Government 320: Public Opinion and Public Choice

Spring 2006
Tuesday and Thursday 2:55–4:10 (MG 165)
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Course web page:
`http://macht.arts.cornell.edu/wrm1/gov320.html`

- heresthetic and the critique of democracy: instability and meaninglessness
- based on social choice theory, in particular on Arrow's Theorem
- assumes individual rationality

- **rationality: transitive and complete individual preference orders**

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- **transitivity:**
  - **transitivity of strict preferences: for every individual** i **and all alternatives** a**,** b **and** c**,** aP_ib **and** bP_ic **implies** aP_ic
  - **transitivity of indifference:** aI_ib **and** bI_ic **implies** aI_ic
  - **transitivity of weak preference:** aR_ib **means** aP_ib **or** aI_ib**;** aR_ib **and** bR_ic **implies** aR_ic

- **rationality: transitive and complete individual preference orders**
- **transitivity:**
  - **transitivity of strict preferences: for every individual i and all alternatives a, b and c, aP_ib and bP_ic implies aP_ic**
  - **transitivity of indifference: aI_ib and bI_ic implies aI_ic**
  - **transitivity of weak preference: aR_ib means aP_ib or aI_ib; aR_ib and bR_ic implies aR_ic**
- **completeness:**
  - **for every individual i and all alternatives a and b, aR_ib or bR_ia**
  - **incompleteness, which implies inability to choose, is not the same as indifference (Buridan's ass example)**

- axiomatic social choice theory
  - state as axioms general properties of any collective choice procedure (or any collective preference procedure)
  - derive characteristic properties of any procedure that the axioms describe
  - this may include showing that some desirable properties are mutually incompatible

- **example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
  - cP_iaP_ib
  - bP_icP_ia

- **example (and point of departure): Condorcet's paradox (a.k.a. the paradox of voting)**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
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- **consider majority rule as the collective choice procedure**
- **because majority rule considers only two alternatives at a time, the three alternatives must be considered in some order**
- **the order in which the votes occur is an agenda**

- **example: the paradox of voting**
- **consider three voters with the following preferences:**
  - aP_ibP_ic
  - cP_iaP_ib
  - bP_icP_ia
- **consider three agendas with pairwise majority votes:**
  - a **versus** b**, then winner versus** c
  - a **versus** c**, then winner versus** b
  - b **versus** c**, then winner versus** a

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  - a **versus** b**, then winner versus** c
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  - b **versus** c**, then winner versus** a
- **the outcome depends on the agenda**

- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **consider three agendas with pairwise majority votes and sophisticated voting:**
- **with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently**

- **consider three voters with the following preferences:**
  - aP_ibP_ic**,** cP_iaP_ib**,** bP_icP_ia
- **consider three agendas with pairwise majority votes and sophisticated voting:**
- **with sophisticated voting, each voter looks ahead: the value of the current vote depends on what will happen subsequently**
- **this is strategic voting only if one considers each step in the agenda a distinct collective choice process**
- **but it is different in that each voter's vote depends on everyone else's preferences**

- **consider three voters with the following preferences:**
  - aP_ibP_ic, cP_iaP_ib, bP_icP_ia
- **pairwise majority votes and sophisticated voting:**
- **everyone knows everyone votes sincerely in the last stage**
  - a **versus** b**, then winner versus** c
    - **the first vote is really** c **versus** b**, because** cPa
    - **so the winner is** b **(voter 1 votes for** b **not** a**)**

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    * **the first vote is really** a **versus** b**, because** bPc
    * **so the winner is** a **(voter 2 votes for** a **not** c**)**
  - b **versus** c**, then winner versus** a
    * **the first vote is really** c **versus** a**, because** aPb
    * **so the winner is** c **(voter 3 votes for** c **not** b**)**

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    - **the winner is** a
  - b **versus** c**, then winner versus** a
    - **the winner is** c

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    - **the winner is** c
- **the outcome depends on the agenda**

