Government 320: Public Opinion and Public Choice

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Tuesday and Thursday 2:55–4:10 (MG 165)

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Course web page:

http://macht.arts.cornell.edu/wrm1/gov320.html

- **heresthetic and the critique of democracy: instability and meaninglessness**
- **based on social choice theory, in particular on Arrow's Theorem**
- **assumes individual rationality**

- **rationality: transitive and complete individual preference orders**

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- **transitivity:**
  - **transitivity of strict preferences: for every individual i and all alternatives a, b and c, aP_ib and bP_ic implies aP_ic**
  - **transitivity of indifference: aI_ib and bI_ic implies aI_ic**
  - **transitivity of weak preference: aR_ib means aP_ib or aI_ib; aR_ib and bR_ic implies aR_ic**

- **rationality: transitive and complete individual preference orders**
- **transitivity:**
  - **transitivity of strict preferences: for every individual i and all alternatives a, b and c, aP_ib and bP_ic implies aP_ic**
  - **transitivity of indifference: aI_ib and bI_ic implies aI_ic**
  - **transitivity of weak preference: aR_ib means aP_ib or aI_ib; aR_ib and bR_ic implies aR_ic**
- **completeness:**
  - **for every individual i and all alternatives a and b, aR_ib or bR_ia**
  - **incompleteness, which implies inability to choose, is not the same as indifference (Buridan's ass example)**

- **axiomatic social choice theory**
  - **state as axioms general properties of any collective choice procedure (or any collective preference procedure)**
  - **derive characteristic properties of any procedure that the axioms describe**
  - **this may include showing that some desirable properties are mutually incompatible**

- **example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
  - cP_iaP_ib
  - bP_icP_ia

- **example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
  - cP_iaP_ib
  - bP_icP_ia
- **consider majority rule as the collective choice procedure**
- **because majority rule considers only two alternatives at a time, the three alternatives must be considered in some order**
- **the order in which the votes occur is an agenda**

- **example: the paradox of voting**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
  - cP_iaP_ib
  - bP_icP_ia
- **consider three agendas with pairwise majority votes:**
  - a **versus** b**, then winner versus** c
  - a **versus** c**, then winner versus** b
  - b **versus** c**, then winner versus** a

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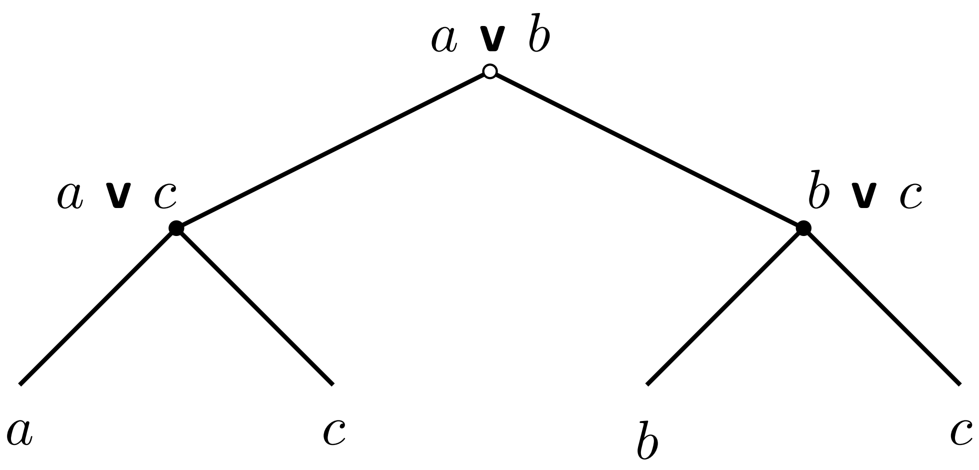
- **consider three agendas with pairwise majority votes:**
  - a **versus** b**, then winner versus** c
  - a **versus** c**, then winner versus** b
  - b **versus** c**, then winner versus** a
- **the outcome depends on the agenda**

- consider three voters with the following preferences:
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- consider three agendas with pairwise majority votes and sophisticated voting:
- with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently

- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **consider three agendas with pairwise majority votes and sophisticated voting:**
- **with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently**
- **this is strategic voting only if each voter's vote depends on the voter's beliefs about everyone else's preferences**
- **implicitly there is usually a "common knowledge" assumption that implies (among other things) that those beliefs are not systematically wrong**

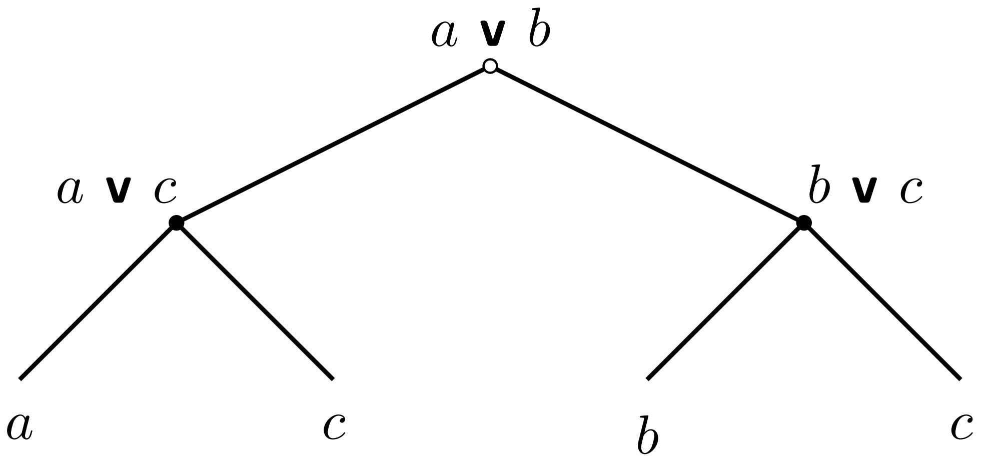
- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **pairwise majority votes and sophisticated voting:**
- **everyone knows everyone votes sincerely in the last stage**
  - a **versus** b**, then winner versus** c **(tree diagrams help)**

Sophisticated Voting

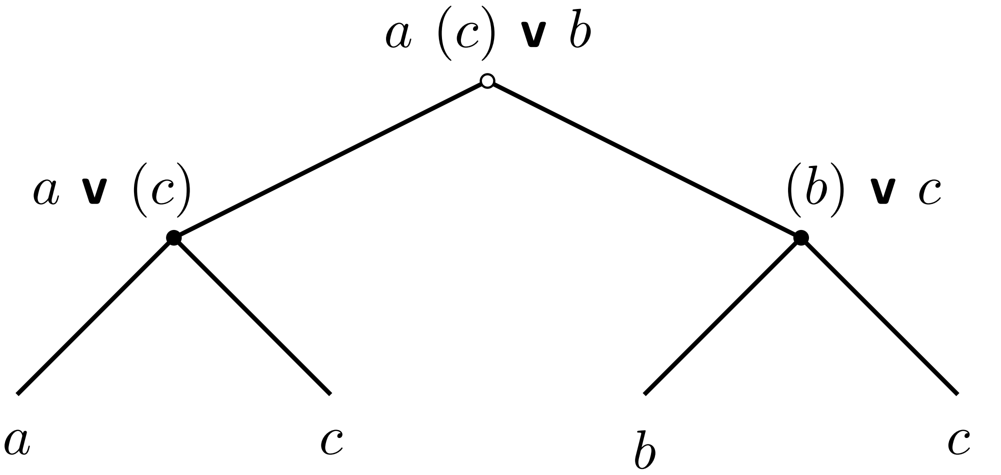


- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **with sincere voting,** a **beats** b**, then** c **beats** a
- **everyone knows everyone votes sincerely in the last stage**
- **so everyone knows** cPa **and** bPc

Sophisticated Voting (Backward Induction)



- **because everyone knows cPa and bPc, everyone knows the situation really is**



- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **pairwise majority votes and sophisticated voting:**
- **everyone knows everyone votes sincerely in the last stage**
  - a **versus** b**, then winner versus** c
    * **the first vote is really** c **versus** b**, because** cPa
    * **so the winner is** b **(voter 1 votes for** b **not** a**)**

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    - **the first vote is really** c **versus** b**, because** cPa
    - **so the winner is** b **(voter 1 votes for** b **not** a**)**
  - a **versus** c**, then winner versus** b
    - **the first vote is really** a **versus** b**, because** bPc
    - **so the winner is** a **(voter 2 votes for** a **not** c**)**

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    * **so the winner is** b **(voter 1 votes for** b **not** a**)**
  - a **versus** c**, then winner versus** b
    * **the first vote is really** a **versus** b**, because** bPc
    * **so the winner is** a **(voter 2 votes for** a **not** c**)**
  - b **versus** c**, then winner versus** a
    * **the first vote is really** c **versus** a**, because** aPb
    * **so the winner is** c **(voter 3 votes for** c **not** b**)**

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    - **the winner is** c

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  - b **versus** c**, then winner versus** a
    - **the winner is** c
- **the outcome depends on the agenda**

- **examples of voting rules**
  - **majority rule**
  - **plurality voting with one winner (winner take all, first past the post)**
  - **plurality voting with** $M > 1$
  - **approval voting**
  - **Borda count**
  - **utilitarian systems**

- examples of voting rules
  - majority rule
  - plurality voting with one winner (winner take all, first past the post)
  - plurality voting with $M > 1$
  - approval voting
  - Borda count
  - utilitarian systems
- there is extensive theory for single districts, little for whole systems

- **heresthetic: changing outcomes without changing preferences**

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- **heresthetic versus rhetoric (i.e., manipulation versus persuasion)**
  - **every individual's preference ordering remaining the same is the definition of no persuasion**
  - **changing people's beliefs about what the decision is is not considered persuasion**
  - **likewise, changing beliefs about what's at stake in a decision, or about the consequences of a decision, is not considered persuasion**

- **heresthetic: three basic kinds of manipulation**
  - **strategic voting**
  - **agenda control**
  - **dimension manipulation**

- **heresthetic: stories grouped by type manipulation (according to Riker)**
  - **strategic voting: Pliny (7); Massachusetts and SC delegates (8); Chicester (9); anti-school aid Republicans (11); Chrystal (5); Morris (4)**
  - **agenda control: Plott and Levine (3); Pliny (7); Reed and Norris (12)**
  - **dimension manipulation: Lincoln (1); DePew (2); Chrystal (5); city manager (6); Magnuson (10); Morris (4)**

- **dimension manipulation illustration: Warren Magnuson**
- **alternatives**
  - o: **leave shells of gas in Okinawa**
  - a: **bring nerve gas to Alaska**
  - G: **detoxify gas "outside the U.S." (Gravel amendment)**
  - d: **support defense department**
  - n: **support Alaskan interests**
  - s: **support constitutional authority of the Senate**

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- **distinguish actions (for or against G) from implications of actions: preferences refer to implications**
  - **before Magnuson maneuver: for G means (o, n); against G means (a, d)**
  - **after Magnuson maneuver: for G means (o, n, s) (to some senators); against G means (a, d)**

- **dimension manipulation illustration: Warren Magnuson**
- **dimension manipulation is covert agenda control (sort of)**
  - **but what about Byrd and Thurmond, who "were unmoved by Magnuson's heresthetic"? likewise the "Church-Cooper supporters"?**
  - **Magnuson's implicit "amendment" via trying to redefine G only partially succeeded; not everyone saw it the same way**
  - **("common knowledge" seems to be lacking)**

- dimension manipulation illustration: Warren Magnuson
- different facts? (Mackie)
  - in Mackie's version the original Magnuson amendment is still on the table and is the actual reversion point if G fails
  - so Mackie says Riker has the legislative situation wrong

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  - **hence according to Mackie there is**
    - w: **no nerve gas "to the United States"**
    - **before Magnuson maneuver: for G means (o, n); against G means (d, w)**
    - **after Magnuson maneuver: the same; s was irrelevant**

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    - **before Magnuson maneuver: for G means (o, n); against G means (d, w)**
    - **after Magnuson maneuver: the same; s was irrelevant**
  - **Mackie says Thurmond and Byrd cared about textiles, not s**
  - **moreover s was not about Senate prerogatives but just a tiff with the defense department**

- **axiomatic social choice theory**
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- **how to generalize from the Condorcet paradox example? Of what is it an example?**
- **examples of properties of social choice rules**
  - **choice from all subsets**
  - **unrestricted preferences**

- **examples of properties of social choice rules**
  - **unanimity: if everyone has identical preferences, including aP_ib, then a is the sole choice from $\{a, b\}$**
  - **weak Pareto: if for everyone aP_ib, then a is the sole choice from $\{a, b\}$ regardless of what their other preferences are**
  - **strong Pareto: if one person has aP_ib and no one prefers b to a, then a is the sole choice from $\{a, b\}$**

- **examples of properties of social choice rules**
  - **independence: the social choice from a set of alternatives depends only on the preferences individuals have about the alternatives in that set**
  - **pairwise independence: the social choice from $\{a, b\}$ depends only on individuals' preferences between a and b**

- **majority voting satisfies pairwise independence**

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- **first-past-the-post (winner is the one with the most votes) does not (Craven's version)**
- **preferences showing first-past-the-post violates independence: choose from $\{b, c\}$ while allowing votes to be cast for any of $\{a, b, c\}$**

individuals	**I**	**II**
1	bP_1cP_1a	aP_1bP_1c
2–10	aP_icP_ib	cP_ibP_ia

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- **this may not be as crazy as it may appear: let a denote abstention**
- **(but, note, "voting" here is not behavioral)**

- **the Borda count does not satisfy independence**
- **preferences showing the Borda count violates independence:**

individuals	I	II
1,2	aP_ibP_ic	aP_icP_ib
3,4	bP_iaP_ic	bP_iaP_ic
5	cP_5bP_5a	cP_5bP_5a

- **the preferences regarding a and b are not changed, but b is chosen in I while a is chosen in II**

- **what properties describe majority rule?**
- **majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence**

- what properties describe majority rule?
- majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence
- majority rule is the only social choice rule that satisfies independence, positive responsiveness, symmetry and anonymity
  - positive responsiveness: if there are two preference profiles R_i and R_i' such that $aP_i'b$ whenever aP_ib and $aR_i'b$ whenever aI_ib, then if $C(a,b) = \{a\}$ with preferences R_i then $C(a,b) = \{a\}$ with preferences R_i'
  - anonymity (exchanging two voters' preferences does not change the social choice)
  - symmetry (exchanging a and b in everyone's preferences means a and b are exchanged in the social choice)

- **an undesirable property: dictatorship**
- **dictatorship: if** $C_i(a, T) = a$**, then** $C(a, T) = a$**, and if** $C_i(U, T) = U$ **then** $C(U, T) \subset U$

- **other desirable properties**
  - **collective rationality**
    - **RC1: if** $C(a, b) = \{a\}$ **and** $C(b, c) = \{b\}$ **then** $C(a, c) = \{a\}$
    - **RC2: if** $C(a, b) = \{a, b\}$ **and** $C(b, c) = \{b, c\}$ **then** $C(a, c) = \{a, c\}$
    - **RC3: if** a **is in** $C(T)$ **and** b **is in** T**, then** a **is in** $C(a, b)$
    - **RC4: if** a **is in** $C(a, b)$ **and** a **is in** $C(T)$**, then** a **is in** $C_(T \cup b)$

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- **majority rule fails collective rationality, since it produces voting cycles**

- **recall that we presume**
  - **individual rationality**
    - **RC1: if** $C_i(a,b) = \{a\}$ **and** $C_i(b,c) = \{b\}$ **then** $C_i(a,c) = \{a\}$
    - **RC2: if** $C_i(a,b) = \{a,b\}$ **and** $C_i(b,c) = \{b,c\}$ **then** $C_i(a,c) = \{a,c\}$
    - **RC3: if** a **is in** $C_i(T)$ **and** b **is in** T**, then** a **is in** $C_i(a,b)$
    - **RC4: if** a **is in** $C_i(a,b)$ **and** a **is in** $C_i(T)$**, then** a **is in** $C_(T \cup b)$

- **Arrow's theorem**
- **generalizes the conclusions about majority rule, almost universally**

- **Arrow's theorem (Vickrey variant)**
- **Theorem 3.3: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  pairwise independence

  weak Pareto

  and if the social choices satisfy rationality conditions RC1 to RC4

- Arrow's theorem (Vickrey variant)
- proof in two steps
  1. epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives
  2. the dictator: show that the fully decisive set can contain only one individual

- **epidemic**
- **set D of individuals is semidecisive for a over b if $C(a,b) = \{a\}$ when aP_ib for everyone in D but bP_ia for everyone else**
- **assume D is semidecisive for a over b**

- **epidemic**
- **set D of individuals is semidecisive for a over b if $C(a,b)=\{a\}$ when aP_ib for everyone in D but bP_ia for everyone else**
- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **epidemic**
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- **unrestricted preferences make all these profiles relevant**
- **pairwise independence means these profiles can be considered in isolation from the rest of the preference orders, which may vary within each set of individuals**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

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in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
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in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **I: $C(a,b) = \{a\}$ (semidecisive assumption); $C(b,c) = \{b\}$ (weak Pareto); $C(a,c) = \{a\}$ (RC1); D is semidecisive for a over c (definition)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **I:** $C(a,b) = \{a\}$ **(semidecisive assumption);** $C(b,c) = \{b\}$ **(weak Pareto);** $C(a,c) = \{a\}$ **(RC1);** D **is semidecisive for** a **over** c **(definition)**
- **II:** $C(a,b) = \{a\}$ **(semidecisive assumption);** $C(d,a) = \{d\}$ **(weak Pareto);** $C(d,b) = \{d\}$ **(RC1);** D **is semidecisive for** d **over** b **(definition)**

- **combining and generalizing examples I and II gives**
- **SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y**
- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**

- combining and generalizing examples I and II gives
- SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y
- SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y
- combining and generalizing examples I and II gives
- SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b

- **combining and generalizing examples I and II gives**
- **SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y**
- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**
- **combining and generalizing examples I and II gives**
- **SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b**
- **SD4: if D is semidecisive for a over b, then SD2 implies that D is semidecisive for a over any alternative y**

- **combining and generalizing examples I and II gives**
- **SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y**
- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**
- **combining and generalizing examples I and II gives**
- **SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b**
- **SD4: if D is semidecisive for a over b, then SD2 implies that D is semidecisive for a over any alternative y**
- **SD5: if D is semidecisive for a over b, then SD3 and SD4 imply that D is semidecisive for any x over any y (except for b over a)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **III:** $C(b,c)=\{b\}$ **(SD5);** $C(a,c)=\{c\}$ **(weak Pareto);** $C(a,b)=\{b\}$ **(RC1);** D **is semidecisive for** b **over** a **(definition)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
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- **III:** $C(b,c) = \{b\}$ **(SD5);** $C(a,c) = \{c\}$ **(weak Pareto);** $C(a,b) = \{b\}$ **(RC1);** D **is semidecisive for** b **over** a **(definition)**
- **if D is semidecisive for a over b, then D is semidecisive for any x over any y**

- **semidecisiveness implies decisiveness**
- D **is a decisive set of individuals if for any alternatives** x **and** y**,** $C(x, y) = \{x\}$ **whenever** xP_iy **for everyone in** D **whatever preferences others have**

- **semidecisiveness implies decisiveness**
- **assume D is semidecisive (i.e., for any x over any y)**

- **semidecisiveness implies decisiveness**
- **assume D is semidecisive (i.e., for any x over any y)**
- **preferences demonstrating decisiveness:**

individuals

in D	dP_ieP_ic
in E	eP_idP_ic
in F	eP_icP_id
rest	eP_icI_id

- $C(d,e)=\{d\}$ **(semidecisiveness); $C(c,e)=\{e\}$ (weak Pareto); $C(c,d)=\{d\}$ (RC1); D is decisive for d over c (definition)**

- extension to decisiveness over choices from sets larger than pairs
- assume D is a decisive set
- assume everyone in D has aP_ib for all other $b \in T$
- then $C(a, b) = \{a\}$ for all $b \in T$

- **extension to decisiveness over choices from sets larger than pairs**

- **assume** D **is a decisive set**

- **assume everyone in** D **has** aP_ib **for all other** $b \in T$

- **then** $C(a,b) = \{a\}$ **for all** $b \in T$

- **let** $b \neq a$ **and** $b \in T$ **and suppose** $b \in C(T)$**; then** $b \in C(a,b)$ **(RC3), which contradicts** $C(a,b) = \{a\}$**; therefore** $b \notin C(T)$**; therefore** $C(T) = \{a\}$

- **extension to decisiveness over choices from sets larger than pairs**
- **assume** D **is a decisive set**
- **assume everyone in** D **has** aP_ib **for all other** $b \in T$
- **then** $C(a, b) = \{a\}$ **for all** $b \in T$
- **let** $b \neq a$ **and** $b \in T$ **and suppose** $b \in C(T)$**; then** $b \in C(a, b)$ **(RC3), which contradicts** $C(a, b) = \{a\}$**; therefore** $b \notin C(T)$**; therefore** $C(T) = \{a\}$
- $C(T) = \{a\}$ **matches the preference of the members of** D

- **extension to decisiveness over choices from sets larger than pairs**

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- $C(T) = \{a\}$ **matches the preference of the members of** D

- **a set** D **is fully decisive if** D **can ensure that the most preferred alternative of all individuals in** D **is the social choice**

- **Arrow's theorem (Vickrey variant)**
- **proof in two steps**
  1. **epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives**
  2. **the dictator: show that the fully decisive set can contain only one individual**

- **the dictator**
- **the collective**

  i. **a collective is the smallest fully decisive set**

  ii. **all sets that include all members of the collective are fully decisive**

  iii. **no set that does not include all members of the collective is fully decisive**

  iv. **there is only one collective**

- **the collective**
- **intersecting fully decisive sets:** D **and** E **are fully decisive sets that have members in common**

in D **and in** E	bP_icP_ia
in D**, not in** E	aP_ibP_ic
in E**, not in** D	cP_iaP_ib
rest	aP_icP_ib

- **the collective**
- **intersecting fully decisive sets:** D **and** E **are fully decisive sets that have members in common**

in D **and in** E	bP_icP_ia
in D**, not in** E	aP_ibP_ic
in E**, not in** D	cP_iaP_ib
rest	aP_icP_ib

- $C(b,c)=\{b\}$ **(**D **decisive);** $C(a,c)=\{c\}$ **(**E **decisive);** $C(a,b)=\{b\}$ **(RC1);** $D\cap E$ **is semidecisive for** b **over** a**; therefore** $D\cap E$ **is a fully decisive set**

- the dictator
- suppose the collective contains two or more members
- let some but not all collective members be in E
- these assumptions lead to a contradiction

  in E $\quad aP_icP_ib$

  rest $\quad cP_ibP_ia$

- $C(a,b) = \{a,b\}$ (neither subset is semidecisive); $C(a,c) = \{a,c\}$ (neither subset is semidecisive); $C(b,c) = \{b,c\}$ (RC2); but $C(b,c) = \{b,c\}$ violates weak Pareto

- **the dictator**
- **suppose the collective contains two or more members**
- **let some but not all collective members be in** E
- **these assumptions lead to a contradiction**

  in E $\quad aP_icP_ib$

  rest $\quad cP_ibP_ia$
- $C(a,b) = \{a,b\}$ **(neither subset is semidecisive);** $C(a,c) = \{a,c\}$ **(neither subset is semidecisive);** $C(b,c) = \{b,c\}$ **(RC2); but** $C(b,c) = \{b,c\}$ **violates weak Pareto**
- **if only one subset contains the collective, there is no contradiction**
- **the collective is impossible to divide only if has only one member**

- **Arrow's theorem (as proven)**
- **Theorem 3.7: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  pairwise independence

  weak Pareto

  and if the social choices satisfy rationality conditions RC1, RC2 and RC3

- **manipulation**

- **manipulation**
- **a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences**

- **manipulation**
- **a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences**
- **a rule is non-manipulable if it is not open to manipulation by any individual on any set of alternatives**

- **Gibbard-Satterthwaite theorem (variant)**
- **Theorem 5.4: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  weak Pareto

  non-manipulability

  and if the social choices satisfy rationality conditions RC1 to RC4

- Riker and Arrow (heresthetic and the general impossibility theorem)
- Arrow's Theorem (variant): There is a dictator if there are at least three alternatives and the social choice rule satisfies: choice from all subsets; unrestricted preferences; pairwise independence; weak Pareto; and if the social choices satisfy rationality conditions RC1, RC2 and RC3
- heresthetic is motivated by the basic principles: avoid dictatorship; ensure that collective choices reflect everyone's preferences; have collective choices be effective

- **Riker and Arrow (heresthetic and the general impossibility theorem)**
- **Arrow's Theorem (variant): There is a dictator if there are at least three alternatives and the social choice rule satisfies: choice from all subsets; unrestricted preferences; pairwise independence; weak Pareto; and if the social choices satisfy rationality conditions RC1, RC2 and RC3**
- **heresthetic is motivated by the basic principles: avoid dictatorship; ensure that collective choices reflect everyone's preferences; have collective choices be effective**
- **agenda control: follows from giving up transitivity**
- **dimension manipulation: follows from giving up independence**
- **strategic voting: follows from giving up independence, via nonmanipulability**

- **restrictions on preferences**
  - **preference restrictions can avoid majority rule voting cycles (see Craven chapter 6)**
  - **recall that majority voting is not manipulable if there are no cycles**
  - **but falsely stating preferences can create a cycle even when no cycle exists in true preferences**

