separately for each legislative district
- outliers are prevalent

## votos nulos studentized residual



Guanajuato



votos nulos studentized residual

- an example from the 2006 Mexican presidential election
  - relationship between presidential votos nulos and senate votos nulos
  - use casilla (ballot box) counts
  - estimate separately for each legislative district
  - outliers are prevalent
    - \* 130,020 casillas are in the analysis (from 299 districts)

proportion of residuals larger than

2	3	4
.11	.06	.04

• checking whether vote counts conform with expected distributions

- checking whether vote counts conform with expected distributions
- digits of vote counts and Benford's Law
  - compare vote counts' second digits to the second digit Benford's Law (2BL)
  - there are strong arguments against expecting vote counts' first digits to satisfy Benford's Law for first digits

Freq	uency (	of First	and S	econd ]	Digits a	accordi	ng to E	Benford	's Law	
digit	0	1	2	3	4	5	6	7	8	9
first		.301	.176	.124	.097	.079	.067	.058	.051	.046
second	.120	.114	.109	.104	.100	.097	.093	.090	.088	.085

• the statistic is

$$X_{B_2}^2 = \sum_{i=0}^9 \frac{(d_{2i} - d_2 q_{B_2 i})^2}{d_2 q_{B_2 i}}$$

where

- $q_{B_{2i}}$  is the expected relative frequency with which the second significant digit is *i* (the values shown in the second line of table of Benford's Law frequencies)
- $d_{2i}$  is the number of times the second digit is *i* among the precincts being considered

$$- d_2 = \sum_{i=0}^{9} d_{2i}$$

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 $- d_2 = \sum_{i=0}^{9} d_{2i}$ 

- with one set of counts (for one office in one area), use the critical value of  $\chi_9^2$  for test level  $\alpha = .05$ , which is 16.9
- looking at multiple sets of counts, control for the false discovery rate (FDR)

- an example from the 2004 American election: Florida, Miami-Dade County
  - vote counts for major party candidates for president (Kerry and Bush) and for the Senate (Castor and Martinez)
  - also vote counts for eight proposed constitutional amendments
  - with 20 tests, the FDR-controlled critical value for  $\chi_9^2$  is 25.5
Florida Constitutional Amendments on the Ballot in 2004

Yes No Parental Notification of a Minor's Termination of 1 4,639,635 2,534,910 Pregnancy 2Constitutional Amendments Proposed by Initiative 4,574,361 2,109,013 The Medical Liability Claimant's Compensation 3 2,622,143 4,583,164 Amendment Authorizes Voters to Approve Slot Machines in 3,512,181 3,631,261 4 Parimutuel Facilities Florida Minimum Wage Amendment 5,198,514 2,097,1515 6 Repeal of High Speed Rail Amendment 4,519,423 2,573,280 Patients' Right to Know About Adverse Medical In-1,358,183 5,849,125 7 cidents Public Protection from Repeated Medical Malprac-2,083,864 8 5,121,841tice

## Miami-Dade Election Day First-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	29.3	Am. 4 Yes	144.8
Kerry	39.9	Am. 4 No	119.6
Martinez	35.6	Am. 5 Yes	115.4
Castor	22.0	Am. 5 No	27.6
Am. 1 Yes	86.2	Am. 6 Yes	98.8
Am. 1 No	80.5	Am. 6 No	84.0
Am. 2 Yes	95.6	Am. 7 Yes	130.3
Am. 2 No	60.0	Am. 7 No	49.9
Am. 3 Yes	60.5	Am. 8 Yes	123.0
Am. 3 No	51.5	Am. 8 No	102.6

Note: n = 757 precincts. Pearson chi-squared statistics, 8 df.

### Miami-Dade Election Day Second-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	7.9	Am. 4 Yes	3.3
Kerry	9.5	Am. 4 No	5.7
Martinez	8.9	Am. 5 Yes	17.9
Castor	12.0	Am. 5 No	5.8
Am. 1 Yes	2.5	Am. 6 Yes	4.3
Am. 1 No	5.5	Am. 6 No	9.1
Am. 2 Yes	16.7	Am. 7 Yes	17.1
Am. 2 No	7.2	Am. 7 No	8.4
Am. 3 Yes	3.3	Am. 8 Yes	12.7
Am. 3 No	12.9	Am. 8 No	6.5

Note: n = 757 precincts. Pearson chi-squared statistics, 9 df.