- **examples of voting rules**
  - **majority rule**
  - **plurality voting with one winner (winner take all, first past the post)**
  - **plurality voting with $M > 1$**
  - **approval voting**
  - **Borda count**
  - **utilitarian systems**

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  - **utilitarian systems**
- **there is extensive theory for single districts, little for whole systems**

- **heresthetic: changing outcomes without changing preferences**

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- **heresthetic versus rhetoric (i.e., manipulation versus persuasion)**
  - **every individual's preference ordering remaining the same is the definition of no persuasion**
  - **changing people's beliefs about what the decision is is not considered persuasion**
  - **likewise, changing beliefs about what's at stake in a decision, or about the consequences of a decision, is not considered persuasion**

- **heresthetic: three basic kinds of manipulation**
  - **strategic voting**
  - **agenda control**
  - **dimension manipulation**

- **heresthetic: stories grouped by type manipulation (according to Riker)**
  - **strategic voting: Pliny (7); Massachusetts and SC delegates (8); Chicester (9); anti-school aid Republicans (11); Chrystal (5); Morris (4)**
  - **agenda control: Plott and Levine (3); Pliny (7); Reed and Norris (12)**
  - **dimension manipulation: Lincoln (1); DePew (2); Chrystal (5); city manager (6); Magnuson (10); Morris (4)**

- **dimension manipulation illustration: Warren Magnuson**
- **alternatives**
  - o: **leave shells of gas in Okinawa**
  - a: **bring nerve gas to Alaska**
  - G: **detoxify gas "outside the U.S." (Gravel amendment)**
  - d: **support defense department**
  - n: **support Alaskan interests**
  - s: **support constitutional authority of the Senate**

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  - s: **support constitutional authority of the Senate**
- **distinguish actions (for or against G) from implications of actions: preferences refer to implications**
  - **before Magnuson maneuver: for G means (o, n); against G means (a, d)**
  - **after Magnuson maneuver: for G means (o, n, s) (to some senators); against G means (a, d)**

- dimension manipulation illustration: Warren Magnuson
- dimension manipulation is covert agenda control (sort of)
  - but what about Byrd and Thurmond, who "were unmoved by Magnuson's heresthetic"? likewise the "Church-Cooper supporters"?
  - Magnuson's implicit "amendment" via trying to redefine G only partially succeeded; not everyone saw it the same way
  - ("common knowledge" seems to be lacking)

- dimension manipulation illustration: Warren Magnuson
- different facts? (Mackie)
  - in Mackie's version the original Magnuson amendment is still on the table and is the actual reversion point if G fails
  - so Mackie says Riker has the legislative situation wrong

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  - hence according to Mackie there is
    - w: no nerve gas "to the United States"
    - before Magnuson maneuver: for G means (o, n); against G means (d, w)
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    - before Magnuson maneuver: for G means (o, n); against G means (d, w)
    - after Magnuson maneuver: the same; s was irrelevant
  - Mackie says Thurmond and Byrd cared about textiles, not s
  - moreover s was not about Senate prerogatives but just a tiff with the defense department

- **axiomatic social choice theory**
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- **how to generalize from the Condorcet paradox example? Of what is it an example?**
- **examples of properties of social choice rules**
  - **choice from all subsets**
  - **unrestricted preferences**

- **examples of properties of social choice rules**
  - **unanimity: if everyone has identical preferences, including aP_ib, then a is the sole choice from $\{a, b\}$**
  - **weak Pareto: if for everyone aP_ib, then a is the sole choice from $\{a, b\}$ regardless of what their other preferences are**
  - **strong Pareto: if one person has aP_ib and no one prefers b to a, then a is the sole choice from $\{a, b\}$**

- **examples of properties of social choice rules**
  - **independence: the social choice from a set of alternatives depends only on the preferences individuals have about the alternatives in that set**
  - **pairwise independence: the social choice from $\{a, b\}$ depends only on individuals' preferences between a and b**