- **potentially manipulable preferences (Table 6.6):**

individuals	**number**	**true**	**false**
E	n_1	aP_ibP_ic	$aP'_ibP'_ic$
F	n_2	bP_iaP_ic	$bP'_iaP'_ic$
G	n_3	bP_icP_ia	$bP'_icP'_ia$
H	n_4	cP_ibP_ia	$cP'_iaP'_ib$

- **if** $n_1 > n_2 + n_3 + n_4$**, then** aPb **and** aPc **(no manipulation)**

- **if** $n_2 + n_3 + n_4 > n_1$**,** $n_1 + n_4 > n_2 + n_3$ **and** $n_3 + n_4 > n_1 + n_2$**, then** bPc**,** bPc **and** cPa **but** $aP'b$**,** $bP'c$ **and** $cP'a$

- **restrictions on preferences**
- **the (one-dimensional) spatial model**

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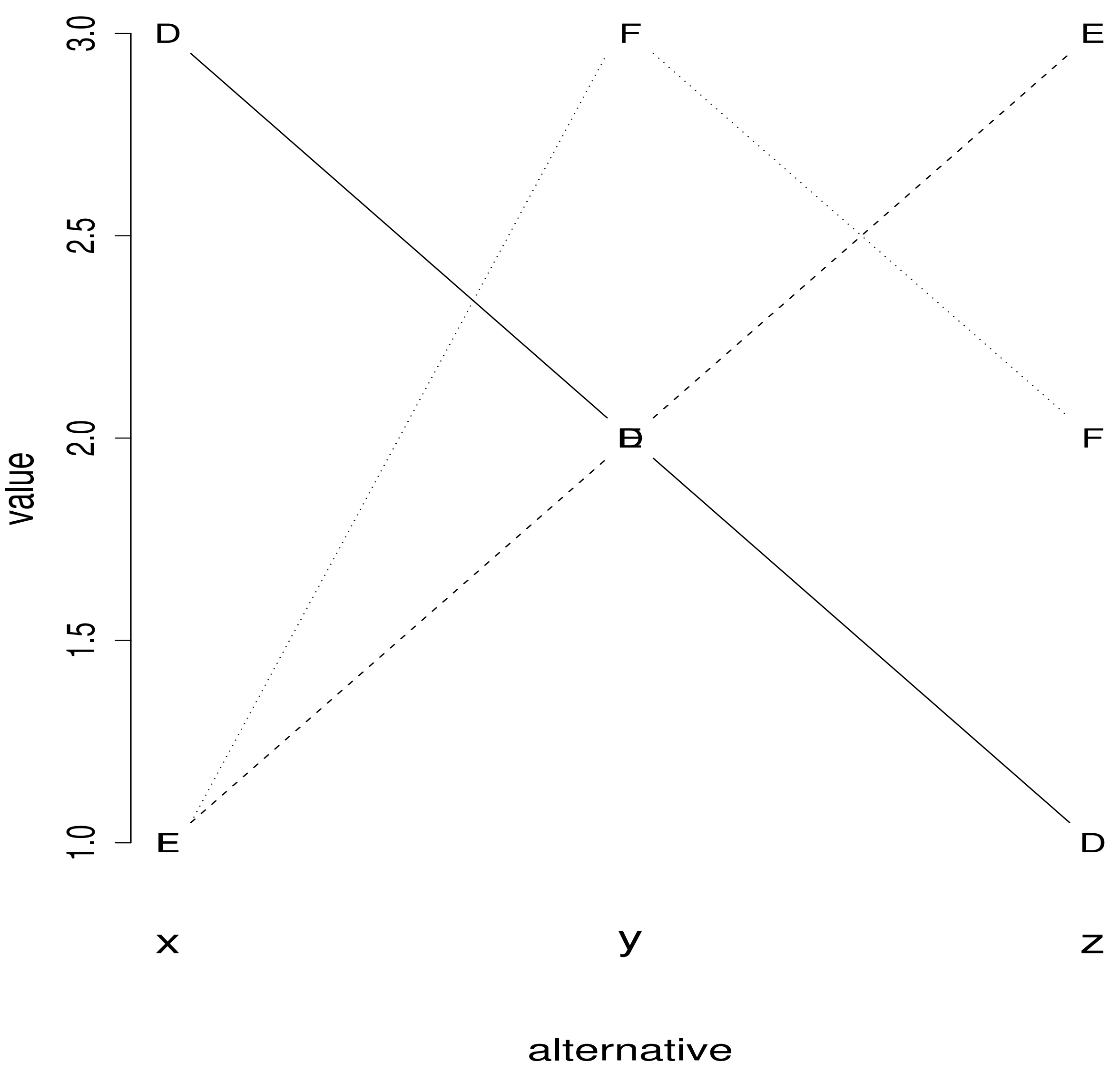
- **the (one-dimensional) spatial model**
  - **are preferences single peaked?**

- **restrictions on preferences**
- **the (one-dimensional) spatial model**
  - **are preferences single peaked?**
  - **example: YES**
  - **preferences:**
    - **person D:** xP_iyP_iz
    - **person E:** zP_iyP_ix
    - **person F:** yP_izP_ix

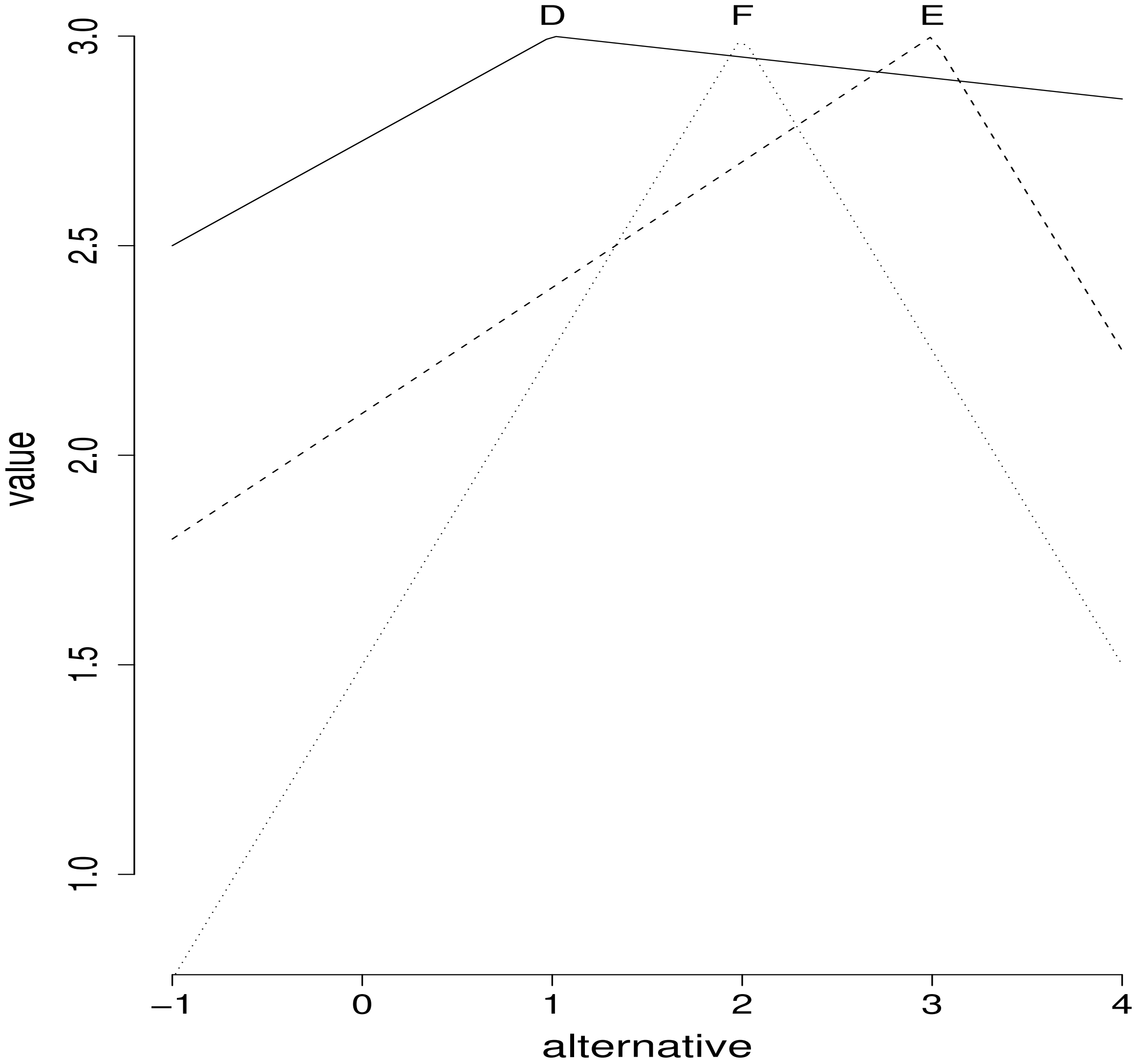
- **restrictions on preferences**
- **the (one-dimensional) spatial model**
  - **are preferences single peaked?**
  - **example: YES**
  - **preferences:**
    - **person D:** xP_iyP_iz
    - **person E:** zP_iyP_ix
    - **person F:** yP_izP_ix
  - **one alternative is never ranked last**
  - **each person has an IDEAL POINT**



value
3.0
2.5
2.0
1.5
1.0
D
F
E
B
F
E
D
x
y
z
alternative

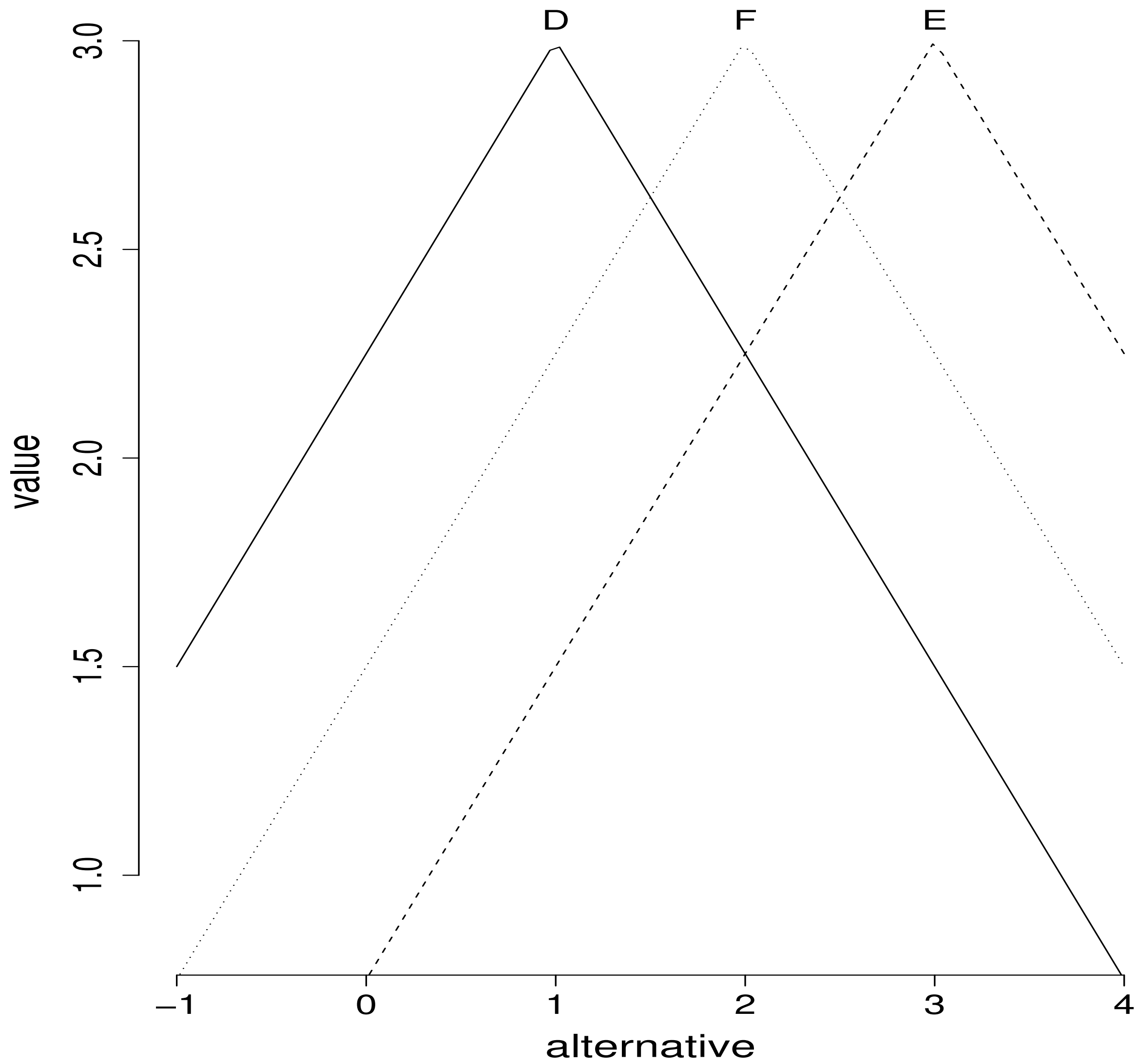
- **the spatial model**
- **single-peaked preferences**

- **the spatial model**
- **single-peaked preferences**
- **which alternative wins in a series of pairwise votes?**


D
F
E
value
3.0
2.5
2.0
1.5
1.0
−1
0
1
2
3
4
alternative

- **the spatial model**
- **single-peaked preferences**
- **which alternative wins in a series of pairwise votes?**
- **with single-peaked preferences, the median is the Condorcet winner**

- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **example with symmetry and same distances for different people**
  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$


D
F
E
value
3.0
2.5
2.0
1.5
1.0
−1
0
1
2
3
4
alternative

- **the spatial model**
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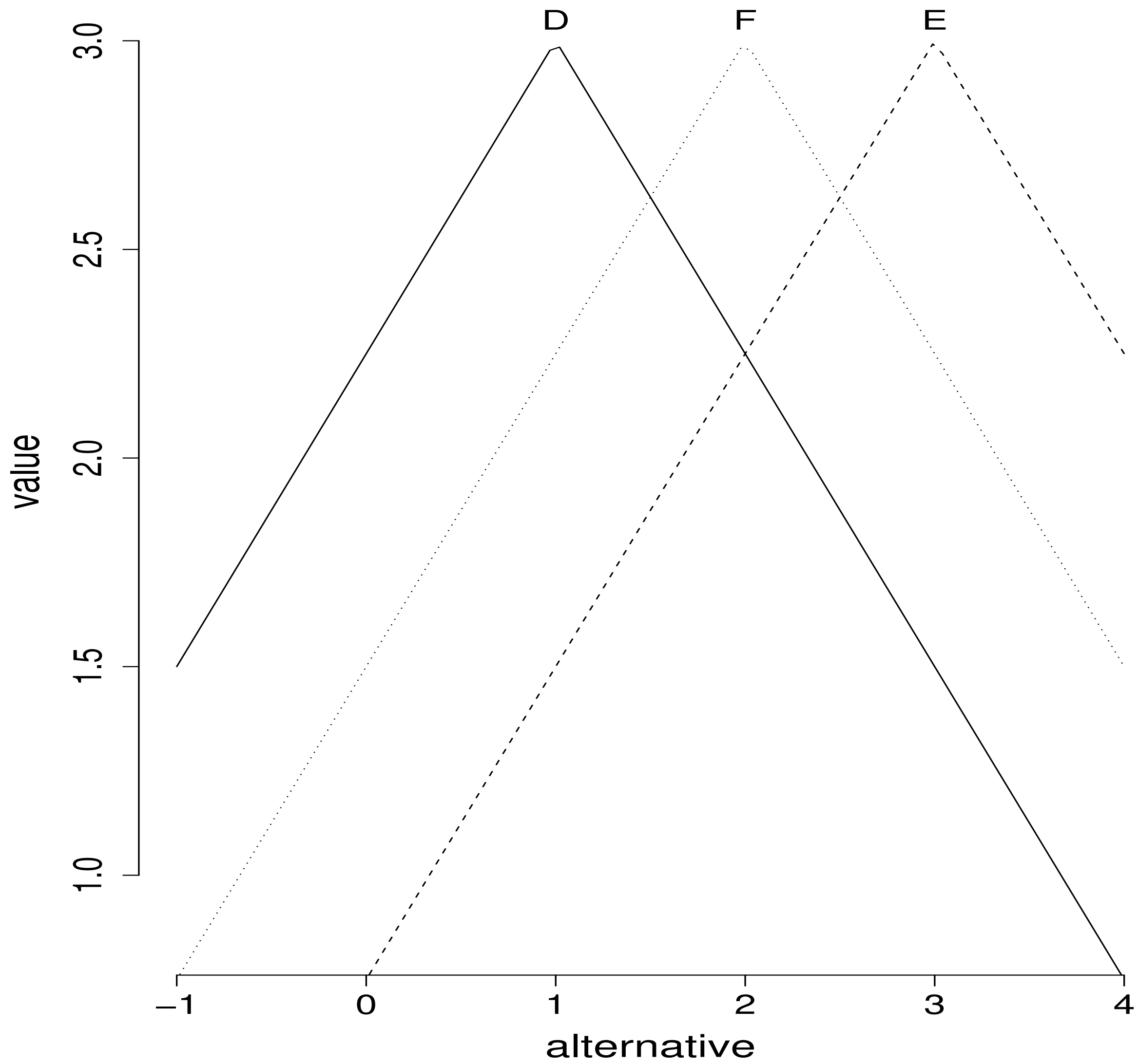
  - **example with symmetry and same distances for different people**
  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$
  - **apply an affine transformation:** $y_i = a + cx_i$

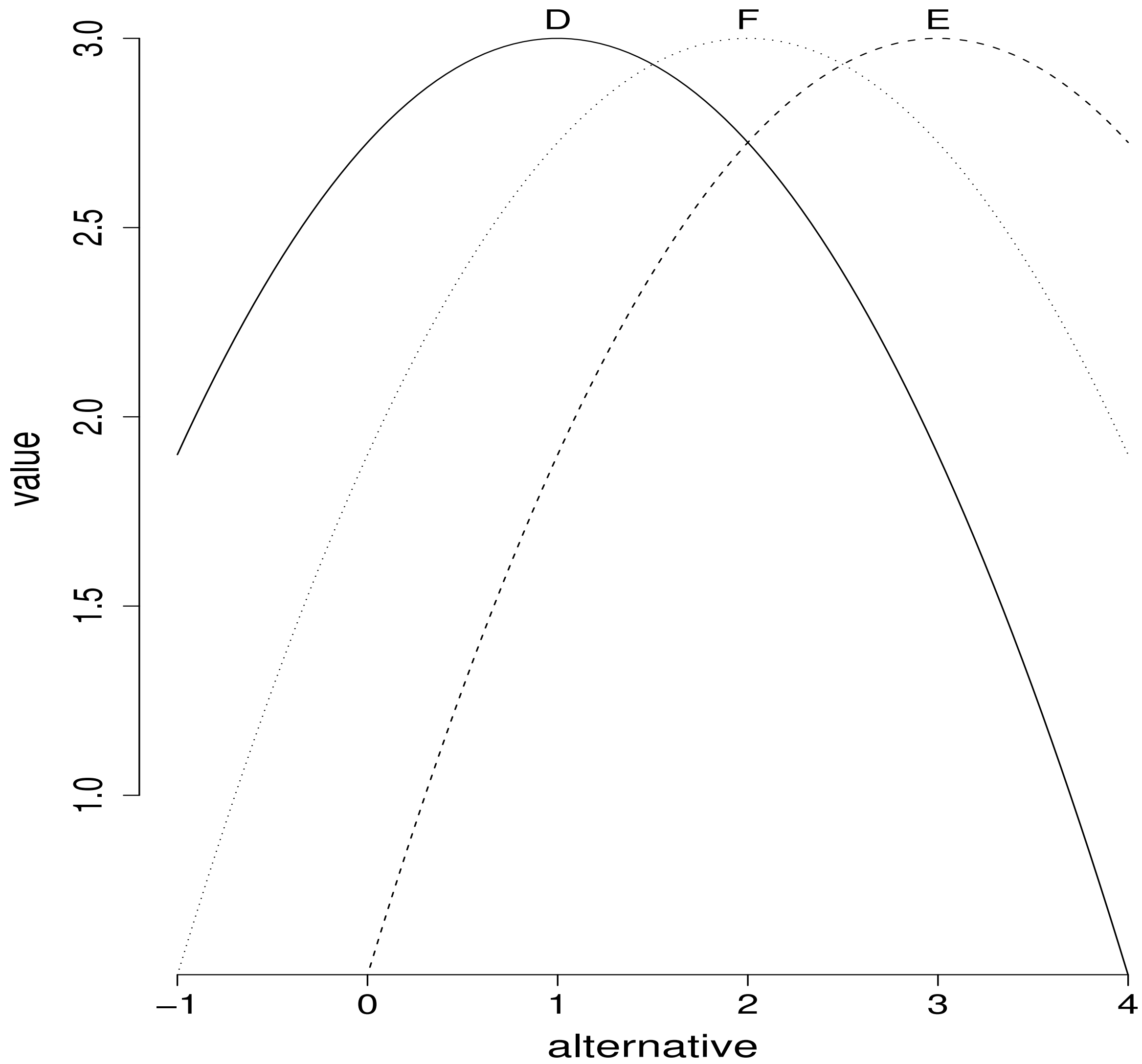
- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **example with symmetry and same distances for different people**
  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$
  - **apply an affine transformation:** $y_i = a + cx_i$

$$\begin{aligned} u_{ik} &= -b|y_k - y_i| \\ &= -b|a + cx_k - (a + cx_i)| \\ &= -bc|x_k - x_i| \end{aligned}$$

  - **the transformation changes the distance measure but not the shape of the preference curves: intensity doesn't matter**


D
F
E
value
3.0
2.5
2.0
1.5
1.0
−1
0
1
2
3
4
alternative

- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **euclidean metrics:** $u_{ik} = -b[(x_k - x_i)^2]^{1/2}$
  - **symmetry**
- **the median remains the Condorcet winner**


D
F
E
3.0
2.5
2.0
1.5
1.0
value
−1
0
1
2
3
4
alternative

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- **single-peaked preferences and measures of distance**
  - **euclidean metrics:** $u_{ik} = -b[(x_k - x_i)^2]^{1/2}$
  - **symmetry**

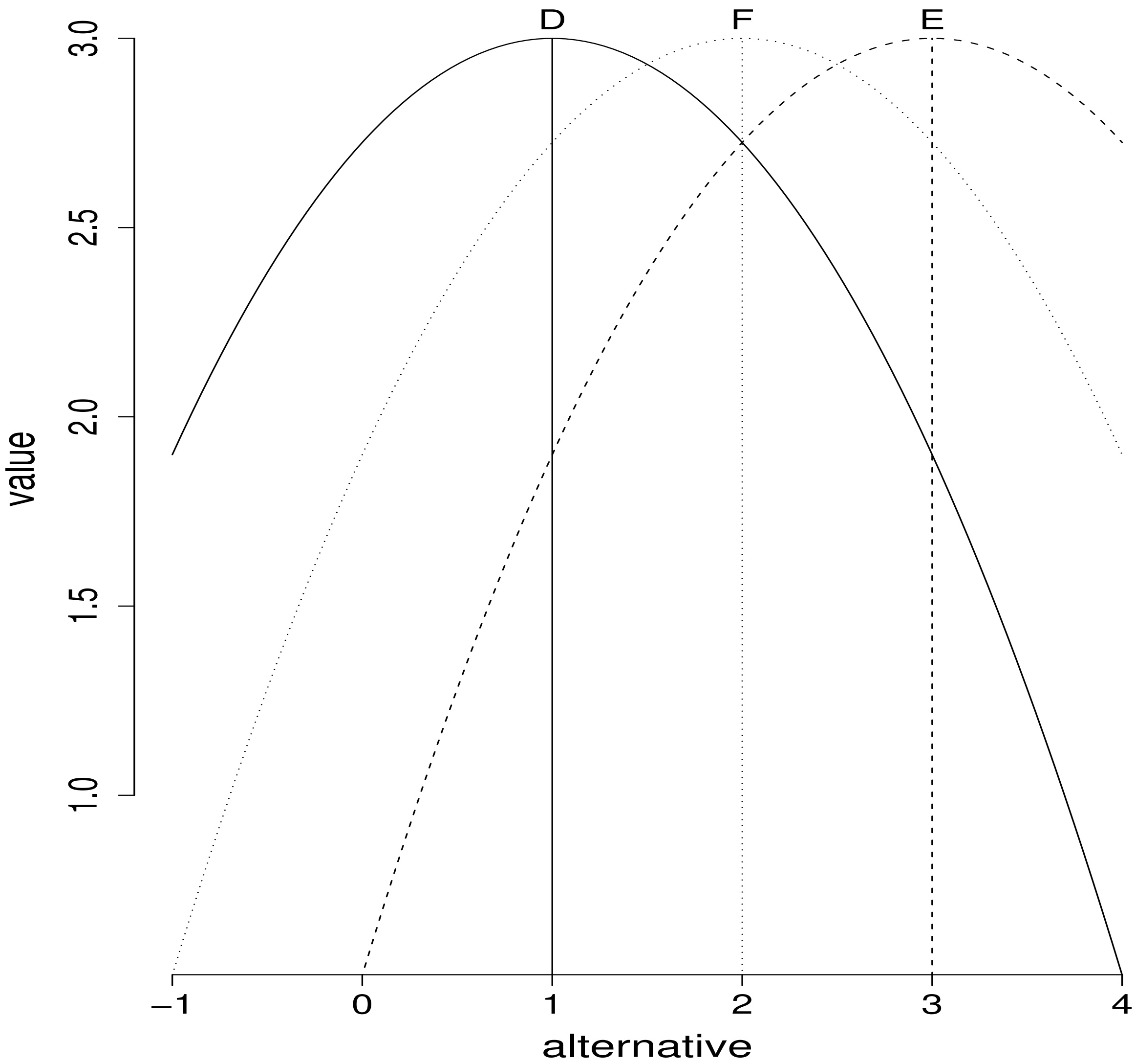
- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **euclidean metrics:** $u_{ik} = -b[(x_k - x_i)^2]^{1/2}$
  - **symmetry**
  - **apply an affine transformation:** $y_i = a + cx_i$**:**