- why should we expect vote counts to satisfy 2BL?
- model vote counts as results of particular mixtures
- at least two mechanisms can generate counts that satisfy 2BL (and not 1BL)
  - mechA: mix support that varies over precincts with a small random frequency of errors
  - mechB: mix support that varies over precincts with varying precinct sizes

Benf.	Size	Benf.	Size	Benf.	Size	Benf.	Size
8.3	$7,\!100$	11.3	$3,\!800$	18.6	1,500	10.3	500
9.1	$7,\!200$	9.2	$3,\!900$	21.6	$1,\!600$	9.5	600
8.9	$7,\!300$	12.2	$4,\!000$	19.9	1,700	10.0	700
9.3	$7,\!400$	10.5	$4,\!100$	17.5	$1,\!800$	9.0	800
7.8	$7,\!500$	10.4	$4,\!200$	14.0	$1,\!900$	10.0	900
7.9	$7,\!600$	9.1	$4,\!300$	14.1	$2,\!000$	9.7	1,000
9.1	7,700	10.2	$4,\!400$	9.7	$2,\!100$	10.4	$1,\!100$
10.9	$7,\!800$	12.3	$4,\!500$	8.7	$2,\!200$	12.0	$1,\!200$
8.7	$7,\!900$	9.9	$4,\!600$	11.6	$2,\!300$	12.3	$1,\!300$
9.0	8,000	11.2	4,700	12.2	$2,\!400$	13.4	$1,\!400$

2BL Tests for Simulated Precinct Vote Counts (First Mechanism)

Note: Chi-squared statistics, 9 df, 25 Monte Carlo replications.

- why should we expect vote counts to satisfy 2BL?
- while precinct vote counts should satisfy 2BL, counts on voting machines used in each precinct should not
  - voting machine counts are subject to "roughly equal division with leftovers" (REDWL)
  - simulations verify the <code>REDWL</code> mechanism

- why should we expect vote counts to satisfy 2BL?
- while precinct vote counts should satisfy 2BL, counts on voting machines used in each precinct should not
  - voting machine counts are subject to "roughly equal division with leftovers" (REDWL)
  - simulations verify the REDWL mechanism
- and actual machine-level vote counts do not satisfy 2BL

### Miami-Dade Election Day Second-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	17.2	Am. 4 Yes	43.5
Kerry	44.0	Am. 4 No	25.4
Martinez	11.5	Am. 5 Yes	57.6
Castor	12.7	Am. 5 No	25.6
Am. 1 Yes	43.6	Am. 6 Yes	29.7
Am. 1 No	19.8	Am. 6 No	15.3
Am. 2 Yes	38.7	Am. 7 Yes	53.2
Am. 2 No	11.9	Am. 7 No	136.7
Am. 3 Yes	78.0	Am. 8 Yes	54.2
Am. 3 No	25.7	Am. 8 No	23.2

Note: n = 7,064 precinct-machines. Pearson chi-squared stats, 9 df.

- the 2BL test can detect artificial manipulations of vote counts that otherwise satisfy 2BL
- simulations show a wide range of ways to manipulate the votes can be detected
  - adding votes
  - subtracting votes
  - switching votes

# Simulated "Repeater" Vote Switching: Receive Votes When Above Expectation

	Receiver (cand. $1$ )			Don	or (can	d. 2)
fraction	500	1000	2000	500	1000	2000
0	9.6	8.7	12.4	11.1	11.9	13.0
0.01	11.2	13.3	15.0	9.3	10.3	11.4
0.02	12.7	17.7	27.1	8.8	12.2	13.2
0.03	15.5	27.2	44.1	10.5	10.7	14.2
0.04	25.6	41.8	68.9	10.9	13.1	16.9
0.05	24.8	38.1	67.2	11.2	13.6	17.1
0.06	23.6	42.2	74.2	12.0	15.1	19.3
0.07	28.2	48.4	89.9	12.9	15.6	22.1
0.08	33.5	58.1	112.8	13.5	17.3	26.5
0.09	32.7	56.5	107.7	12.9	18.0	29.3

# Simulated "Repeater" Vote Switching: Receive Votes When Below Expectation

	Receiver (cand. $1$ )			Don	or (can	d. 2)
fraction	500	1000	2000	500	1000	2000
0	9.6	10.3	12.8	9.7	10.3	12.2
0.01	10.0	13.1	15.0	10.4	11.4	14.3
0.02	12.6	18.3	28.0	11.8	12.7	19.9
0.03	18.6	26.8	50.3	13.5	18.3	22.8
0.04	25.9	44.5	80.0	12.4	19.4	26.7
0.05	26.5	45.4	74.8	16.1	21.5	31.4
0.06	28.5	46.6	87.1	14.8	21.5	37.9
0.07	33.1	57.1	102.2	17.0	24.9	42.1
0.08	39.0	71.8	128.4	16.8	26.3	45.4
0.09	38.0	68.1	126.9	19.6	27.0	40.9

- wider application of the 2BL test: recent American presidential votes
  - precinct vote counts in the 2000 and 2004 elections, separately for the precincts in each county
  - impose FDR-control using the number of counties in each state
    - \* (see maps [in showmappbenf0004fdr.R])

# Counties with Signficant 2BL Tests using State-specific FDR Adjustment: 2000

		Gore	votes	Bush	votes
County	J	$d_2$	$X_{B_2}^2$	$d_2$	$X_{B_2}^2$
Los Angeles, CA	$5,\!045$	$5,\!011$	54.8	$4,\!930$	20.3
Kent, DE	61	61	9.0	61	22.2
Latah, ID	34	31	36.7	34	3.8
Cook, IL	$5,\!179$	$5,\!097$	46.7	$4,\!145$	24.4
Dupage, IL	714	714	28.0	714	41.6
Lake, IL	403	403	33.7	402	16.1
Passaic, NJ	295	295	27.7	294	5.6
Hamilton, OH	$1,\!025$	$1,\!020$	48.7	988	8.9
Hancock, OH	67	67	34.3	67	9.9
Summit, OH	624	624	31.6	612	11.6
Philadelphia, PA	$1,\!681$	$1,\!680$	29.5	$1,\!249$	34.7
King, WA	$2,\!683$	$2,\!665$	27.0	$2,\!641$	8.9