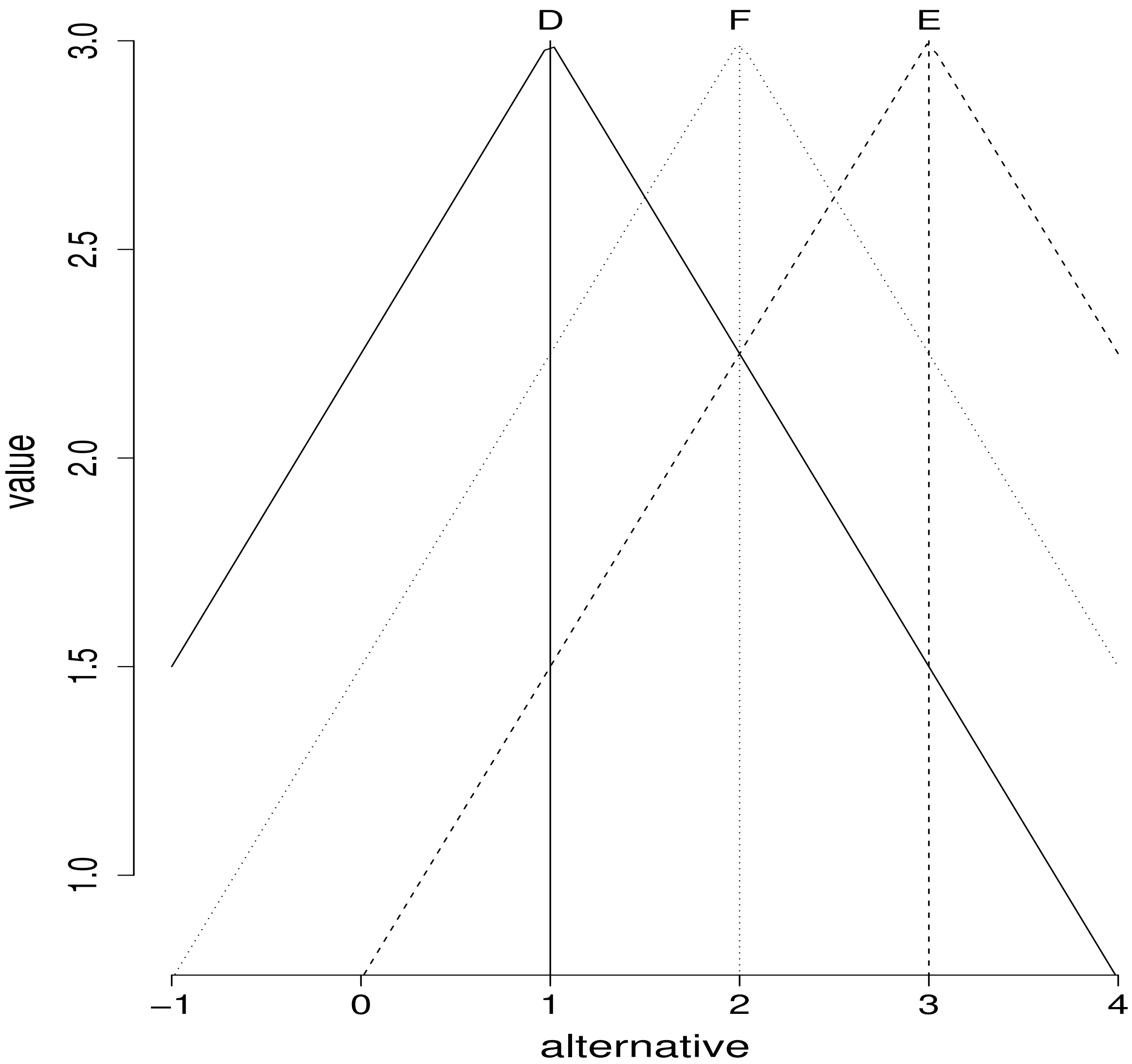
$$
\begin{aligned}
u_{ik} &= -b[(y_k - y_i)^2]^{1/2} \\
&= -b[(a + cx_k - (a + cx_i))^2]^{1/2} \\
&= -bc[(x_k - x_i)^2]^{1/2}
\end{aligned}
$$

  - **the transformation changes the distance measure but not the shape of the indifference curves**

- **the spatial model**
- **with symmetric single-peaked preferences**
- **which is to say, in a pure one-dimensional spatial model...**

- **the spatial model**
- **with symmetric single-peaked preferences**
- **which is to say, in a pure one-dimensional spatial model...**
- **all information about the chooser's preferences that is relevant for choice behavior is summarized by the chooser's ideal point**


D
F
E
value
3.0
2.5
2.0
1.5
1.0
−1
0
1
2
3
4
alternative



D
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2.5
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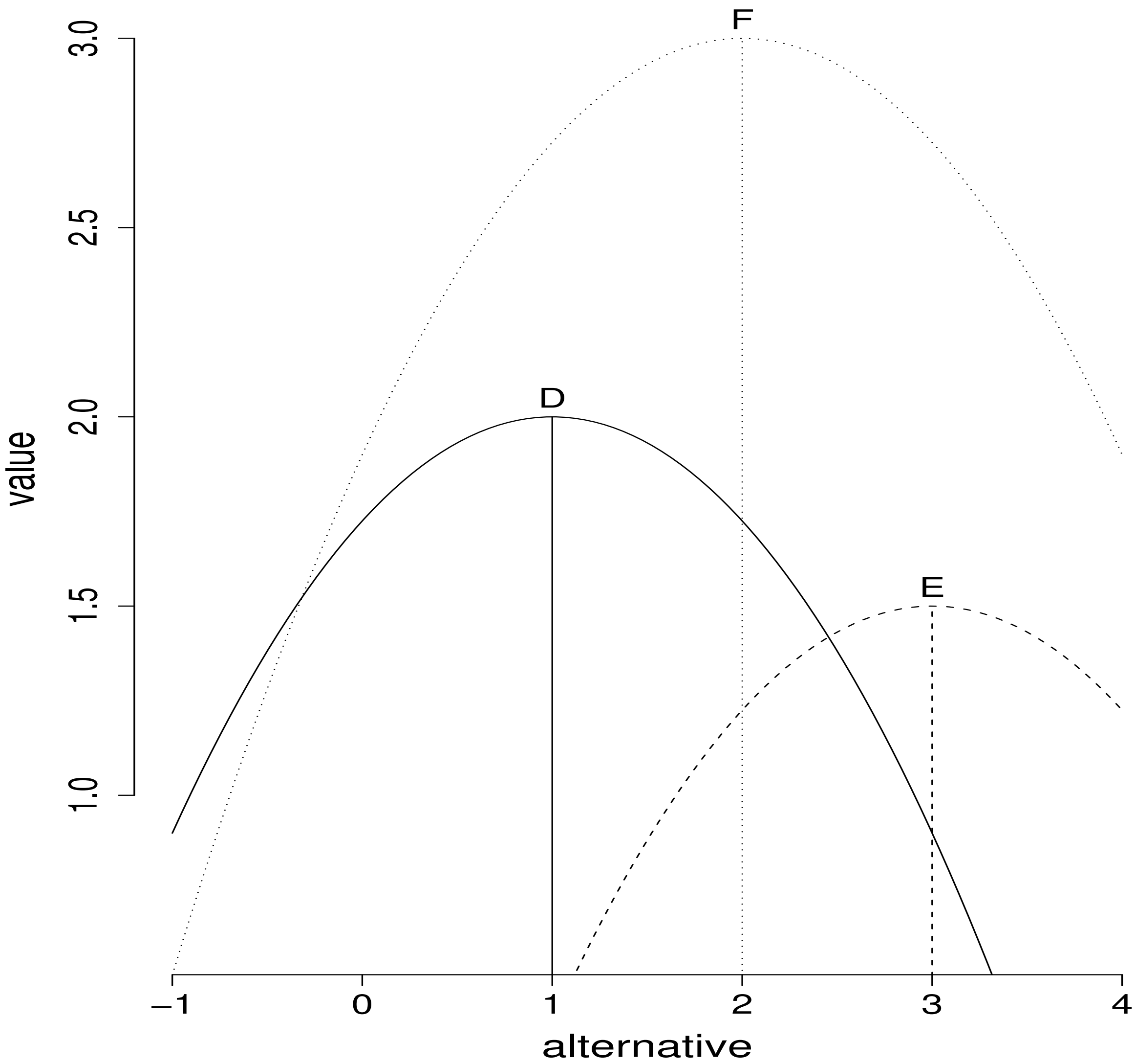
1.5
1.0
value
−1
0
1
2
3
4
alternative

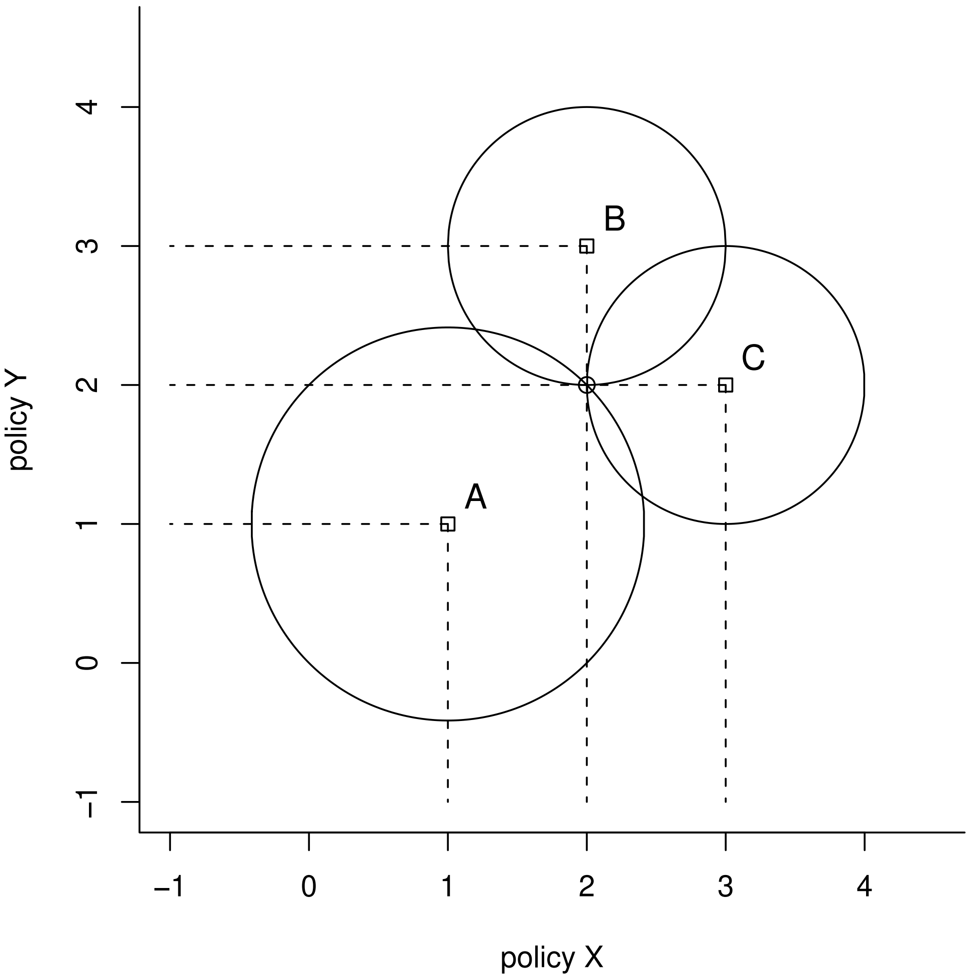

F
D
E
3.0
2.5
2.0
1.5
1.0
value
−1
0
1
2
3
4
alternative

- **a two-dimensional spatial model**
- **two-dimensional separable euclidean preferences:**

  $$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$$

- **an example with three voters**

Three Voters with Separable Euclidean Preferences



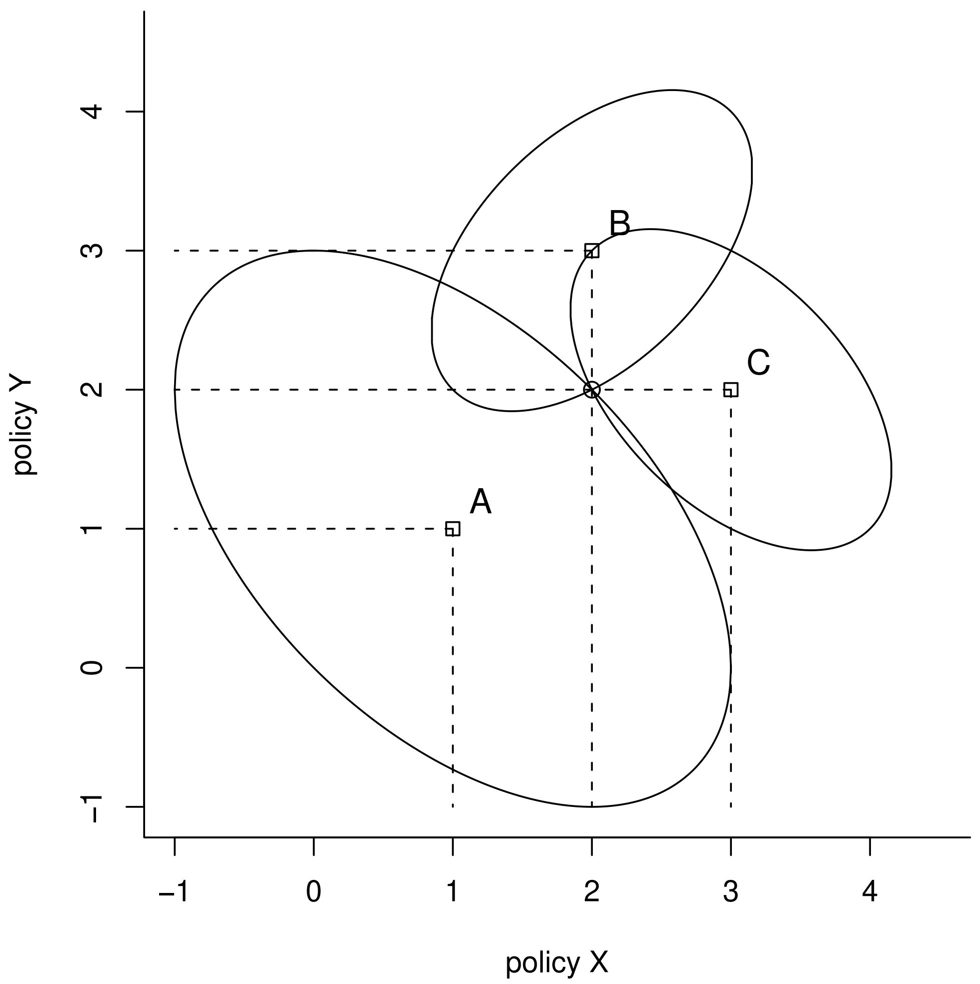
- **a two-dimensional spatial model**
- **two-dimensional separable euclidean preferences:**

  $u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$
- **with two-dimensional spatial preferences, in general the winset of any point** x **is not empty**
- **with separable preferences, the median on one dimension can be defeated by alternatives that shift along both dimensions**
- **with nonseparable preferences, a one-dimensional median is even more unstable**

- **two-dimensional nonseparable euclidean preferences:**

$$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$$

Three Voters with Nonseparable Euclidean Preferences



- **two-dimensional nonseparable euclidean preferences:**

$$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$$

- **with two-dimensional spatial preferences, whether separable or nonseparable, in general the winset of any point x is not empty**
- **with nonseparable preferences, the median on one dimension can be defeated even by alternatives that shift along the same dimension**

- **instability with multidimensional spatial preferences**
- **the chaos theorems**

- **instability with multidimensional spatial preferences**
- **the chaos theorems**
  - **there is a finite path of pairwise majority votes from any alternative to any other alternative, and back**
  - **power of the agenda setter given sincere voters**

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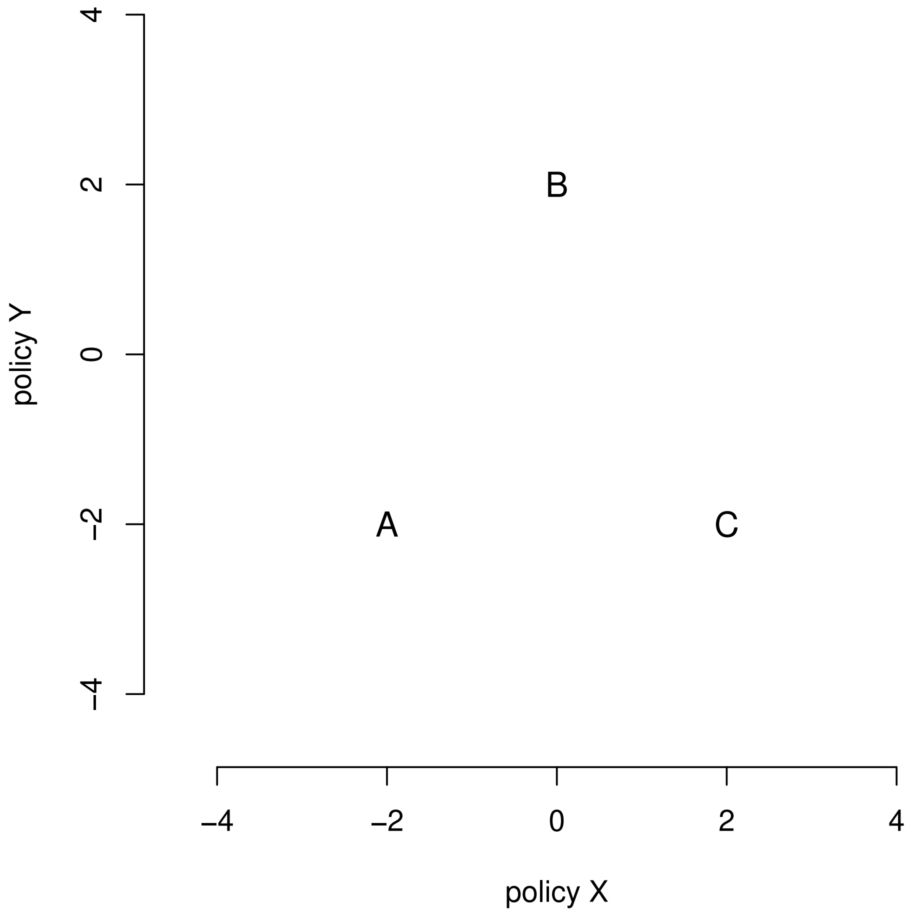
- **the chaos theorems**
  - **there is a finite path of pairwise majority votes from any alternative to any other alternative, and back**
  - **power of the agenda setter given sincere voters**
- **an example with three voters**

Three Voters



- **Pareto set:**
  - **the set of all points that are not unanimously inferior to any other point**
- **each point in the Pareto set**
  1. **is not unanimously inferior to any other point**
  2. **is unanimously superior to at least one other point**

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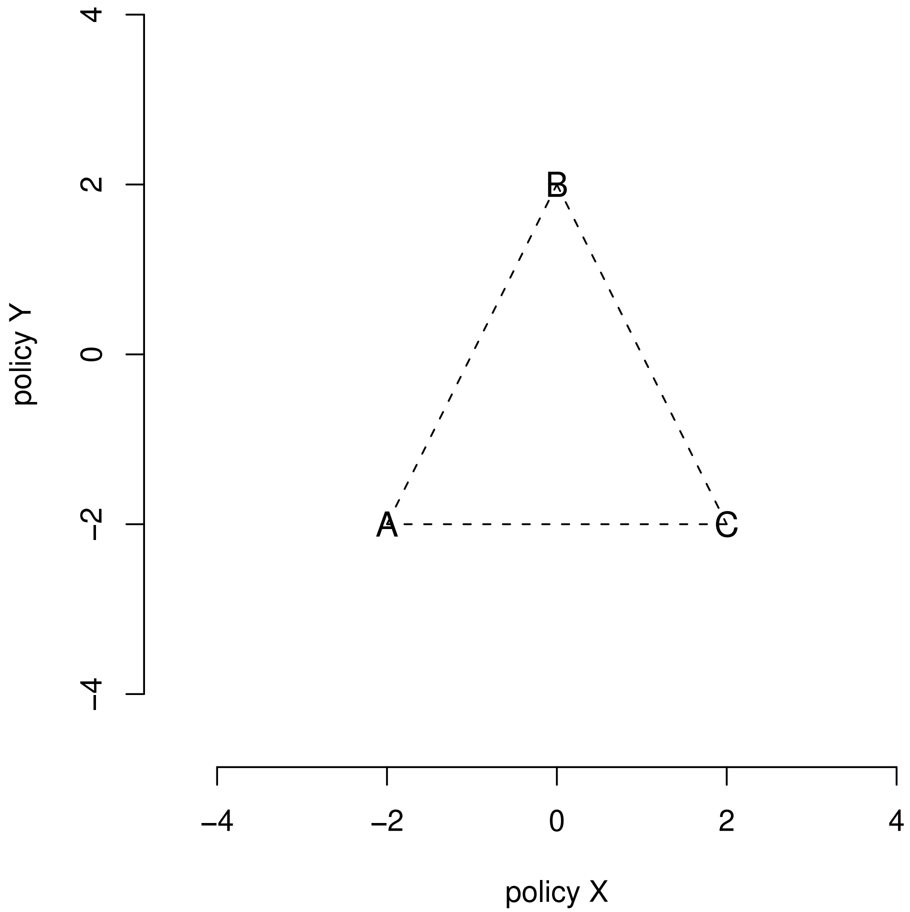
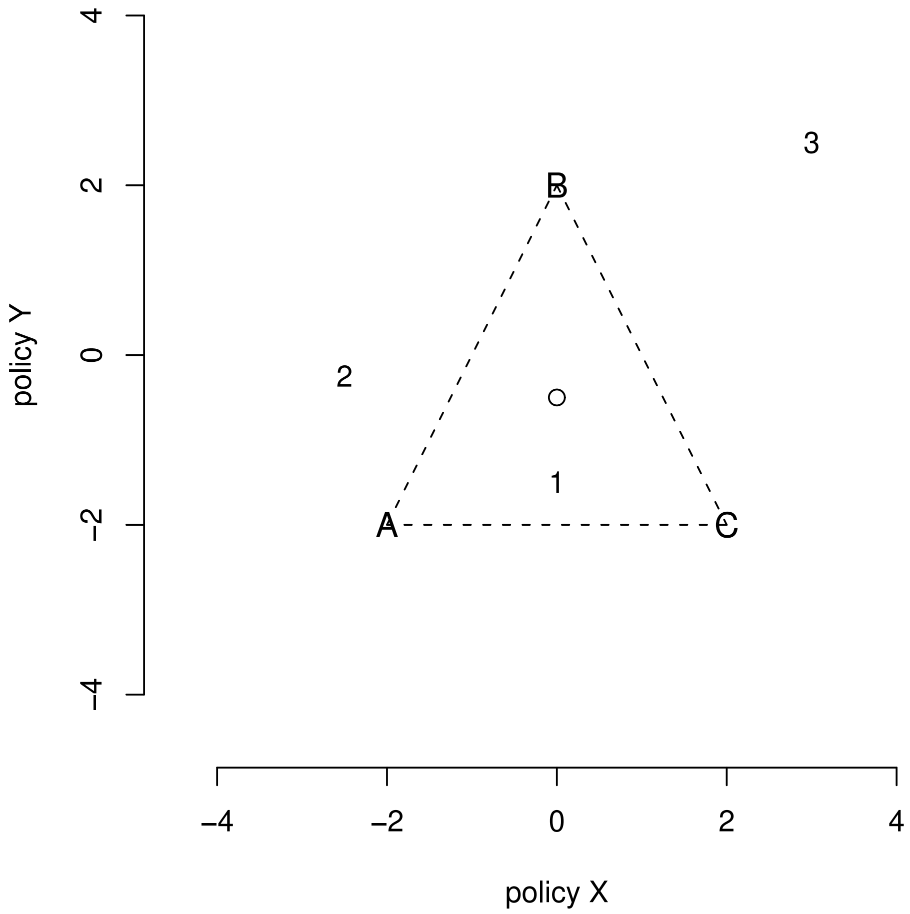
  - **the set of all points that are not unanimously inferior to any other point**
- **each point in the Pareto set**
  1. **is not unanimously inferior to any other point**
  2. **is unanimously superior to at least one other point**
- **with three voters, the boundaries of the Pareto set coincide with the median lines**
- **a median line connects two voters such that half (or half plus one for an odd number of voters) of the remaining voters are on either side of the line**

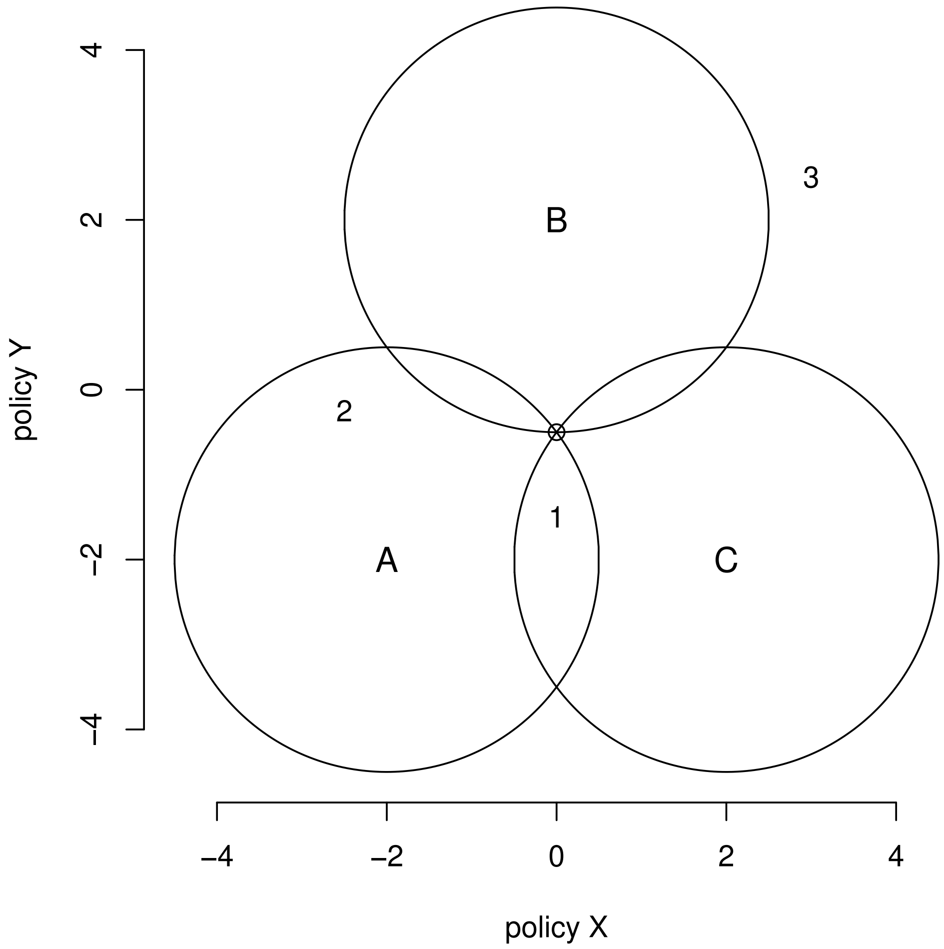
Three Voters with Pareto Set



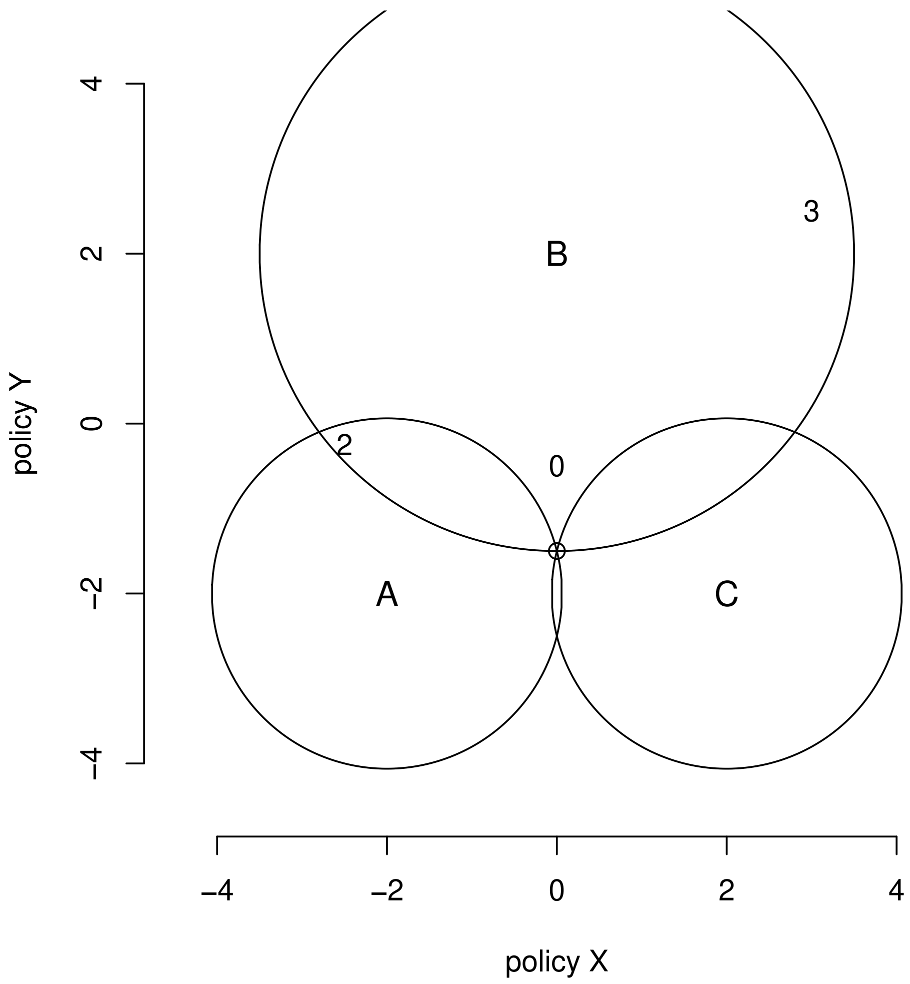
Three Voters Facing an Agenda



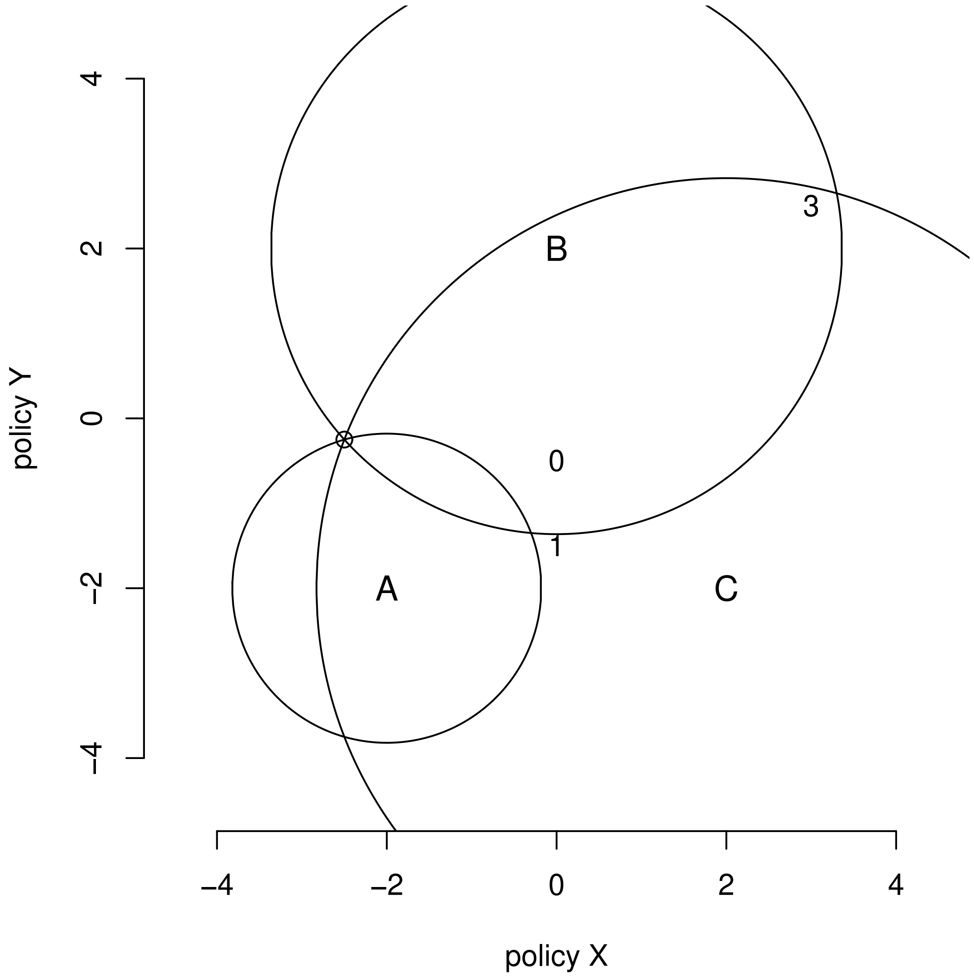
Indifference Curves At Status Quo

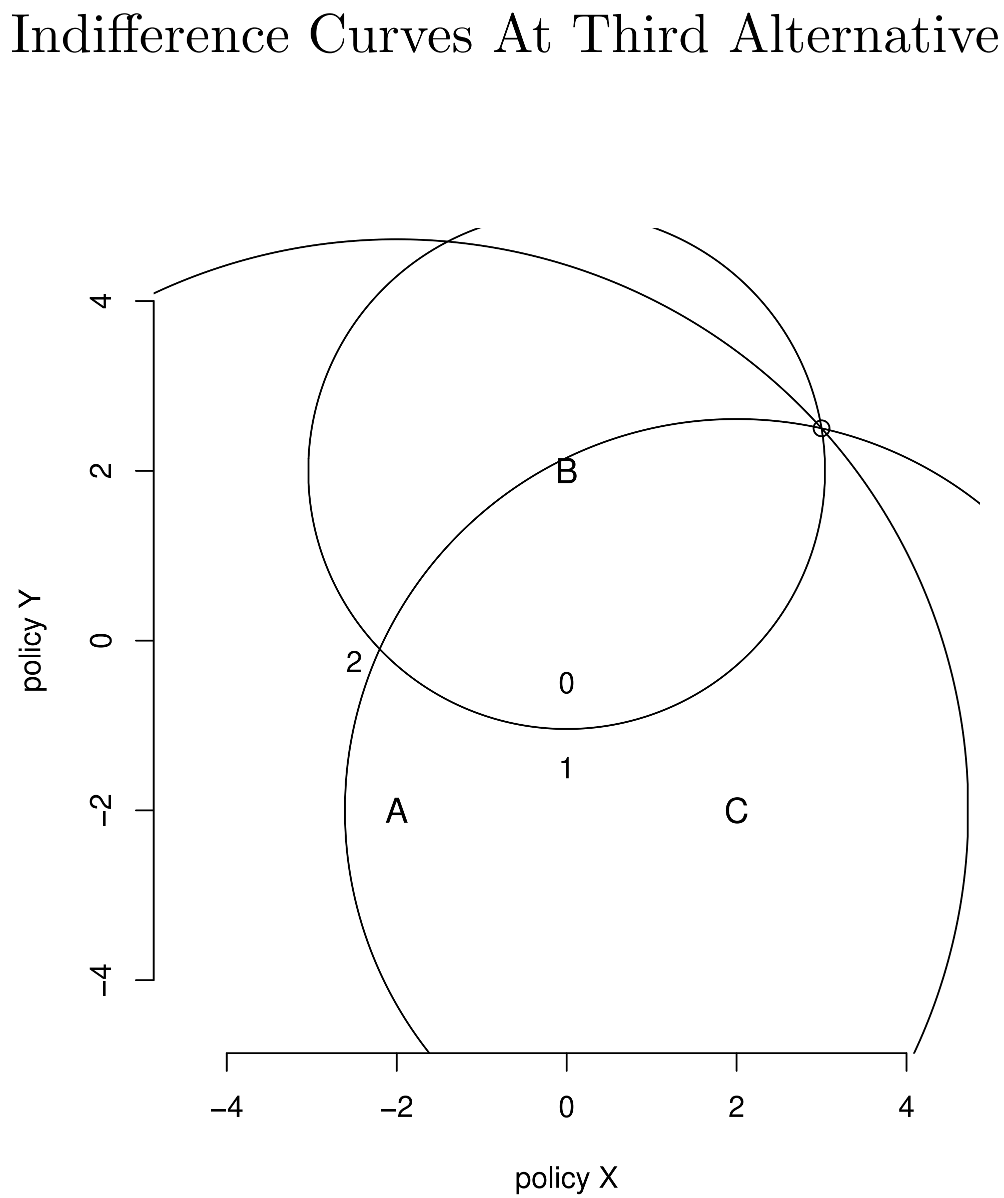


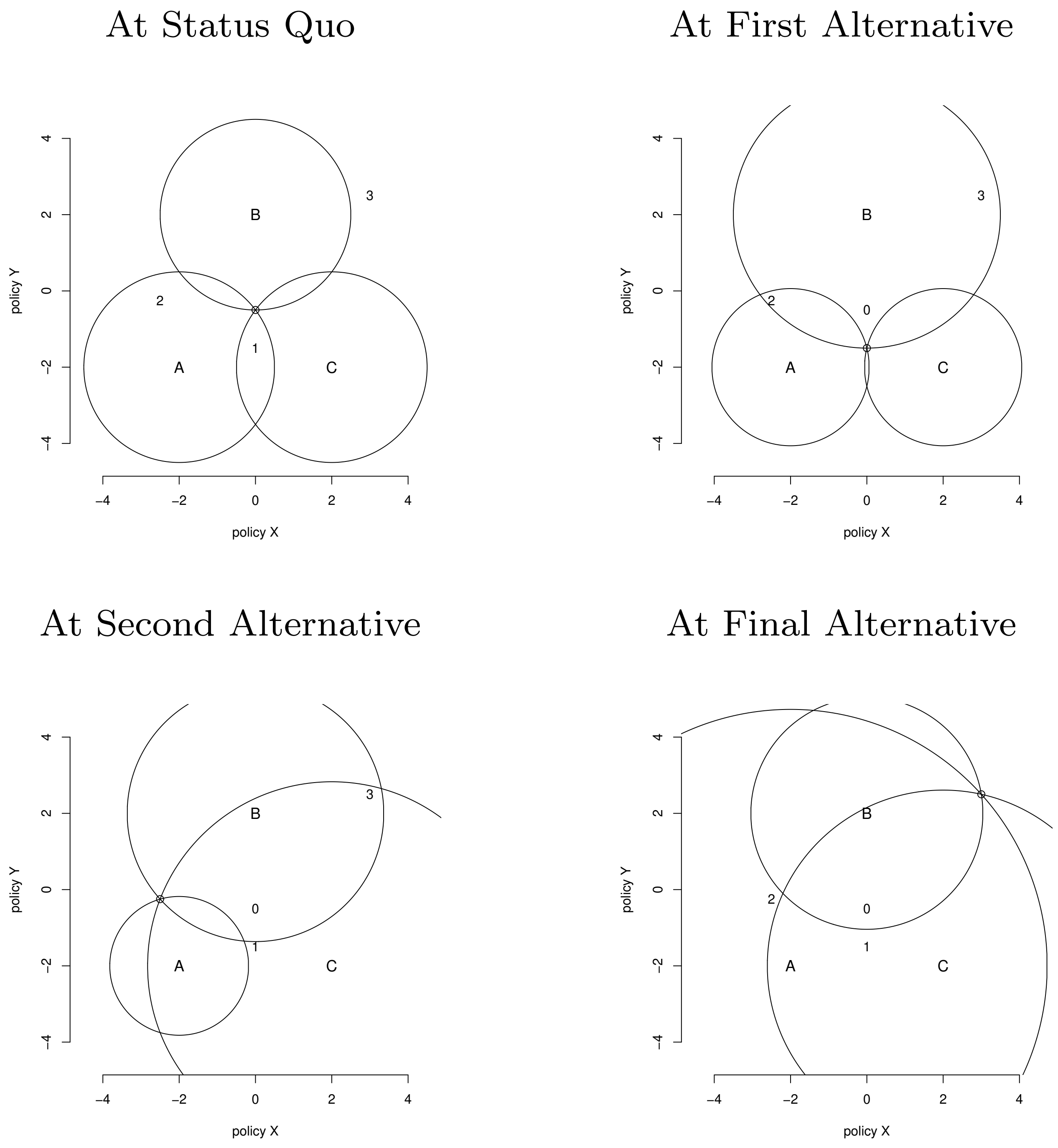
Indifference Curves At First Alternative



Indifference Curves At Second Alternative




Indifference Curves At Third Alternative
B
2
0
1
A
C
policy Y
4
2
0
−2
−4
−4
−2
0
2
4
policy X

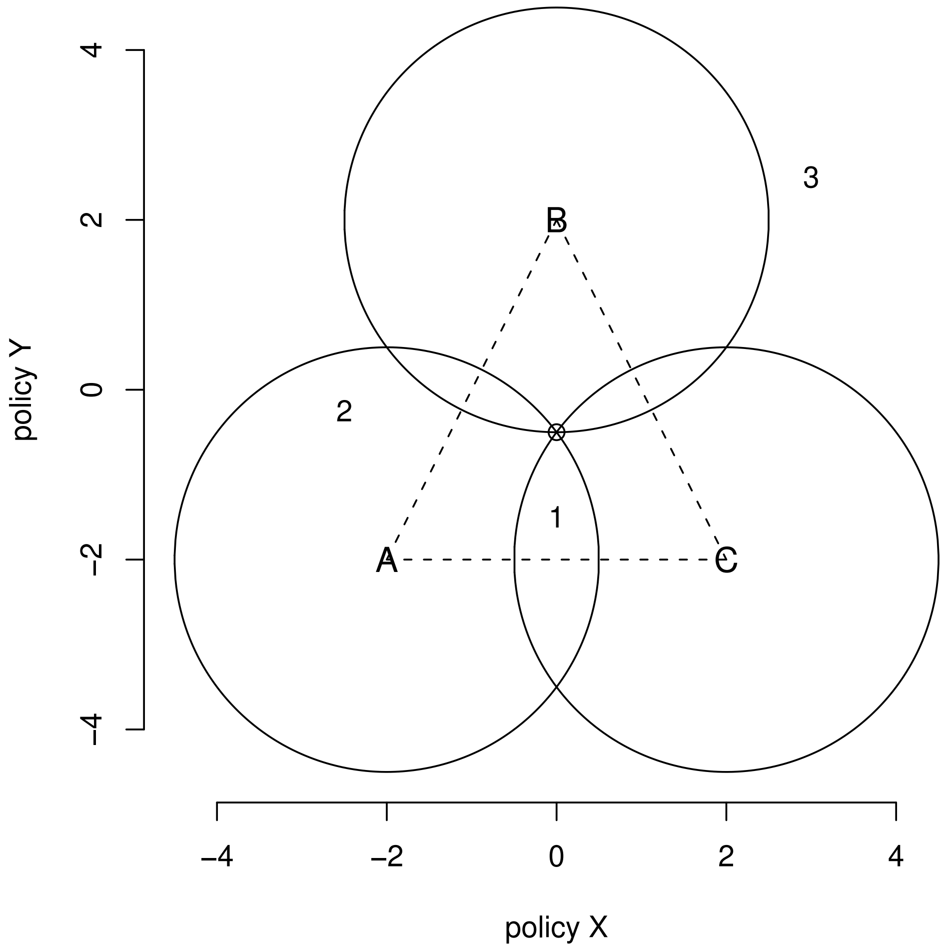

At Status Quo
At First Alternative
At Second Alternative
At Final Alternative
policy X
policy Y
A
B
C
0
1
2
3

- **instability with multidimensional spatial preferences**
- **the chaos theorems**
  - **there is a finite path of pairwise majority votes from any alternative to any other alternative, and back**
  - **power of the agenda setter given sincere voters**

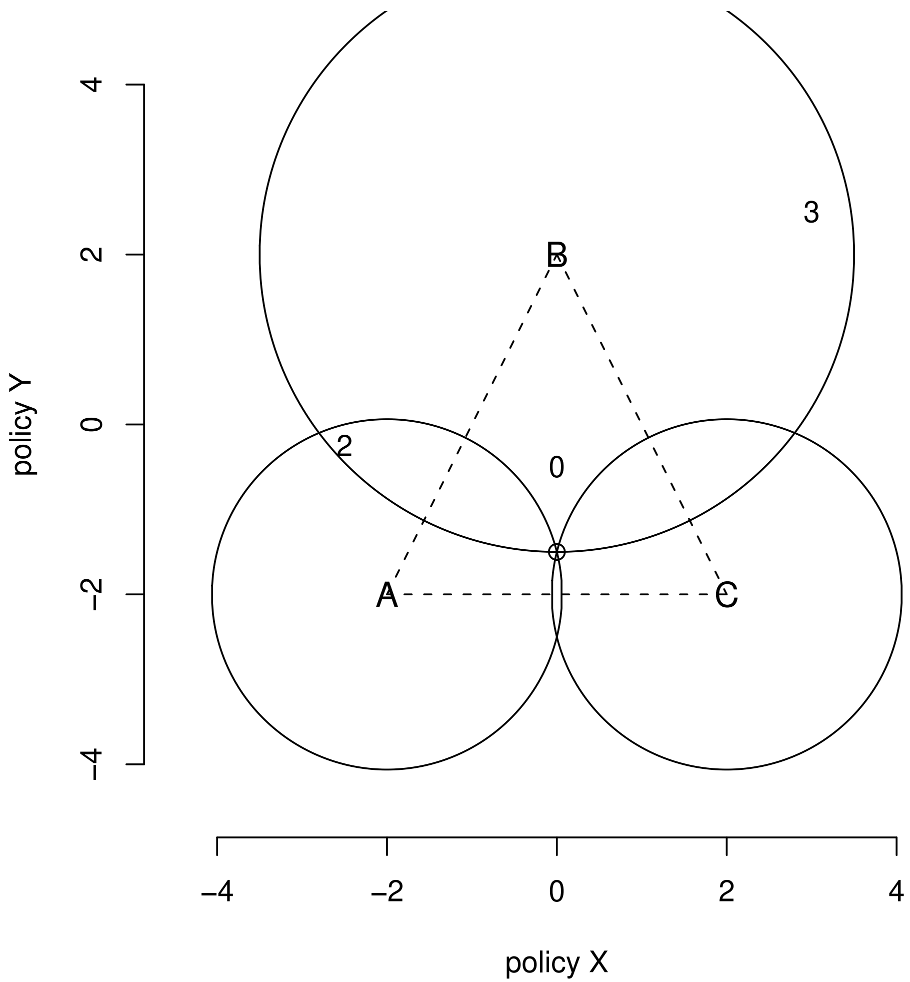
- instability with multidimensional spatial preferences
- the chaos theorems
  - there is a finite path of pairwise majority votes from any alternative to any other alternative, and back
  - power of the agenda setter given sincere voters
- chaos is not possible only if there are Condorcet winners, in other words, only if the core (the set of Condorcet winners) is not empty

- instability with multidimensional spatial preferences
- the chaos theorems
  - there is a finite path of pairwise majority votes from any alternative to any other alternative, and back
  - power of the agenda setter given sincere voters
- chaos is not possible only if there are Condorcet winners, in other words, only if the core (the set of Condorcet winners) is not empty
- under certain conditions on the type of agenda, strategic voters can keep outcomes inside the Pareto set

Indifference Curves At Status Quo, with Pareto Set



Indifference Curves At First Alternative, with Pareto Set



- **instability with multidimensional spatial preferences**
- **strategic voters can keep outcomes inside the Pareto set**
  - **technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"**

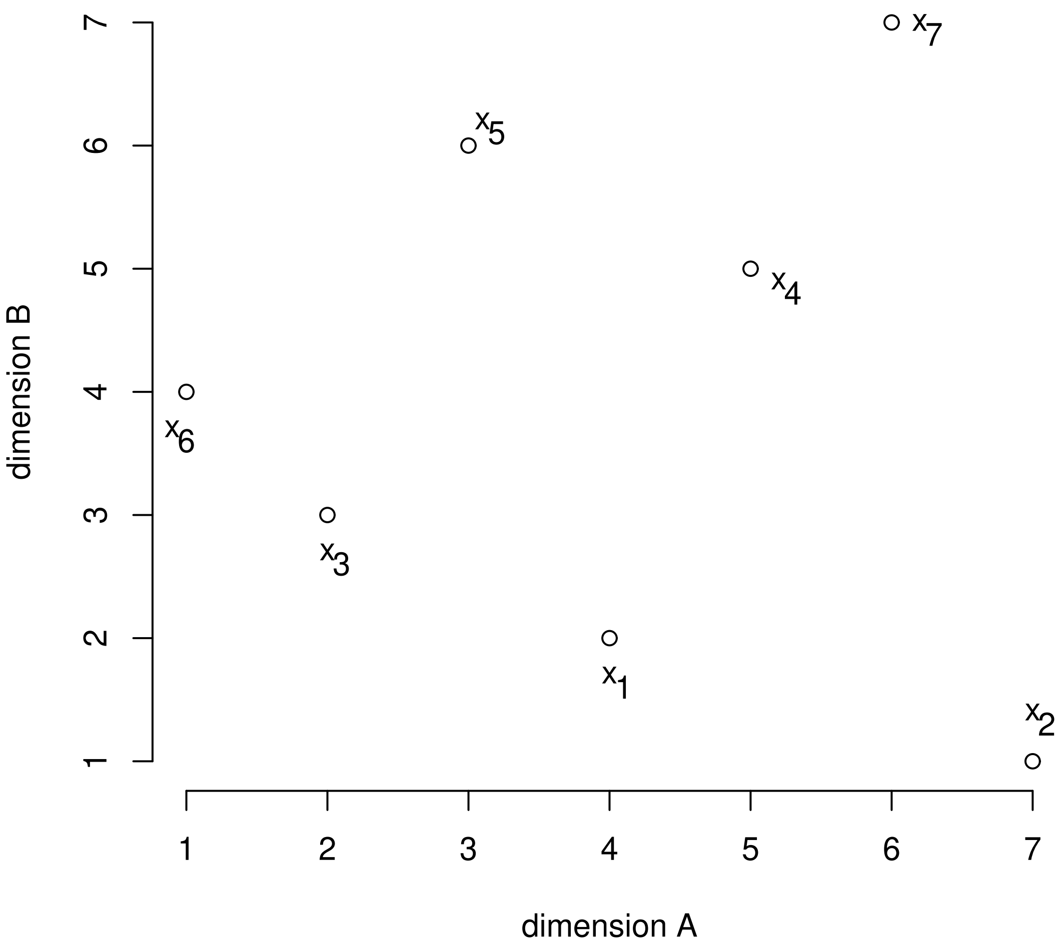
- **instability with multidimensional spatial preferences**
- **strategic voters can keep outcomes inside the Pareto set**
  - **technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"**
- **if the agenda is endogenous, which means the voters propose alternatives, then strategic voting can keep outcomes inside the uncovered set**