# Counties with Significant 2BL Tests using State-specific FDR Adjustment: 2004

		Kerry votes		Bush votes	
County	J	$d_2$	$X_{B_2}^2$	$d_2$	$X_{B_2}^2$
Los Angeles, CA	$4,\!984$	$4,\!951$	70.2	$4,\!929$	12.4
Orange, CA	$1,\!985$	$1,\!887$	26.2	$1,\!904$	32.6
Jefferson, CO	324	323	30.0	323	10.4
Kootenai, ID	75	75	30.9	75	12.1
Cook, IL	$4,\!562$	$4,\!561$	44.5	$4,\!026$	27.8
DuPage, IL	732	732	35.2	732	9.1
Clay, MO	76	76	28.4	76	4.0
Summit, OH	475	475	42.7	474	21.0
Davis, UT	213	212	42.6	213	6.0
Utah, UT	247	241	9.2	246	27.6
Benton, WA	177	168	29.2	173	14.8

- the 2BL test applied to votes for president in the 2006 Mexican election
  - seccion vote counts, separately for the secciones in each legislative district
  - over all 300 districts, the FDR-controlled critical value for  $\chi_9^2$  is 32.4
  - over 1500 district-party combinations, the FDR-controlled critical value for  $\chi_9^2$  is 36.4



2BL test statistic

- the statistical tests and the partial recount done of votes for president in the 2006 Mexican election
  - the original count included 41,791,322 ballots
  - 40,588,729 votes were recorded for one of the parties
  - the original difference between the PAN and PBT vote totals was 243,934 votes, which is 0.58 percent of the ballots cast

- the statistical tests and the partial recount done of votes for president in the 2006 Mexican election
  - the original count included 41,791,322 ballots
  - 40,588,729 votes were recorded for one of the parties
  - the original difference between the PAN and PBT vote totals was 243,934 votes, which is 0.58 percent of the ballots cast
- the recount
  - about nine percent of the casillas were manually recounted
  - I use data from 11,651 recounted casillas (which I think is all of them)

Net Vote Count Changes in the Mexico 2006 Recount

 $\begin{array}{c|ccccccccccccc} PAN & APM & PBT & NA. & ASDC \\ \mbox{original} & 15,000,284 & 9,301,441 & 14,756,350 & 401,804 & 1,128,850 \\ \mbox{change} & -13,333 & -1,885 & -58 & -1,578 & 1,836 \\ \end{array}$ 

Note: Some of the recounted votes included here are from casillas that were canceled in the final official results.

- relationship between the 2006 Mexican recount changes and the two kinds of statistical tests
- definitions for casilla-level variables

$$CHANGE = \begin{cases} 1, & \text{if the vote count changed for any party} \\ 0, & \text{otherwise} \end{cases}$$
$$NULOS2 = \begin{cases} 1, & \text{if the votos nulos } |\text{residual}| \ge 2 \end{cases}$$

$$\mathsf{IULOS2} = egin{cases} 1, & \mathsf{if} \ \mathsf{the} \ \mathsf{votos} \ \mathsf{nulos} \ |\mathsf{residual}| \geq 2 \ 0, & \mathsf{otherwise} \end{cases}$$

• definitions for district-level variable

$$\mathbf{2BL} = \begin{cases} 1, & \text{if the 2BL statistic for any party} \geq 16.9 \\ 0, & \text{otherwise} \end{cases}$$

#### Recount Changes and Test Statistics

## CHANGE

NULOS2	0	1	n
0	0.33	0.67	$9,\!200$
1	0.28	0.72	$2,\!215$

Pearson chi-squared = 20.1

# CHANGE2BL01n00.290.715,00110.330.676,650Pearson chi-squared = 21.5

- relationship between the 2006 Mexican recount changes and the two kinds of statistical tests
  - unusually large votos nulos counts for a casilla are associated with more vote count changes if that casilla is recounted
  - unusually large 2BL test statistics for a district are associated with fewer vote count changes when casillas in that district are recounted
- does this mean that the 2BL test is picking up the fact that votes were faked, in ways that the recount did not detect?

- relationship between the 2006 Mexican recount changes and the two kinds of statistical tests
- is the 2BL test picking up the fact that votes were faked, in ways that the recount did not detect?
- consider the possibility of strategic voting (to mw07.pdf)

- is election manipulation election fraud?
- are either election manipulation or election fraud heresthetic?
  - election manipulation as dimension manipulation (unlikely)
  - election manipulation as agenda control
  - election manipulation as strategic voting
- the key issue is dictatorship (or oligarchy), which heresthetic (via Arrow's theorem) is normatively justified to oppose
- election fraud seems intuitively to be dictatorial, but why is that?