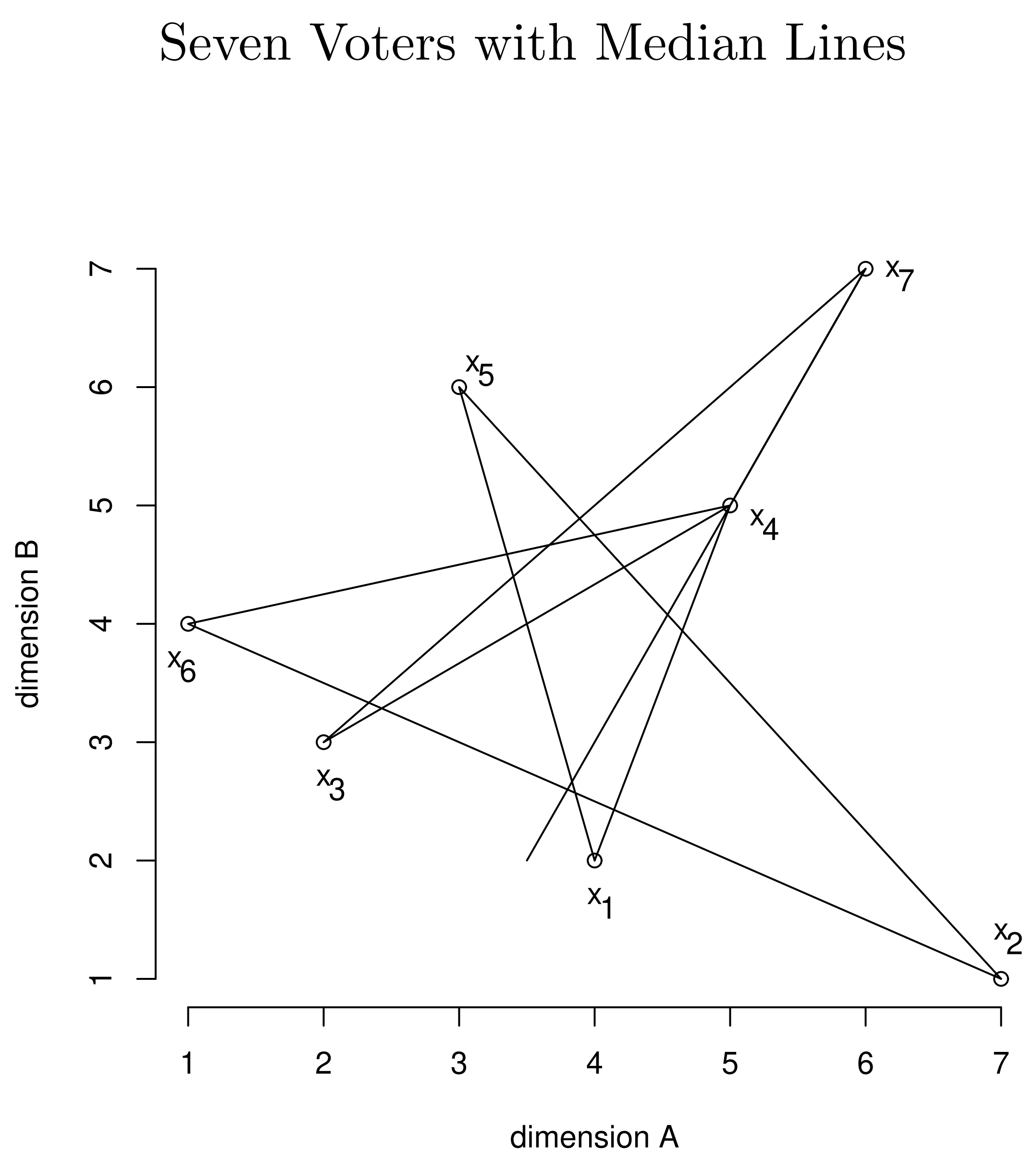
- the uncovered set
- McKelvey, Richard D. 1986. “Covering, Dominance, and Institution-Free Properties of Social Choice.” American Journal of Political Science 30: 283–314.
- x covers y, denoted xCy, if xPy and for all z such that zPx, zPy
- the uncovered set is the set of all alternatives w such xCw is false for all x

- **the uncovered set and the yolk (generalized median set)**
- **the generalized median set (or the yolk) is the smallest ball that intersects all median lines**

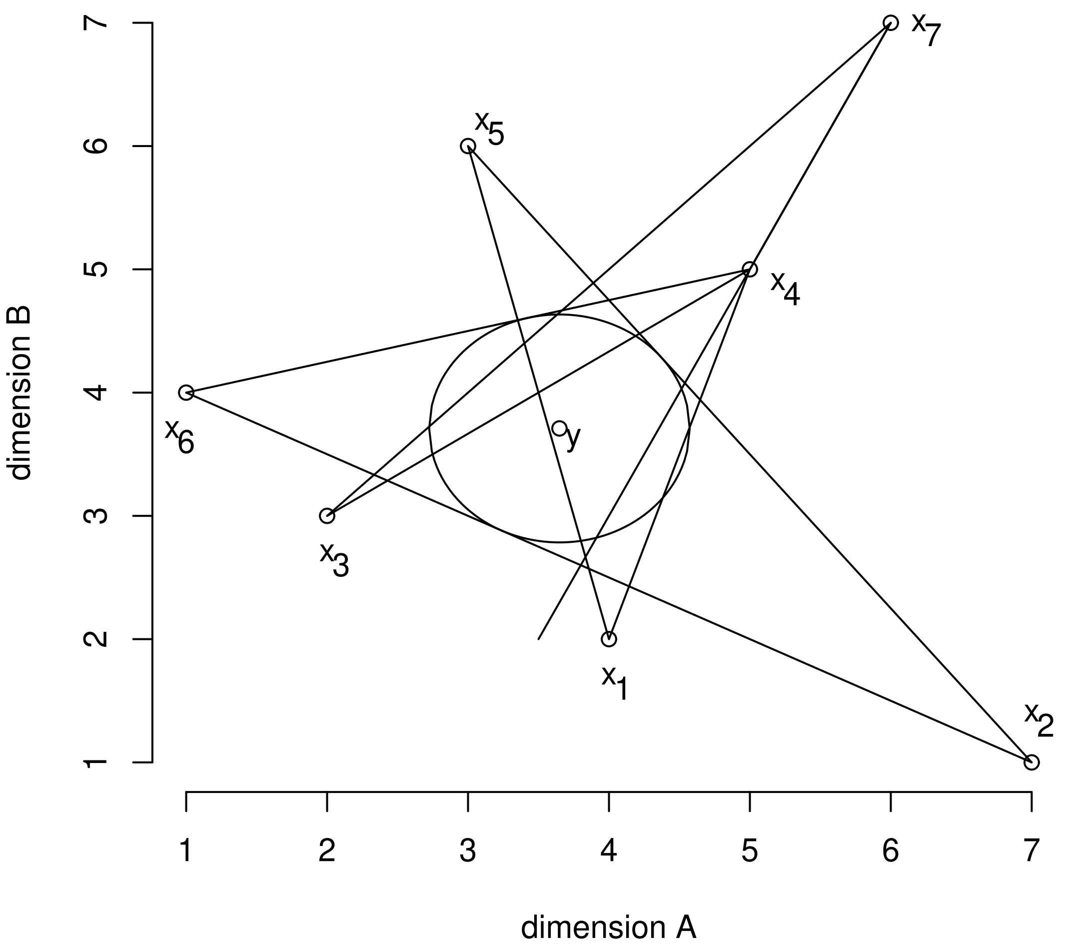
- **the uncovered set and the yolk (generalized median set)**
- **the generalized median set (or the yolk) is the smallest ball that intersects all median lines**
- **the generalized median is the center point of the yolk**
- **the uncovered set is contained in the a region four times the yolk's radius**
- **in general, the yolk shrinks as the number of voters increases (assuming sufficient dispersion)**

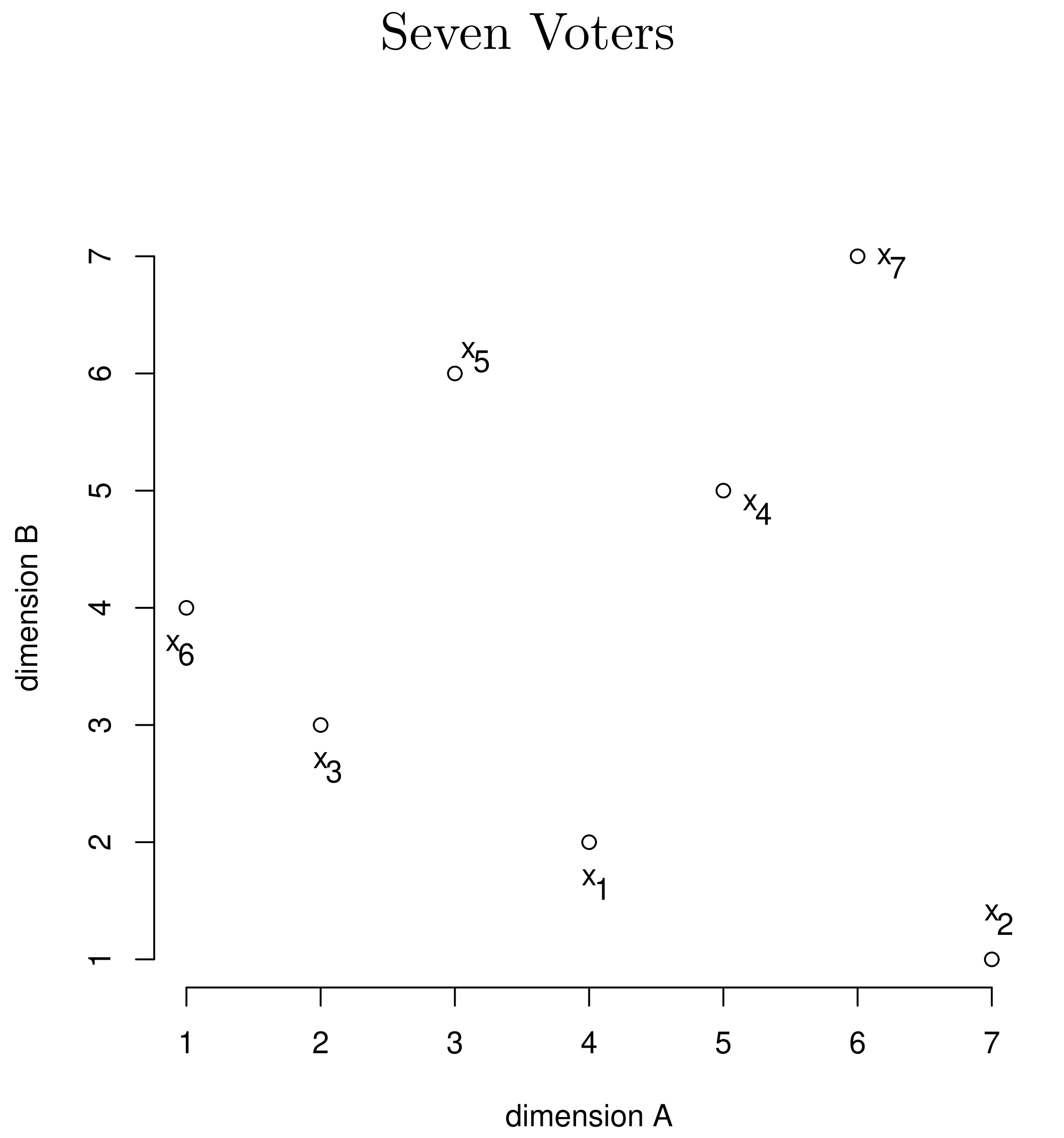
Seven Voters

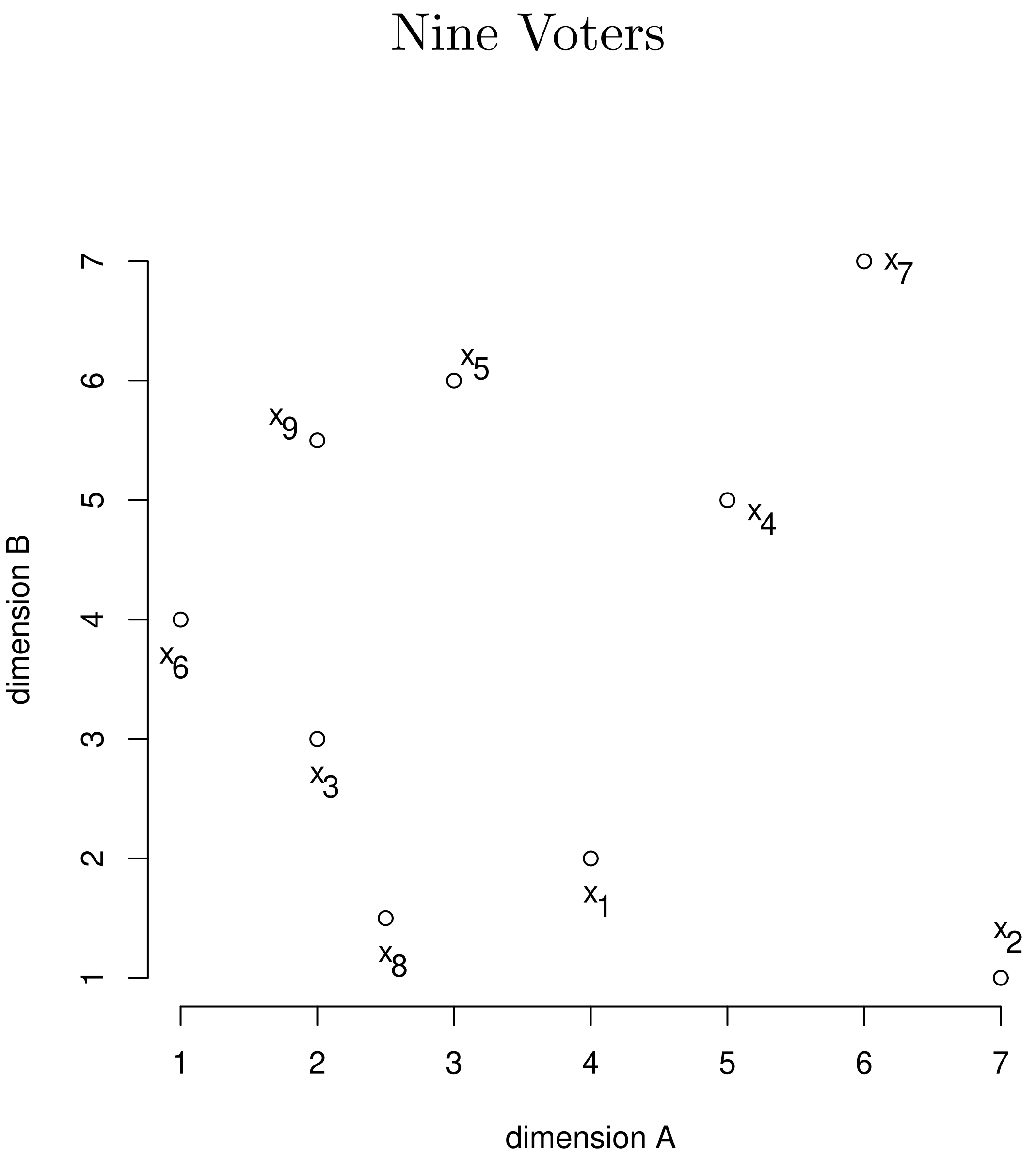



Seven Voters with Median Lines
dimension A
dimension B
1
2
3
4
5
6
7
x1
x2
x3
x4
x5
x6
x7

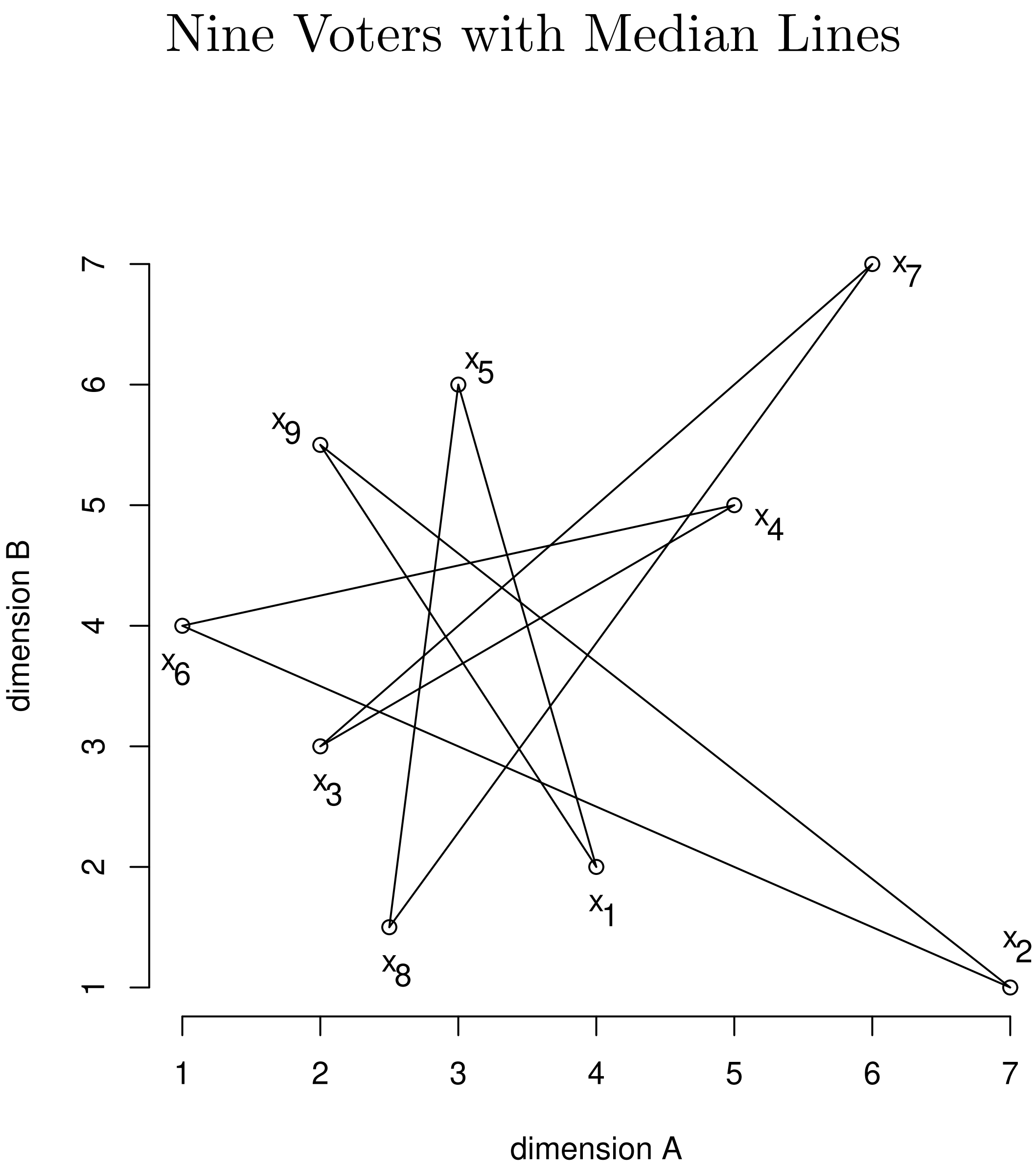
Seven Voters with the Yolk




Seven Voters
dimension A
dimension B
1
2
3
4
5
6
7
x_1
x_2
x_3
x_4
x_5
x_6
x_7

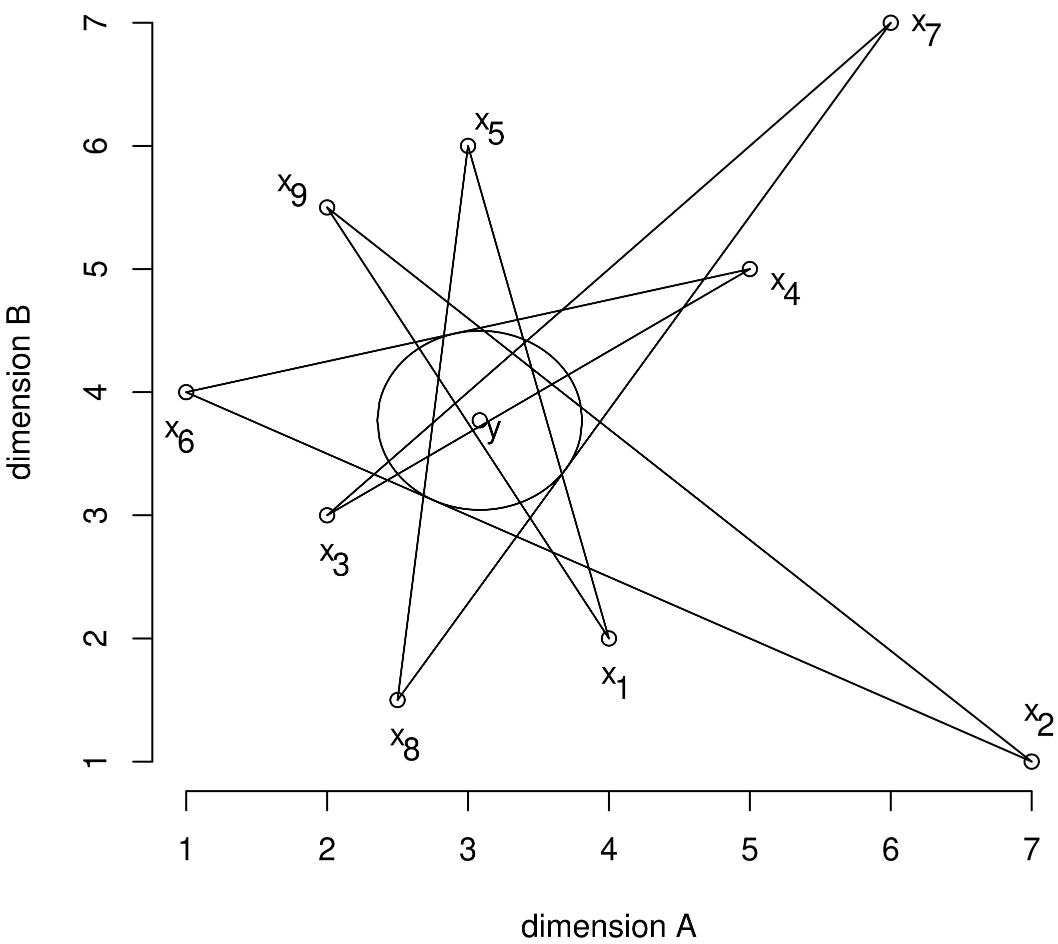


Nine Voters
x1
x2
x3
x4
x5
x6
x7
x8
x9
1
2
3
4
5
6
7
1
2
3
4
5
6
7
dimension A
dimension B

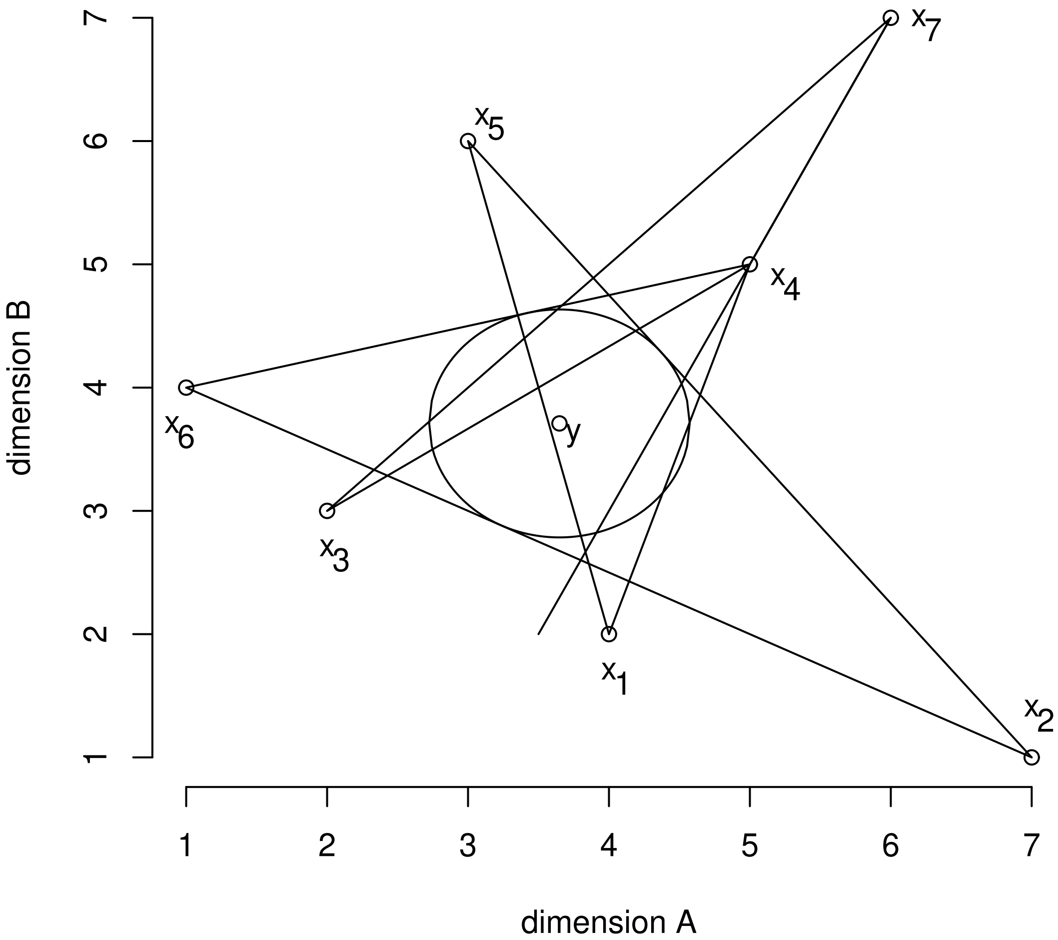


Nine Voters with Median Lines
dimension B
dimension A
1
2
3
4
5
6
7
1
2
3
4
5
6
7
x_1
x_2
x_3
x_4
x_5
x_6
x_7
x_8
x_9

Nine Voters with the Yolk

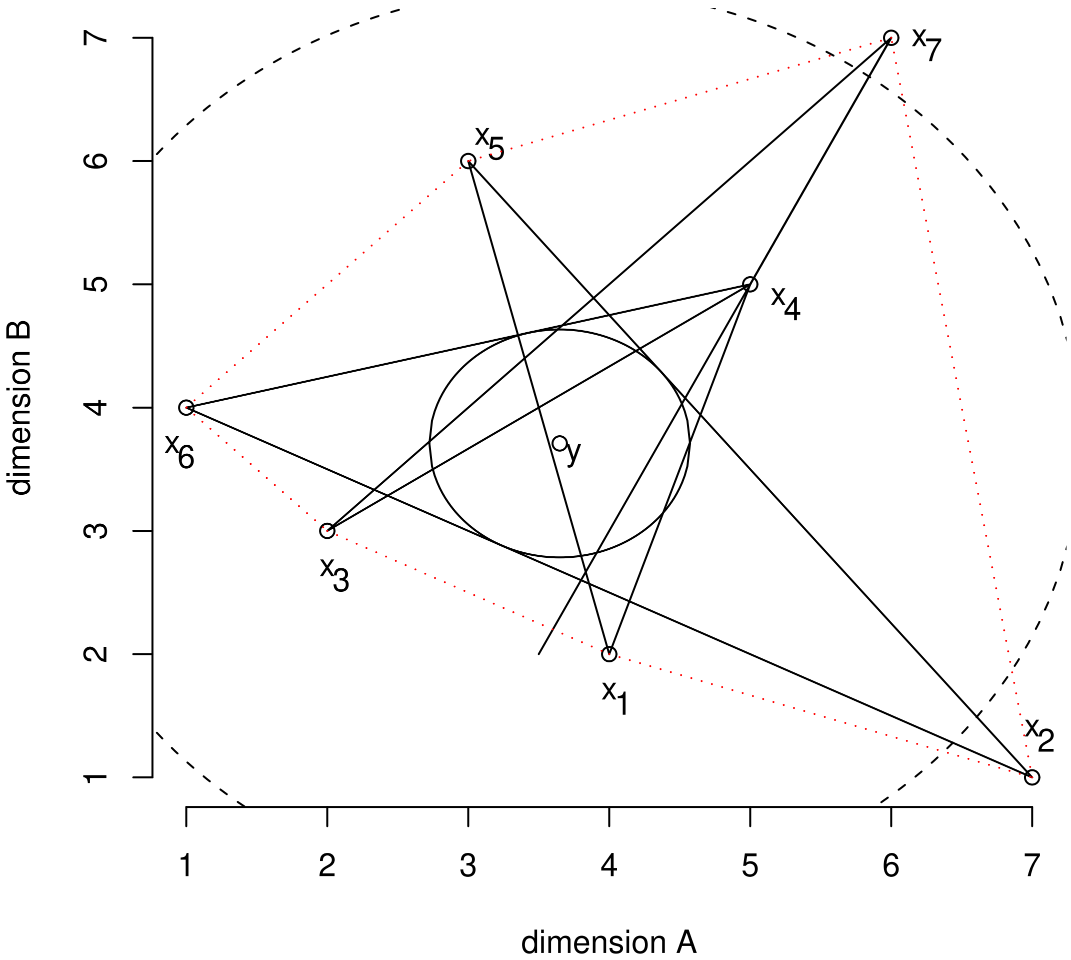


Seven Voters with the Yolk

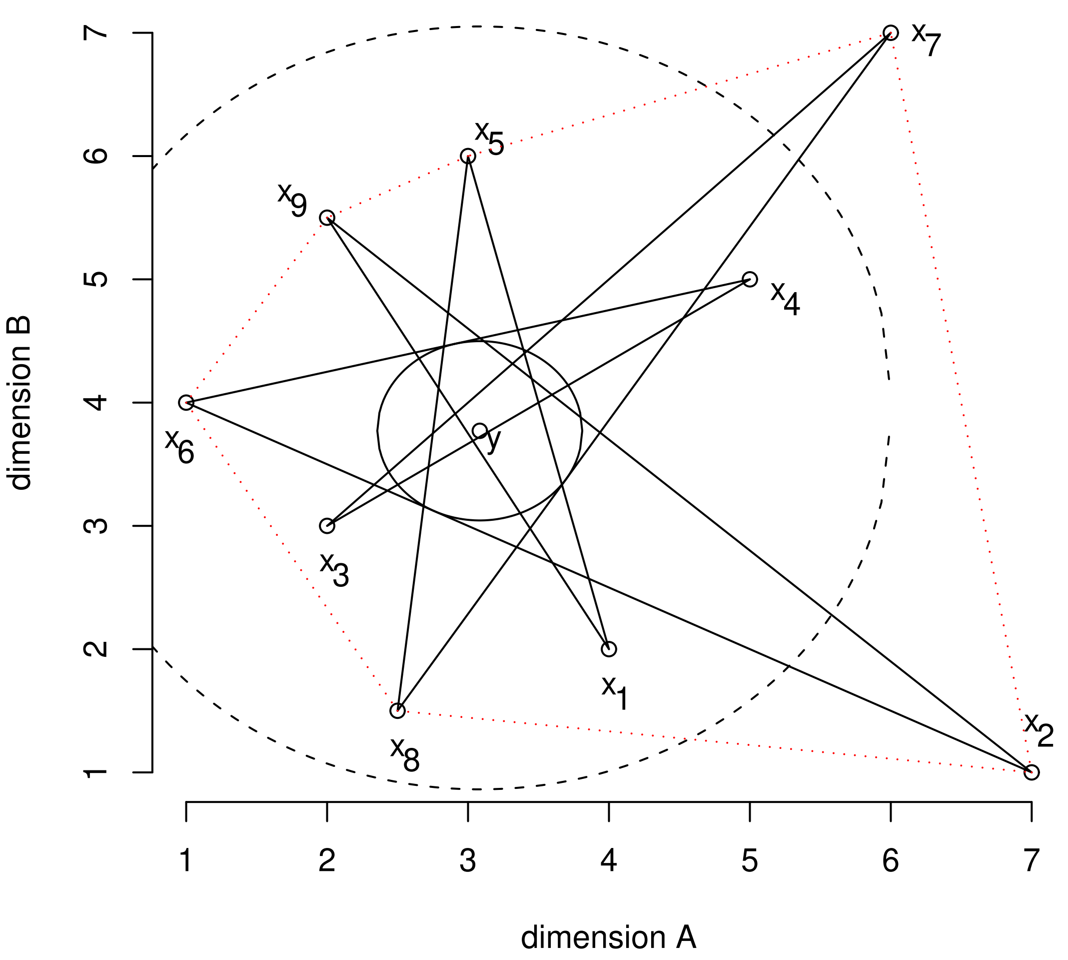


- **the Pareto set and four times the yolk**
- **the uncovered set is contained in the intersection of these two sets**

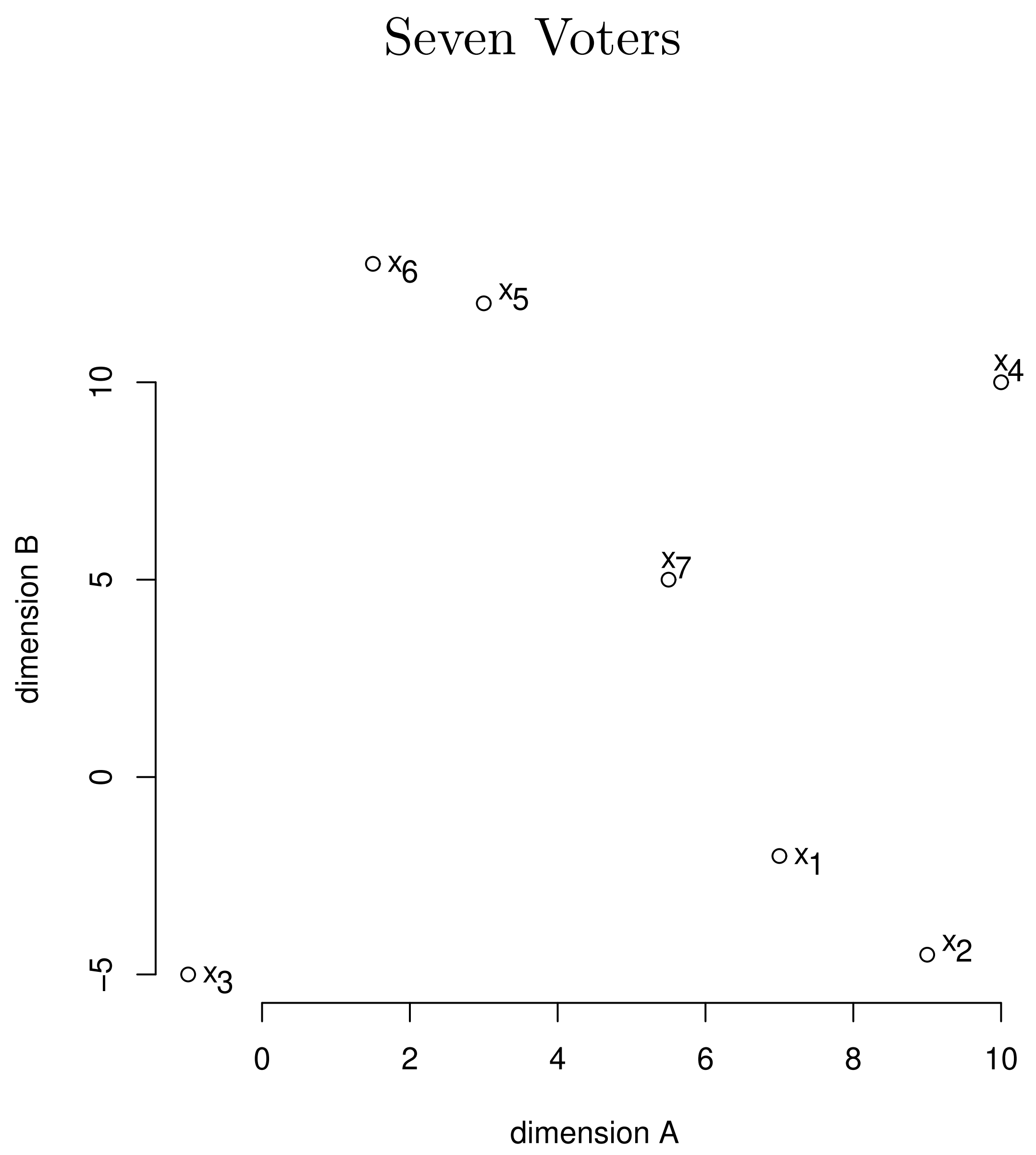
Seven Voters with Uncovered Set Bounds



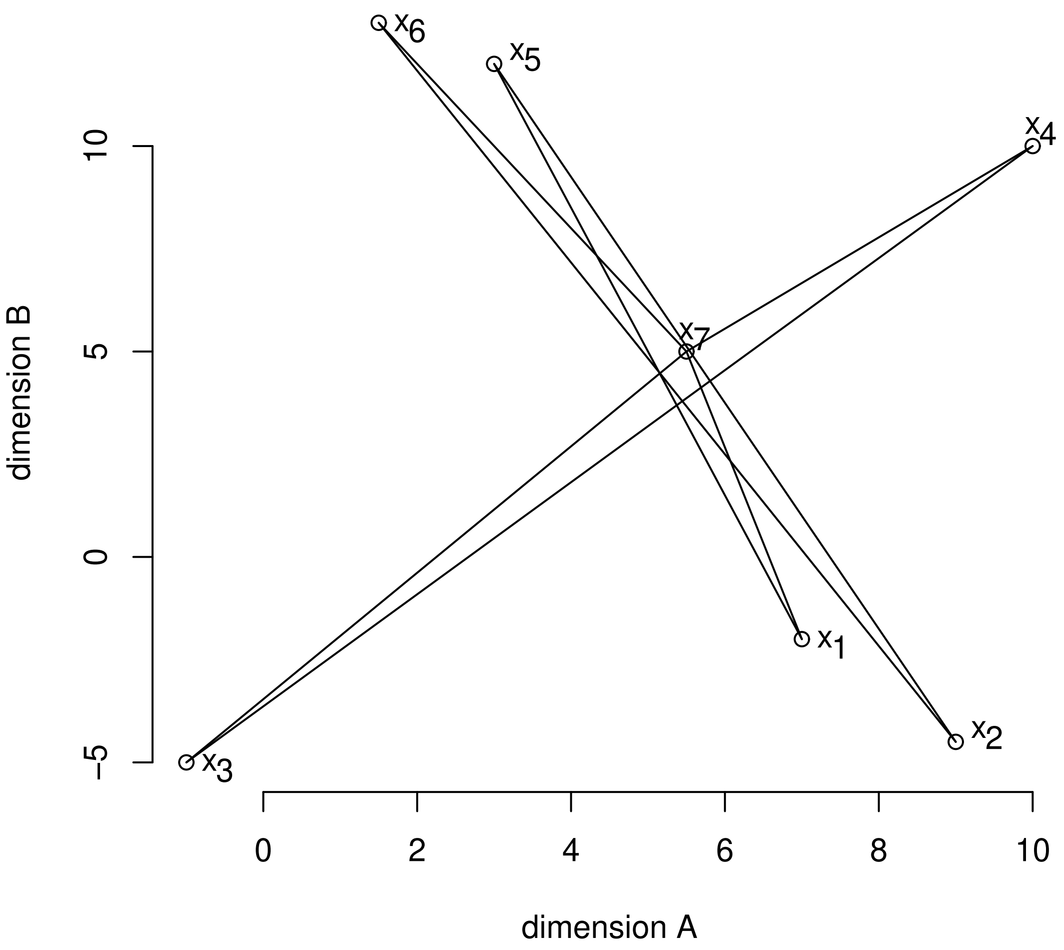
Nine Voters with Uncovered Set Bounds



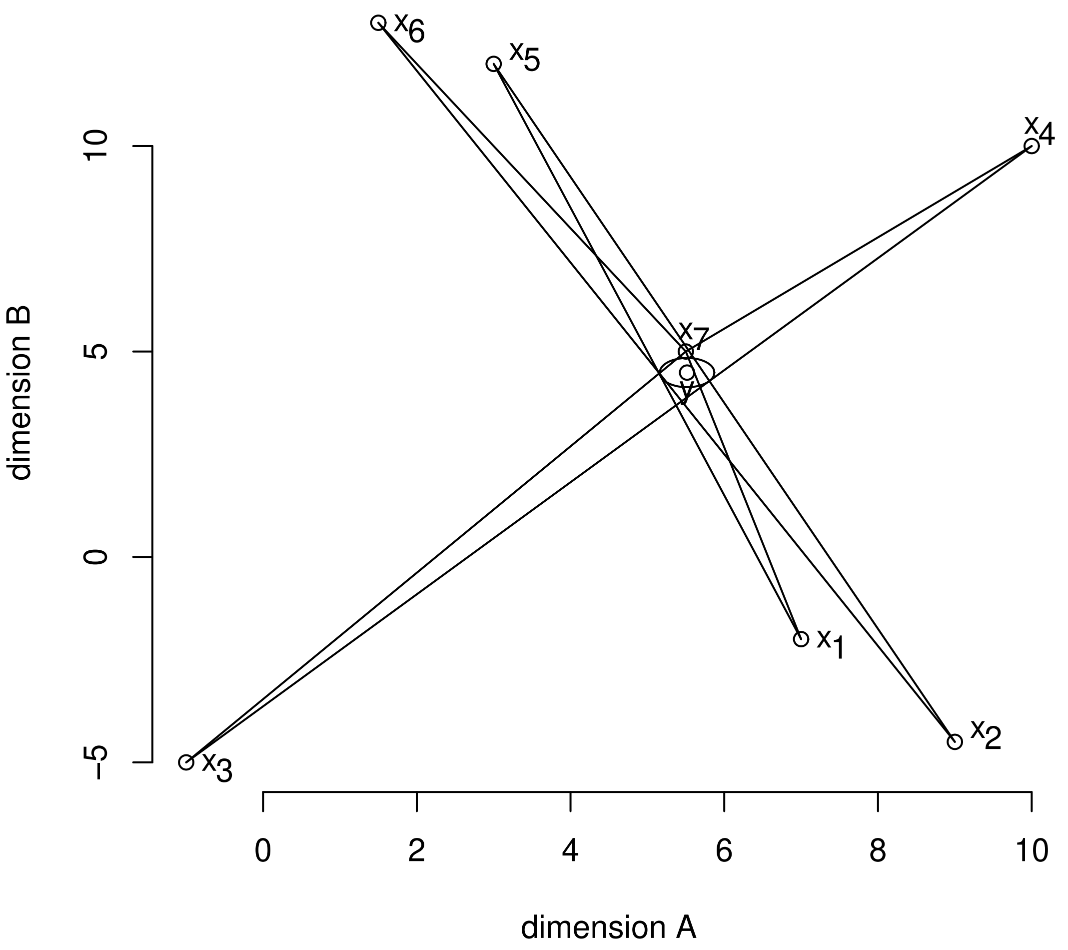
- **the yolk is small when a Condorcet winner almost exists**
- **in this case, strategic, open-agenda outcomes are near where the Condorcet winner would be (if it existed)**


Seven Voters
x6
x5
x4
x7
x1
x2
x3
10
5
0
−5
0
2
4
6
8
10
dimension B
dimension A

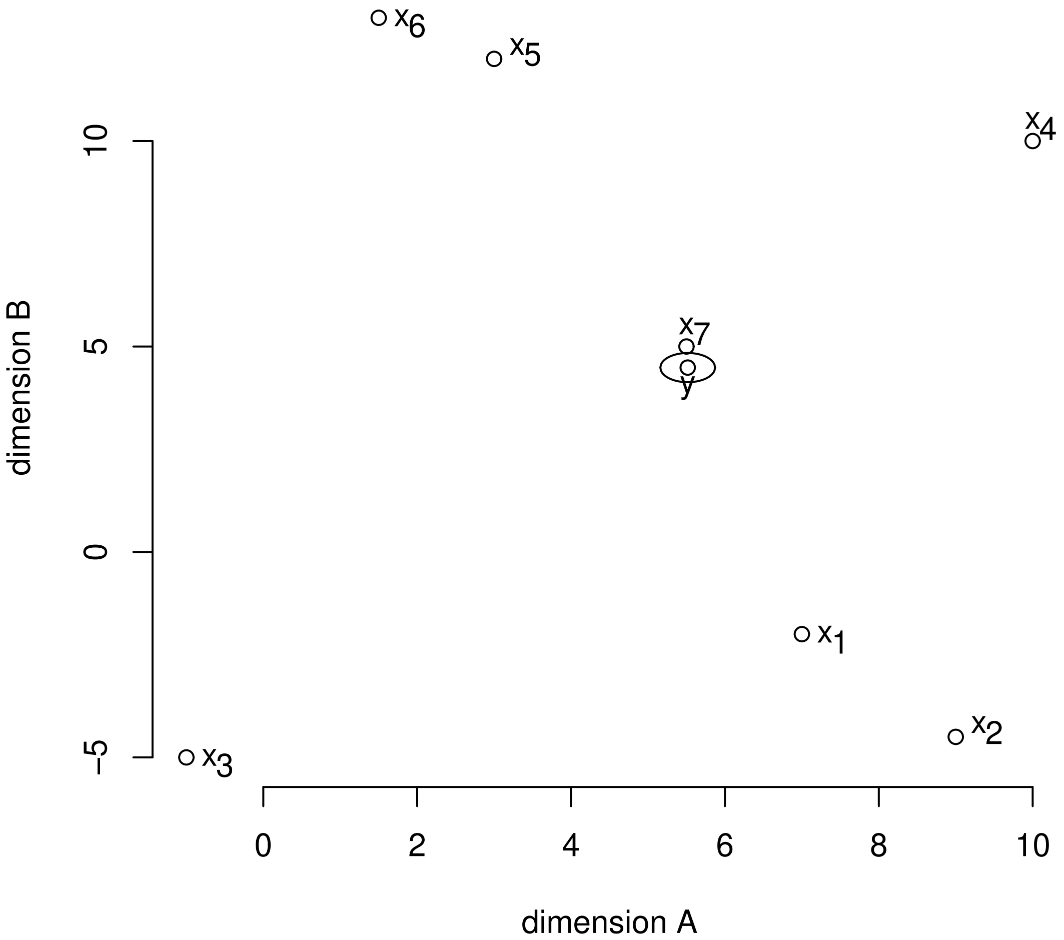
Seven Voters with Median Lines



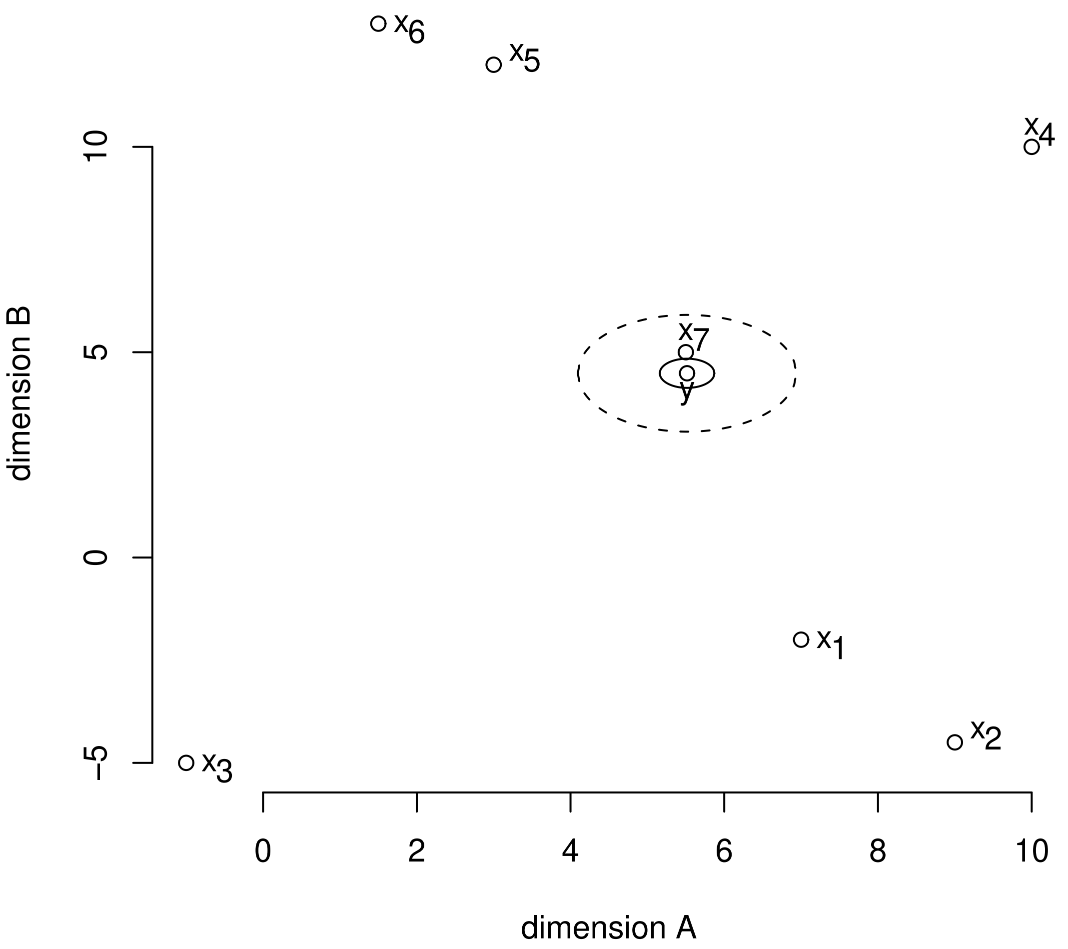
Seven Voters with Yolk



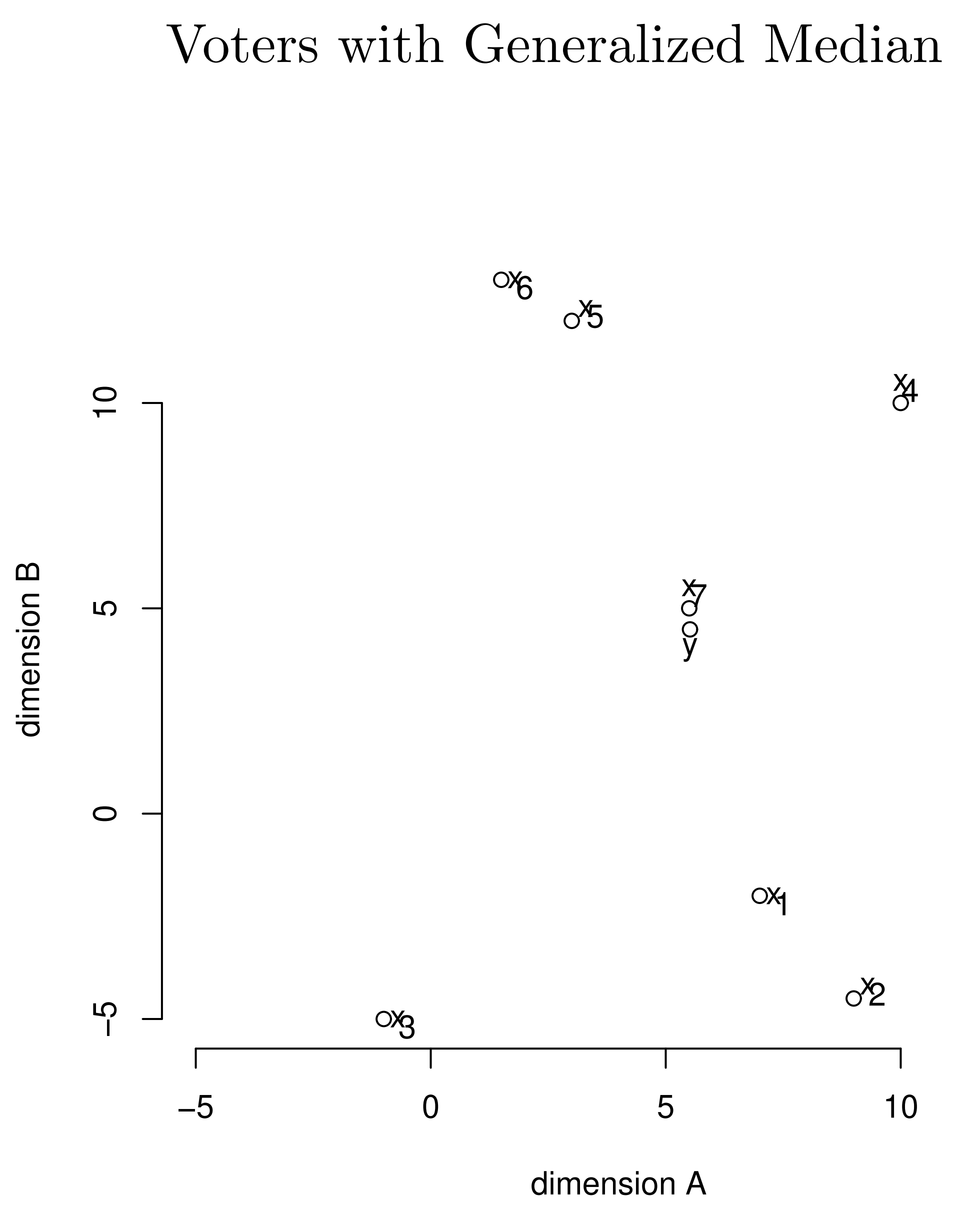
Seven Voters with Yolk



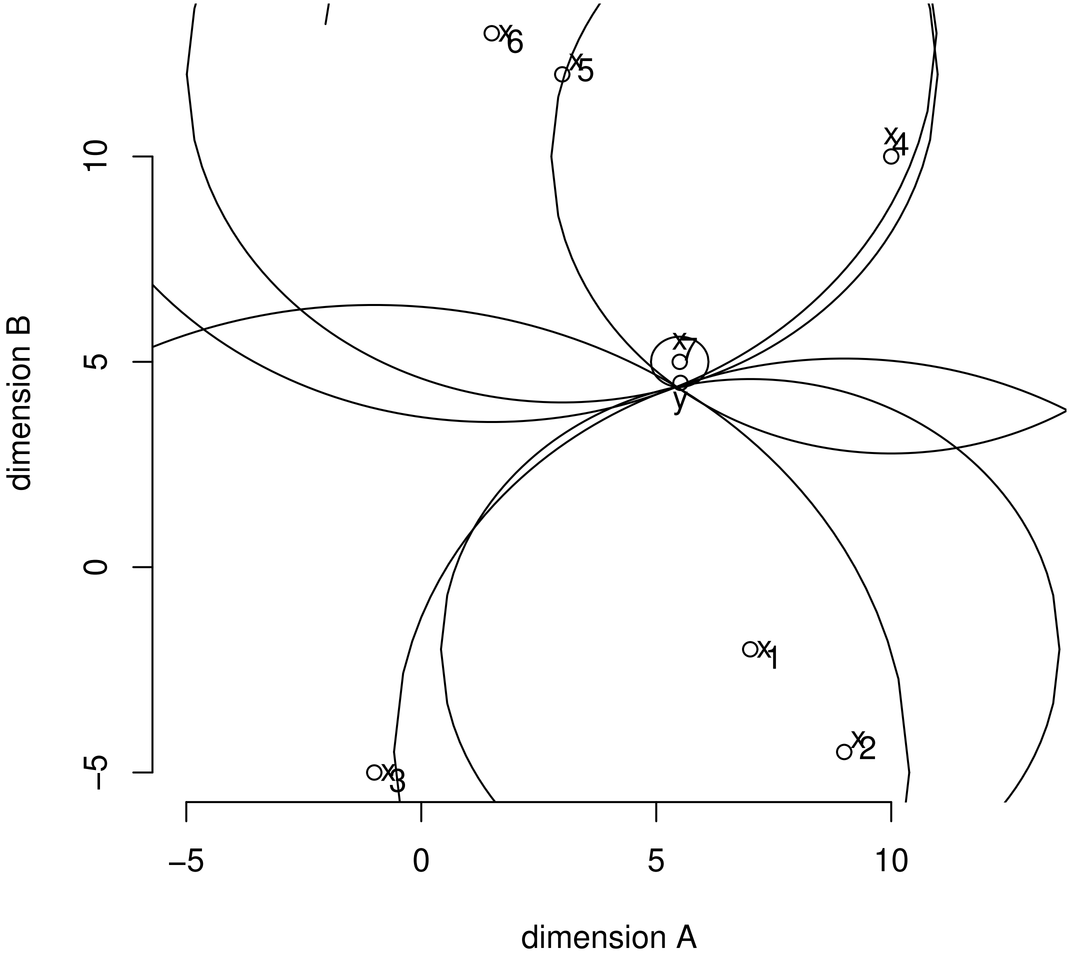
Seven Voters with Uncovered Set Range



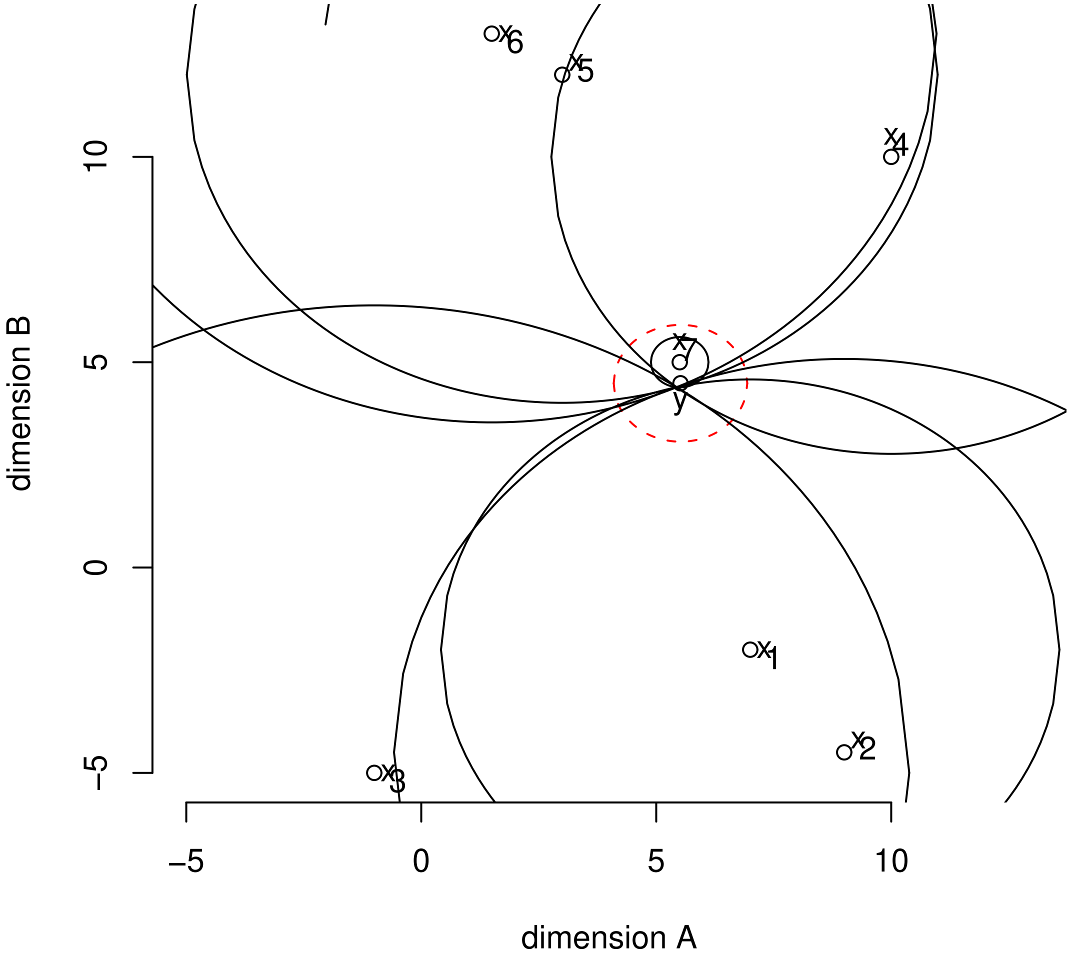
- **the generalized median and indifference curves**


Voters with Generalized Median
dimension B
dimension A
10
5
0
−5
−5
0
5
10
x
6
x
5
x
4
x
7
y
x
1
x
2
x
3

Indifference Curves to Generalized Median



Indifference Curves and Uncovered Set Range



- **instability when there is no Condorcet winner**
- **an agenda setter can produce any result with sincere voters (short of victory for an alternative that loses to every other possible alternative; with spatial preferences there is no such alternative)**

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  - **an agenda is an amendment agenda iff it is symmetric and continuous**
  - **a symmetric agenda is nonrepetitive, complete and uniform**

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  - **an agenda is an amendment agenda iff it is symmetric and continuous**
  - **a symmetric agenda is nonrepetitive, complete and uniform**
- **for other agendas, strategic voting can confine outcomes to alternatives that are dominant or are in the dominant cycle**

- **“But agenda power has two sources: the ability to choose a procedure for selecting from fixed set of alternatives and the ability to choose that set.” (Ordeshook and Schwartz 1987, 194)**

- **political parties: solutions to coordination or collective action problems?**

BoS

	$Bach$	$Cage$
$Bach$	$2, 1$	$0, 0$
$Cage$	$0, 0$	$1, 2$

BoS

	$Bach$	$Cage$
$Bach$	$2, 1$	$0, 0$
$Cage$	$0, 0$	$1, 2$

Coordination

	$Melissa$	$Mariah$
$Melissa$	$2, 2$	$0, 0$
$Mariah$	$0, 0$	$1, 1$

PD

	$Don't\ Confess$	$Confess$
$Don't\ Confess$	$3, 3$	$0, 4$
$Confess$	$4, 0$	$1, 1$

PD

	Don't Confess	*Confess*
Don't Confess	3, 3	0, 4
Confess	4, 0	1, 1

Hawk-Dove

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

- **collective action problem (Aldrich Table 2.1)**

	Bill		
Legislator	**X**	**Y**	**Z**
A	**4**	**3**	**-9**
B	**3**	**-9**	**4**
C	**-9**	**4**	**3**

- **collective action problem (Aldrich Table 2.3)**

	Bill		
Legislator	**X**	**Y**	**Z**
A	3	-1	-1
B	-1	3	-1
C	-1	-1	3

- **why parties?**
- **parties may help solve coordination dilemmas**
  - **electoral (party-in-elections)**
  - **legislative (party-in-government)**
  - **one role for activists: high intensity or high resource people who work or spend to promote focal points**
  - **party labels**

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  - **one role for activists: high intensity or high resource people who work or spend to promote focal points**
  - **party labels**
- **parties may help deploy resources efficiently**
  - **information about policies and candidates (cheap for voters and candidates)**

- **strategic voting in electoral systems**
- **examples of electoral system features**
  - **districts and district magnitude (M)**
  - **presidents versus parliaments**
  - **proportional representation (PR)**
    - **with quotas (e.g., Hare: $Q = V/M$)**
    - **with averages (divisor methods) (e.g., d'Hondt: $a_i(t) = v_i/(s_i(t) + 1)$)**
  - **party lists**
  - **runoffs (dual ballots)**

- **examples of voting rules**
  - **majority rule**
  - **plurality voting with one winner (winner take all, first past the post)**
  - **plurality voting with $M > 1$**
  - **Borda count**
  - **instant runoff voting (Hare systems)**
  - **approval voting**
  - **utilitarian systems**
  - **range voting**

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  - **instant runoff voting (Hare systems)**
  - **approval voting**
  - **utilitarian systems**
  - **range voting**
- **there is extensive theory for single districts, little for whole systems**

- **current “election reform” debates about voting rules**
- **Borda count**
  - **favored by Saari (perhaps with different weights)**
  - **problem is positional methods are manipulable**

- current “election reform” debates about voting rules
- Borda count
  - favored by Saari (perhaps with different weights)
  - problem is positional methods are manipulable
- approval voting
  - claimed to have many virtues (see Brams et al.)
  - among the flaws is it need not select a Condorcet winner (SC&W election example)
  - outcomes depend almost entirely on strategies, in particular on how many alternatives each voter decides to approve

- **current “election reform” debates about voting rules**
- **instant runoff voting (Hare systems)**
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- **instant runoff voting (Hare systems)**
  - **claimed to have many virtues (see Richie et al.)**
  - **major flaw is it violates monotonicity**
- **range voting**
  - **has strong advocates (see RangeVoting.html)**
  - **is approval voting with more degrees of freedom**

- Duverger's Law: electoral systems cause party systems
  - plurality voting (with $M = 1$) favors a two-party system
  - PR favors a multiparty system
- motivated by strategic voting (voters) and coalition formation (elites)

- **Duverger's Law: electoral systems cause party systems**
  - **plurality voting (with $M = 1$) favors a two-party system**
  - **PR favors a multiparty system**
- **motivated by strategic voting (voters) and coalition formation (elites)**
- **generalizing Duverger's Law for $M > 1$**
  - **the $M + 1$ rule**
  - **non-Duvergerian outcomes: parties finishing below the $M + 1$-th place get equal numbers of votes greater than zero**
  - **leads to the bimodality hypothesis, which Cox tests using the SF ratio: $v_{M+2}/v_{M+1} = 0$ or , $v_{M+2}/v_{M+1} = 1$**

- **strategic voting under plurality voting (Cox)**
- **preferences: von Neumann-Morgenstern utilities** $u_i = (u_{i1}, \ldots, u_{iK})'$**,** $\max(u_i) = 1$**,** $\min(u_i) = 0$
- **beliefs:** $F_i = F$ **(common knowledge)**
- **expectations: proportions** $\pi_i = (\pi_{i1}, \ldots, \pi_{iK})'$
- **publicly generated expectations:** $\pi_i = \pi$
- **vote to maximize expected utility** $\sum_{k=1}^{K} \pi_{ik} u_{ik}$

- strategic voting under plurality voting (Cox)
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- **rational expectations condition: if given beliefs** F **everyone votes optimally in light of** π**, the result is expected vote shares that equal** π
- **Theorem 1: If** $0 < \pi_j < \pi_{M+1}$ **for** $j > M + 1$**, then** π **is not a limit of rational expectations**
- **Corollary: If** π **is a limit of rational expectations, then** $\pi_j \in \{0, \pi_{M+1}\}$ **for all** $j > M + 1$

- **Duverger's Law, the $M+1$ rule and other strategic voting: examples (using $>$ to denote P_i), find the rational expectations Nash equilibrium vote distributions**

  $1/8 \quad A > B > C$

  $4/8 \quad B > C > A$

  $3/8 \quad A > C > B$

- **simple plurality**
  - **(**A**,** $1/2$**;** B**,** $1/2$**;** C**,** 0**)**
  - **(**A**,** $1/2$**;** B**,** 0**;** C**,** $1/2$**)**

- **plurality with** $M = 2$

- **two-stage with plurality** $M = 2$ **in first stage**

- **Borda count**
  - **consider coalition-proof refinement to Nash equilibrium**

- **Duverger's Law, the $M+1$ rule and other strategic voting: examples (using $>$ to denote P_i)**

$1/8$	$A > B > C > D$
$2/8$	$B > C > D > A$
$2/8$	$D > B > C > A$
$1/8$	$A > C > D > B$
$2/8$	$D > A > C > B$

- **simple plurality**
  - **(**A**,** $1/2$**;** B**,** $1/2$**;** C**,** 0**;** D**,** 0**)**
  - **(**A**,** 0**;** B**,** 0**;** C**,** $1/2$**;** D**,** $1/2$**)**
- **plurality with $M = 2$**

- **why parties: an informational rationale (Snyder and Ting)**
  - **candidates: one-dimensional policy plus benefits from holding office**
  - **benefit to candidate from holding office:** $w > 0$
  - **candidate ideal point:** $z \in [-1, 1]$
  - **party:** $i \in \{L, R\}$
  - **party position:** $x_i \in [-1, 1]$
  - **cost of party affiliation:** $c \in [0, w)$
  - **policy loss scale factor:** $\alpha > 0$
  - **affiliated candidate utility:** $w - \alpha(x_i - z)^2 - c$
  - **relative benefit of holding office as a party member:** $\theta = \sqrt{(w-c)/\alpha}$**; assume** $\theta < 1$

- **why parties: an informational rationale (Snyder and Ting)**
  - **voters: one-dimensional policy given uncertainty about candidates**
  - **ideal point of the median voter in a district:** y
  - **mean of ideal points of candidates affiliated with party** i**:** μ_i
  - **variance of ideal points of candidates affiliated with party** i**:** σ_i^2
  - **expected utility of the median voter in a district for a candidate affiliated with party** i**:** $-(y-\mu_i)^2 - \sigma_i^2$
  - **expected utility of the median voter in a district for an unaffiliated candidate :** $-y^2 - 1/3$ **(i.e.,** $\mu_U = 0$ **and** $\sigma_U^2 = 1/3$**)**

- **why parties: an informational rationale (Snyder and Ting)**
  - **sequence of game play (page 96)**
  - **platform and affiliation choices when parties maximize the share of offices won (page 100)**
  - **platforms when parties maximize total net benefits of their members (page 102)**

- **simple agenda setting: the setter model (Rosenthal)**
  - **one-dimensional spatial setting**
  - **median voter**
  - **reversion or status quo point**
  - **game is: proposal followed by accept or reject**

- **gerrymanders and partisan bias**
- **a formal model (Cox and Katz 1999)**
  - ν: **average vote share,** $0 < \nu < 1$
  - ρ: **responsiveness,** $\rho \in (-\infty, \infty)$
  - λ: **partisan bias parameter,** $\lambda \in (-\infty, \infty)$
  - s: **seat share; as a function of average vote share**

$$s(\nu; \rho, \lambda) = \frac{\exp\left[\rho \log\left(\frac{\nu}{1-\nu}\right)\right]}{\exp[-\lambda] + \exp\left[\rho \log\left(\frac{\nu}{1-\nu}\right)\right]}$$

  - **partisan bias measure:** $\exp[\lambda]/(\exp[\lambda] + 1) - 0.5$
  - **responsiveness: if** $\rho < 1$**, then the smaller party is overrepresented (**$\nu < s$**)**

- **a formal model (Cox and Katz 1999)**
  - ν**: average vote share,** $0 < \nu < 1$
  - ρ**: responsiveness,** $\rho \in (-\infty, \infty)$
  - λ**: partisan bias parameter,** $\lambda \in (-\infty, \infty)$
  - $s(\nu; \rho, \lambda)$**: seat share, a function of average vote share**
- **feasible plans: a smooth, convex subset of** (ρ, λ) **space**
- **hence the strong party must tradeoff responsiveness and bias**

- **cube law**

$$\frac{S}{1-S} = \left(\frac{V}{1-V}\right)^3$$

- **some Voting Right Act concepts**
- **racially polarized voting**
- **minority vote dilution**
  - **gerrymander**
  - **at-large election plans**
  - **majority-runoff requirements**
  - **anti-single-shot devices (defeats “bullet votes”)**

- some Voting Right Act concepts
- racially polarized voting
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  - gerrymander
  - at-large election plans
  - majority-runoff requirements
  - anti-single-shot devices (defeats “bullet votes”)
- Section 5 preclearance
- Section 2, Mobile v. Bolden, and the 1982 VRA extension
- 1975 VRA extension and language minority provisions

- **electoral systems (Grofman and Davidson, 7)**
  - **at-large**
  - **single-member districts**
  - **mixed systems**

- electoral systems (Grofman and Davidson, 7)
- at-large
  - "In an at-large system, all the contested seats on a governmental body, such as a city council, county commission, or school board, are filled by voters in the jurisdiction at large. If there are eight seats to be filled, all voters have eight votes and theoretically have a chance to influence who gets elected to all eight seats."

- **electoral systems (Grofman and Davidson, 7)**
- **single-member districts**
  - **“the city is divided into geographical districts, and voters in each district, like voters in congressional elections, are limited to a vote for a single candidate running to represent their district”**
- **mixed systems**
  - **“some of the seats are voted on at large, and some by district”**

- **districting in House elections under the Voting Rights Act**
  - **majority minority districts: changes over time**
  - **“bleaching” districts: partisan gerrymander disguised as nondilution**

- **districting in House elections under the Voting Rights Act**
  - **majority minority districts: changes over time**
  - **“bleaching” districts: partisan gerrymander disguised as nondilution**
- **substantive representation versus(?) symbolic representation**
  - **is the 65% majority-minority “rule” appropriate?**
  - **Cameron, Epstein, O’Halloran suggest no for U.S. House districts**
  - **Mapps suggests no for state legislative districts**

- **election fraud: is fraud (legitimate) political manipulation?**
- **detecting anomalies**
- **distinguishing anomalies from fraud**
- **diagnosing fraud**

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- **diagnosing fraud**
- **history of fraudulent elections in the United States**
- **elsewhere (and election monitoring: observers, PVT)**

- **detecting anomalies**
- **Florida 2000: wrong outcome, but why?**
  - **ex-felon lists**
  - **butterfly ballot**
  - **other machines and ballots**

- **detecting anomalies**
- **Florida 2000: wrong outcome, but why?**
  - **ex-felon lists**
  - **butterfly ballot**
  - **other machines and ballots**
- **Florida 2004: fraud alleged**
  - **conservative Democrats**
  - **hacked machines?**

- **Election Forensics**
  - **statistically analyzing recorded vote counts to detect anomalies and try to diagnose fraud**
- **regularities and departures from regularities**
  - **using relationships with covariates to detect outliers**
  - **checking whether vote counts match expected distributions**

- **election forensics and recounts**
  - **two kinds of errors (or frauds) in vote counts**
    - **miscounting the ballots that were cast**
    - **counting falsified ballots**

- **election forensics and recounts**
  - **two kinds of errors (or frauds) in vote counts**
    - **miscounting the ballots that were cast**
    - **counting falsified ballots**
- **recounts can detect the first kind but not the second kind**
  - **exception: physically inspecting ballots may spot signs that some or all are fake**
  - **this depends on there being physical ballots to inspect**
- **statistical analysis may be able to detect both kinds of distortions**

- an example from the 2006 Mexican presidential election
  - relationship between presidential votos nulos and senate votos nulos
  - use casilla (ballot box) counts
  - the linear predictor is

$$Z_i = d_0 + d_1 \textbf{logitz}(\textbf{SenateVN}_i)$$

SenateVN represents the proportion of votos nulos for senate votes at casilla i **logitz**(p) denotes the log-odds function adjusted to handle zero counts (add $1/2$ to each count before computing p)

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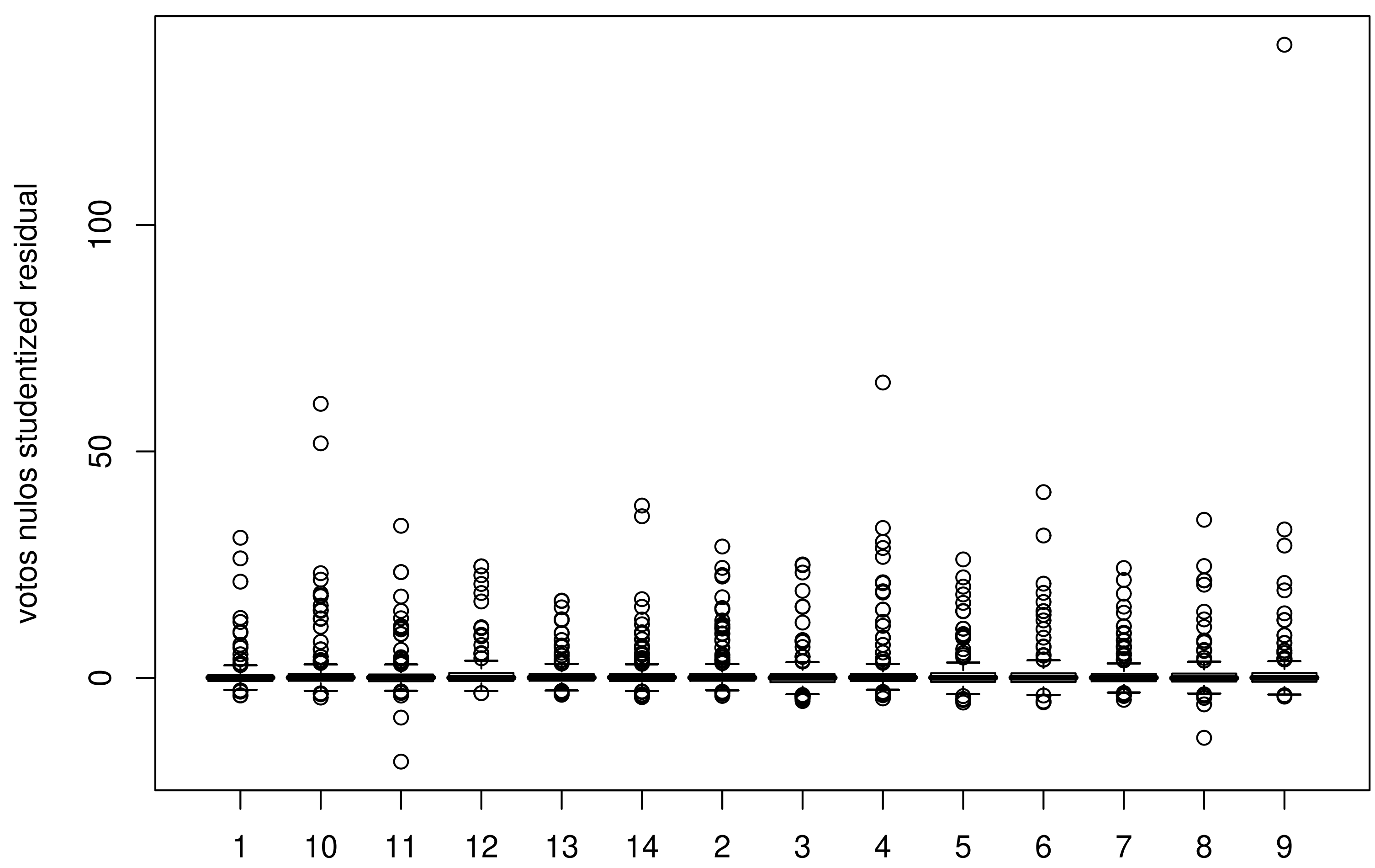
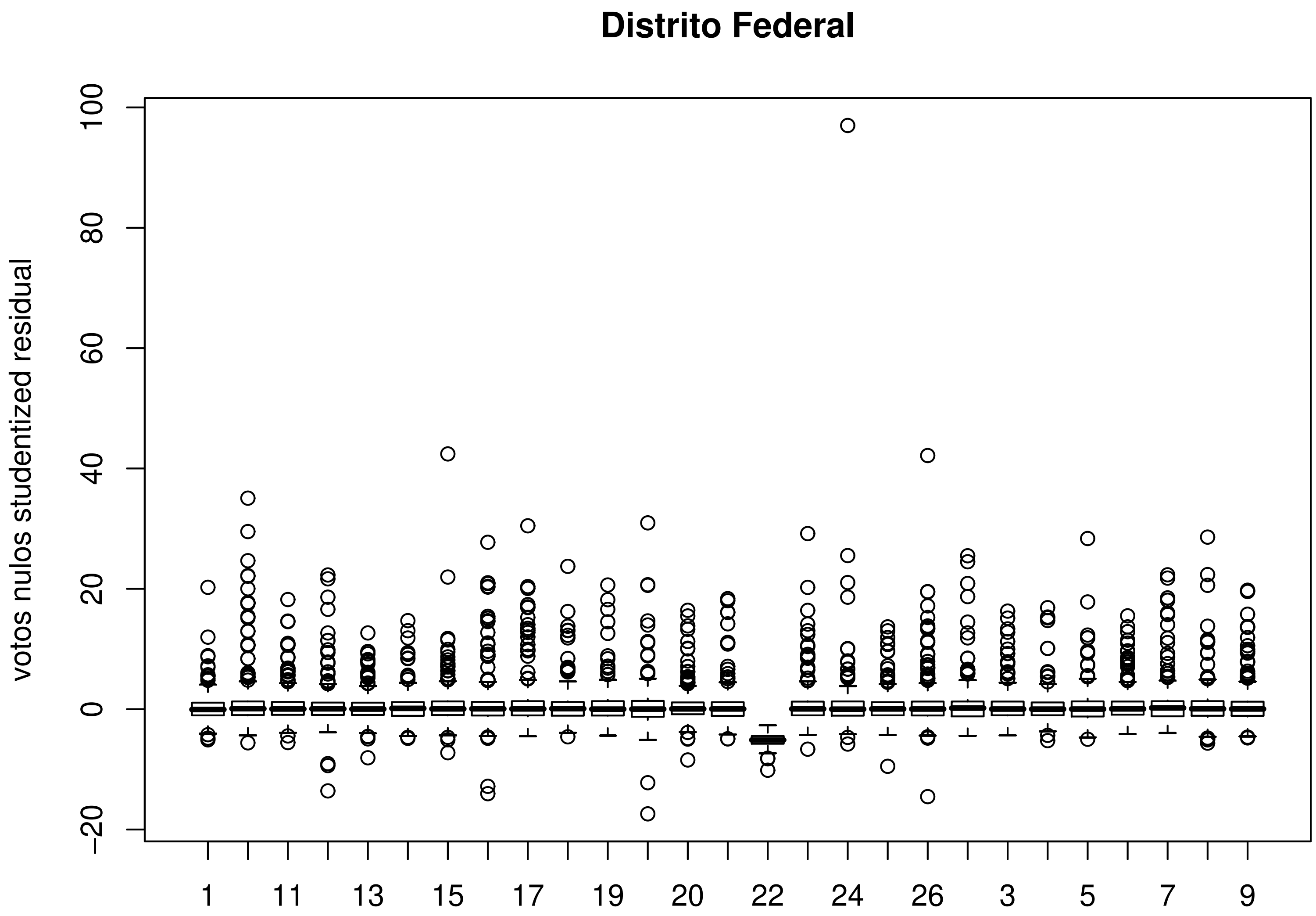
  - estimate separately for each legislative district
  - outliers are prevalent

Guanajuato


votos nulos studentized residual
0
50
100
1
10
11
12
13
14
2
3
4
5
6
7
8
9



Distrito Federal
votos nulos studentized residual
−20
0
20
40
60
80
100
1
11
13
15
17
19
20
22
24
26
3
5
7
9

- **an example from the 2006 Mexican presidential election**
  - **relationship between presidential votos nulos and senate votos nulos**
  - **use casilla (ballot box) counts**
  - **estimate separately for each legislative district**
  - **outliers are prevalent**
    - **130,020 casillas are in the analysis (from 299 districts)**

proportion of residuals larger than

2	3	4
.11	.06	.04

- **checking whether vote counts conform with expected distributions**

- **checking whether vote counts conform with expected distributions**
- **digits of vote counts and Benford's Law**
  - **compare vote counts' second digits to the second digit Benford's Law (2BL)**
  - **there are strong arguments against expecting vote counts' first digits to satisfy Benford's Law for first digits**

Frequency of First and Second Digits according to Benford's Law

digit	0	1	2	3	4	5	6	7	8	9
first	—	.301	.176	.124	.097	.079	.067	.058	.051	.046
second	.120	.114	.109	.104	.100	.097	.093	.090	.088	.085

- **the statistic is**

$$X^2_{B_2} = \sum_{i=0}^{9} \frac{(d_{2i} - d_2 q_{B_2 i})^2}{d_2 q_{B_2 i}}$$

where

- $q_{B_{2i}}$ **is the expected relative frequency with which the second significant digit is** i **(the values shown in the second line of table of Benford's Law frequencies)**
- d_{2i} **is the number of times the second digit is** i **among the precincts being considered**
- $d_2 = \sum_{i=0}^{9} d_{2i}$

- **the statistic is**

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- **with one set of counts (for one office in one area), use the critical value of** χ^2_9 **for test level** $\alpha = .05$**, which is 16.9**

- **looking at multiple sets of counts, control for the false discovery rate (FDR)**

- **an example from the 2004 American election: Florida, Miami-Dade County**
  - **vote counts for major party candidates for president (Kerry and Bush) and for the Senate (Castor and Martinez)**
  - **also vote counts for eight proposed constitutional amendments**
  - **with 20 tests, the FDR-controlled critical value for χ^2_9 is 25.5**

Florida Constitutional Amendments on the Ballot in 2004

		Yes	No
1	Parental Notification of a Minor's Termination of Pregnancy	4,639,635	2,534,910
2	Constitutional Amendments Proposed by Initiative	4,574,361	2,109,013
3	The Medical Liability Claimant's Compensation Amendment	4,583,164	2,622,143
4	Authorizes Voters to Approve Slot Machines in Parimutuel Facilities	3,631,261	3,512,181
5	Florida Minimum Wage Amendment	5,198,514	2,097,151
6	Repeal of High Speed Rail Amendment	4,519,423	2,573,280
7	Patients' Right to Know About Adverse Medical Incidents	5,849,125	1,358,183
8	Public Protection from Repeated Medical Malpractice	5,121,841	2,083,864

Miami-Dade Election Day First-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	29.3	Am. 4 Yes	144.8
Kerry	39.9	Am. 4 No	119.6
Martinez	35.6	Am. 5 Yes	115.4
Castor	22.0	Am. 5 No	27.6
Am. 1 Yes	86.2	Am. 6 Yes	98.8
Am. 1 No	80.5	Am. 6 No	84.0
Am. 2 Yes	95.6	Am. 7 Yes	130.3
Am. 2 No	60.0	Am. 7 No	49.9
Am. 3 Yes	60.5	Am. 8 Yes	123.0
Am. 3 No	51.5	Am. 8 No	102.6

Note: $n = 757$ precincts. Pearson chi-squared statistics, 8 df.

Miami-Dade Election Day Second-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	7.9	Am. 4 Yes	3.3
Kerry	9.5	Am. 4 No	5.7
Martinez	8.9	Am. 5 Yes	17.9
Castor	12.0	Am. 5 No	5.8
Am. 1 Yes	2.5	Am. 6 Yes	4.3
Am. 1 No	5.5	Am. 6 No	9.1
Am. 2 Yes	16.7	Am. 7 Yes	17.1
Am. 2 No	7.2	Am. 7 No	8.4
Am. 3 Yes	3.3	Am. 8 Yes	12.7
Am. 3 No	12.9	Am. 8 No	6.5

Note: $n = 757$ precincts. Pearson chi-squared statistics, 9 df.

- **why should we expect vote counts to satisfy 2BL?**
- **model vote counts as results of particular mixtures**
- **at least two mechanisms can generate counts that satisfy 2BL (and not 1BL)**
  - **mechA: mix support that varies over precincts with a small random frequency of errors**
  - **mechB: mix support that varies over precincts with varying precinct sizes**

2BL Tests for Simulated Precinct Vote Counts (First Mechanism)

Size	Benf.	Size	Benf.	Size	Benf.	Size	Benf.
500	10.3	1,500	18.6	3,800	11.3	7,100	8.3
600	9.5	1,600	21.6	3,900	9.2	7,200	9.1
700	10.0	1,700	19.9	4,000	12.2	7,300	8.9
800	9.0	1,800	17.5	4,100	10.5	7,400	9.3
900	10.0	1,900	14.0	4,200	10.4	7,500	7.8
1,000	9.7	2,000	14.1	4,300	9.1	7,600	7.9
1,100	10.4	2,100	9.7	4,400	10.2	7,700	9.1
1,200	12.0	2,200	8.7	4,500	12.3	7,800	10.9
1,300	12.3	2,300	11.6	4,600	9.9	7,900	8.7
1,400	13.4	2,400	12.2	4,700	11.2	8,000	9.0

Note: Chi-squared statistics, 9 df, 25 Monte Carlo replications.

- **why should we expect vote counts to satisfy 2BL?**
- **while precinct vote counts should satisfy 2BL, counts on voting machines used in each precinct should not**
  - **voting machine counts are subject to "roughly equal division with leftovers" (REDWL)**
  - **simulations verify the REDWL mechanism**

- **why should we expect vote counts to satisfy 2BL?**
- **while precinct vote counts should satisfy 2BL, counts on voting machines used in each precinct should not**
  - **voting machine counts are subject to "roughly equal division with leftovers" (REDWL)**
  - **simulations verify the REDWL mechanism**
- **and actual machine-level vote counts do not satisfy 2BL**

Miami-Dade Election Day Second-digit Benford's Law Tests

item	Benf.	item	Benf.
Bush	17.2	Am. 4 Yes	43.5
Kerry	44.0	Am. 4 No	25.4
Martinez	11.5	Am. 5 Yes	57.6
Castor	12.7	Am. 5 No	25.6
Am. 1 Yes	43.6	Am. 6 Yes	29.7
Am. 1 No	19.8	Am. 6 No	15.3
Am. 2 Yes	38.7	Am. 7 Yes	53.2
Am. 2 No	11.9	Am. 7 No	136.7
Am. 3 Yes	78.0	Am. 8 Yes	54.2
Am. 3 No	25.7	Am. 8 No	23.2

Note: $n = 7,064$ precinct-machines. Pearson chi-squared stats, 9 df.

- **the 2BL test can detect artificial manipulations of vote counts that otherwise satisfy 2BL**
- **simulations show a wide range of ways to manipulate the votes can be detected**
  - **adding votes**
  - **subtracting votes**
  - **switching votes**

Simulated "Repeater" Vote Switching: Receive Votes When Above Expectation

	Receiver (cand. 1)			Donor (cand. 2)		
fraction	500	1000	2000	500	1000	2000
0	9.6	8.7	12.4	11.1	11.9	13.0
0.01	11.2	13.3	15.0	9.3	10.3	11.4
0.02	12.7	17.7	27.1	8.8	12.2	13.2
0.03	15.5	27.2	44.1	10.5	10.7	14.2
0.04	25.6	41.8	68.9	10.9	13.1	16.9
0.05	24.8	38.1	67.2	11.2	13.6	17.1
0.06	23.6	42.2	74.2	12.0	15.1	19.3
0.07	28.2	48.4	89.9	12.9	15.6	22.1
0.08	33.5	58.1	112.8	13.5	17.3	26.5
0.09	32.7	56.5	107.7	12.9	18.0	29.3

Simulated “Repeater” Vote Switching: Receive Votes When Below Expectation

	Receiver (cand. 1)			Donor (cand. 2)		
fraction	500	1000	2000	500	1000	2000
0	9.6	10.3	12.8	9.7	10.3	12.2
0.01	10.0	13.1	15.0	10.4	11.4	14.3
0.02	12.6	18.3	28.0	11.8	12.7	19.9
0.03	18.6	26.8	50.3	13.5	18.3	22.8
0.04	25.9	44.5	80.0	12.4	19.4	26.7
0.05	26.5	45.4	74.8	16.1	21.5	31.4
0.06	28.5	46.6	87.1	14.8	21.5	37.9
0.07	33.1	57.1	102.2	17.0	24.9	42.1
0.08	39.0	71.8	128.4	16.8	26.3	45.4
0.09	38.0	68.1	126.9	19.6	27.0	40.9

- **wider application of the 2BL test: recent American presidential votes**
  - **precinct vote counts in the 2000 and 2004 elections, separately for the precincts in each county**
  - **impose FDR-control using the number of counties in each state**
    - **(see maps [in showmappbenf0004fdr.R])**

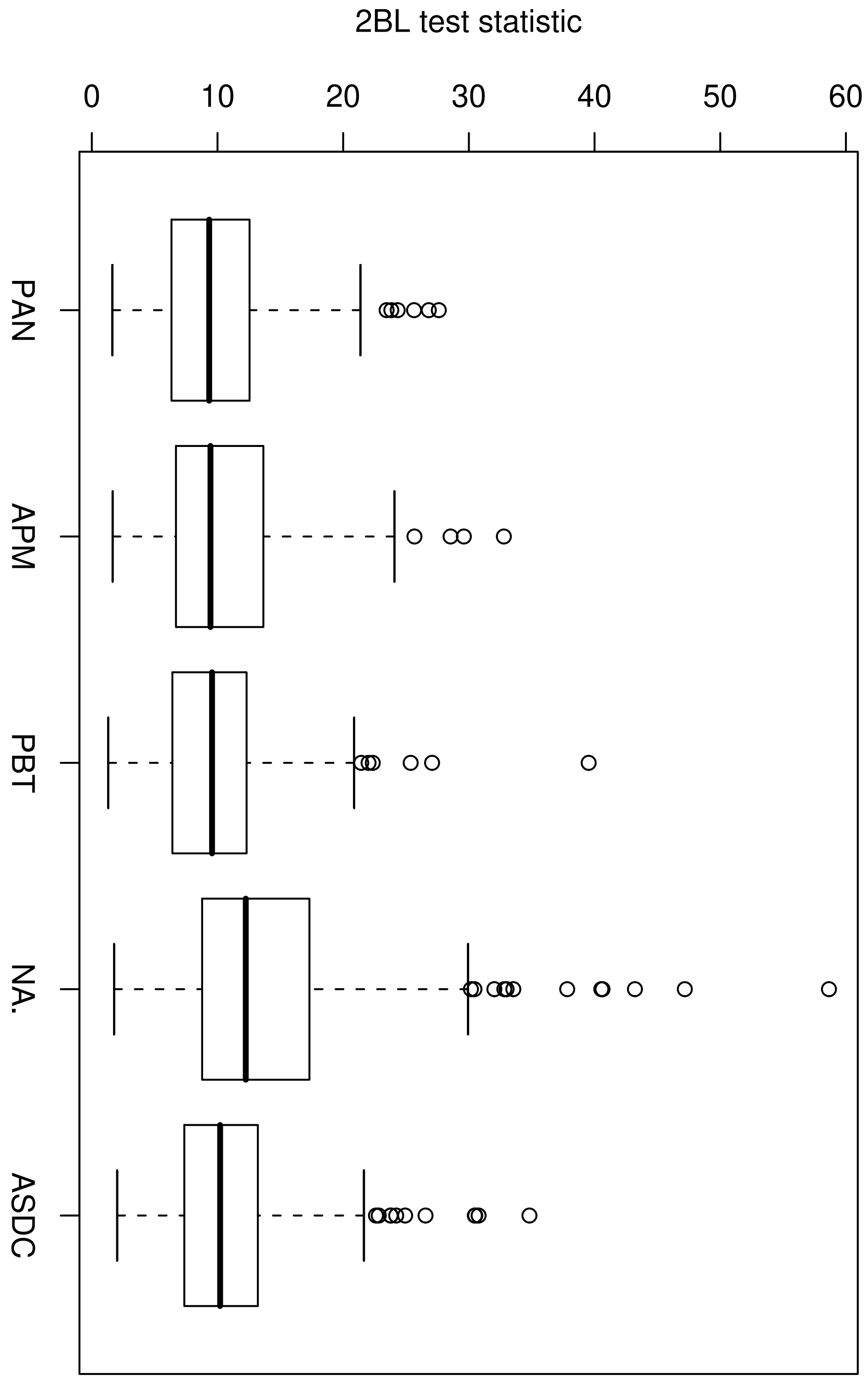
Counties with Signficant 2BL Tests using State-specific FDR Adjustment: 2000

		Gore votes		Bush votes	
County	J	d_2	$X^2_{B_2}$	d_2	$X^2_{B_2}$
Los Angeles, CA	5,045	5,011	54.8	4,930	20.3
Kent, DE	61	61	9.0	61	22.2
Latah, ID	34	31	36.7	34	3.8
Cook, IL	5,179	5,097	46.7	4,145	24.4
Dupage, IL	714	714	28.0	714	41.6
Lake, IL	403	403	33.7	402	16.1
Passaic, NJ	295	295	27.7	294	5.6
Hamilton, OH	1,025	1,020	48.7	988	8.9
Hancock, OH	67	67	34.3	67	9.9
Summit, OH	624	624	31.6	612	11.6
Philadelphia, PA	1,681	1,680	29.5	1,249	34.7
King, WA	2,683	2,665	27.0	2,641	8.9

Counties with Signficant 2BL Tests using State-specific FDR Adjustment: 2004

		Kerry votes		Bush votes	
County	J	d_2	$X^2_{B_2}$	d_2	$X^2_{B_2}$
Los Angeles, CA	4,984	4,951	70.2	4,929	12.4
Orange, CA	1,985	1,887	26.2	1,904	32.6
Jefferson, CO	324	323	30.0	323	10.4
Kootenai, ID	75	75	30.9	75	12.1
Cook, IL	4,562	4,561	44.5	4,026	27.8
DuPage, IL	732	732	35.2	732	9.1
Clay, MO	76	76	28.4	76	4.0
Summit, OH	475	475	42.7	474	21.0
Davis, UT	213	212	42.6	213	6.0
Utah, UT	247	241	9.2	246	27.6
Benton, WA	177	168	29.2	173	14.8

- **the 2BL test applied to votes for president in the 2006 Mexican election**
  - **seccion vote counts, separately for the secciones in each legislative district**
  - **over all 300 districts, the FDR-controlled critical value for χ^2_9 is 32.4**
  - **over 1500 district-party combinations, the FDR-controlled critical value for χ^2_9 is 36.4**


2BL test statistic
0
10
20
30
40
50
60
PAN
APM
PBT
NA.
ASDC

- **the statistical tests and the partial recount done of votes for president in the 2006 Mexican election**
  - **the original count included 41,791,322 ballots**
  - **40,588,729 votes were recorded for one of the parties**
  - **the original difference between the PAN and PBT vote totals was 243,934 votes, which is 0.58 percent of the ballots cast**

- **the statistical tests and the partial recount done of votes for president in the 2006 Mexican election**
  - **the original count included 41,791,322 ballots**
  - **40,588,729 votes were recorded for one of the parties**
  - **the original difference between the PAN and PBT vote totals was 243,934 votes, which is 0.58 percent of the ballots cast**
- **the recount**
  - **about nine percent of the casillas were manually recounted**
  - **I use data from 11,651 recounted casillas (which I think is all of them)**

Net Vote Count Changes in the Mexico 2006 Recount

	PAN	APM	PBT	NA.	ASDC
original	15,000,284	9,301,441	14,756,350	401,804	1,128,850
change	$-13,333$	$-1,885$	-58	$-1,578$	$1,836$

Note: Some of the recounted votes included here are from casillas that were canceled in the final official results.

- **relationship between the 2006 Mexican recount changes and the two kinds of statistical tests**
- **definitions for casilla-level variables**

$$\textsf{\textbf{CHANGE}} = \begin{cases} 1, & \textsf{\textbf{if the vote count changed for any party}} \\ 0, & \textsf{\textbf{otherwise}} \end{cases}$$

$$\textsf{\textbf{NULOS2}} = \begin{cases} 1, & \textsf{\textbf{if the votos nulos }} |\textsf{\textbf{residual}}| \geq 2 \\ 0, & \textsf{\textbf{otherwise}} \end{cases}$$

- **definitions for district-level variable**

$$\textsf{\textbf{2BL}} = \begin{cases} 1, & \textsf{\textbf{if the 2BL statistic for any party}} \geq 16.9 \\ 0, & \textsf{\textbf{otherwise}} \end{cases}$$

Recount Changes and Test Statistics

	CHANGE		
NULOS2	0	1	n
0	0.33	0.67	9,200
1	0.28	0.72	2,215

Pearson chi-squared = 20.1

	CHANGE		
2BL	0	1	n
0	0.29	0.71	5,001
1	0.33	0.67	6,650

Pearson chi-squared = 21.5

- **relationship between the 2006 Mexican recount changes and the two kinds of statistical tests**
  - **unusually large votos nulos counts for a casilla are associated with more vote count changes if that casilla is recounted**
  - **unusually large 2BL test statistics for a district are associated with fewer vote count changes when casillas in that district are recounted**
- **does this mean that the 2BL test is picking up the fact that votes were faked, in ways that the recount did not detect?**

- **relationship between the 2006 Mexican recount changes and the two kinds of statistical tests**
- **is the 2BL test picking up the fact that votes were faked, in ways that the recount did not detect?**
- **consider the possibility of strategic voting (to mw07.pdf)**

- **is election manipulation election fraud?**
- **are either election manipulation or election fraud heresthetic?**
  - **election manipulation as dimension manipulation (unlikely)**
  - **election manipulation as agenda control**
  - **election manipulation as strategic voting**
- **the key issue is dictatorship (or oligarchy), which heresthetic (via Arrow's theorem) is normatively justified to oppose**
- **election fraud seems intuitively to be dictatorial, but why is that?**