- **majority voting satisfies pairwise independence**

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- **first-past-the-post (winner is the one with the most votes) does not (Craven's version)**
- **preferences showing first-past-the-post violates independence: choose from $\{b, c\}$ while allowing votes to be cast for any of $\{a, b, c\}$**

individuals	**I**	**II**
1	bP_1cP_1a	aP_1bP_1c
2–10	aP_icP_ib	cP_ibP_ia

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1	bP_1cP_1a	aP_1bP_1c
2–10	aP_icP_ib	cP_ibP_ia

- **this may not be as crazy as it may appear: let a denote abstention**

- **the Borda count does not satisfy independence**
- **preferences showing the Borda count violates independence:**

individuals	**I**	**II**
1,2	aP_ibP_ic	aP_icP_ib
3,4	bP_iaP_ic	bP_iaP_ic
5	cP_5bP_5a	cP_5bP_5a

- **the preferences regarding a and b are not changed, but b is chosen in I while a is chosen in II**

- **what properties describe majority rule?**
- **majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence**

- **what properties describe majority rule?**
- **majority rule satisfies choice from all subsets, unrestricted preferences, weak Pareto and pairwise independence**
- **majority rule is the only social choice rule that satisfies independence, positive responsiveness, symmetry and anonymity**
  - **positive responsiveness: if there are two preference profiles R_i and R_i' such that $aP_i'b$ whenever aP_ib and $aR_i'b$ whenever aI_ib, then if $C(a,b) = \{a\}$ with preferences R_i then $C(a,b) = \{a\}$ with preferences R_i'**
  - **anonymity (exchanging two voters' preferences does not change the social choice)**
  - **symmetry (exchanging a and b in everyone's preferences means a and b are exchanged in the social choice)**

- **an undesirable property: dictatorship**
- **dictatorship: if** $C_i(a, T) = a$**, then** $C(a, T) = a$**, and if** $C_i(U, T) = U$ **then** $C(U, T) \subset U$

- **other desirable properties**
  - **rationality (individual)**
    - **RC1: if** $C_i(a,b) = \{a\}$ **and** $C_i(b,c) = \{b\}$ **then** $C_i(a,c) = \{a\}$
    - **RC2: if** $C_i(a,b) = \{a,b\}$ **and** $C_i(b,c) = \{b,c\}$ **then** $C_i(a,c) = \{a,c\}$
    - **RC3: if** a **is in** $C_i(T)$ **and** b **is in** T**, then** a **is in** $C_i(a,b)$
    - **RC4: if** a **is in** $C_i(a,b)$ **and** a **is in** $C_i(T)$**, then** a **is in** $C_(T \cup b)$

- **other desirable properties**
  - **collective rationality**
    - **RC1: if** $C(a, b) = \{a\}$ **and** $C(b, c) = \{b\}$ **then** $C(a, c) = \{a\}$
    - **RC2: if** $C(a, b) = \{a, b\}$ **and** $C(b, c) = \{b, c\}$ **then** $C(a, c) = \{a, c\}$
    - **RC3: if** a **is in** $C(T)$ **and** b **is in** T**, then** a **is in** $C(a, b)$
    - **RC4: if** a **is in** $C(a, b)$ **and** a **is in** $C(T)$**, then** a **is in** $C_(T \cup b)$

- **other desirable properties**
  - **collective rationality**
    - **RC1: if $C(a, b) = \{a\}$ and $C(b, c) = \{b\}$ then** $C(a, c) = \{a\}$
    - **RC2: if $C(a, b) = \{a, b\}$ and $C(b, c) = \{b, c\}$ then** $C(a, c) = \{a, c\}$
    - **RC3: if a is in $C(T)$ and b is in T, then a is in $C(a, b)$**
    - **RC4: if a is in $C(a, b)$ and a is in $C(T)$, then a is in** $C_(T \cup b)$
- **majority rule fails collective rationality, since it produces voting cycles**

- **Arrow's theorem**
- **generalizes the conclusions about majority rule, almost universally**

- **Arrow's theorem (Vickrey variant)**
- **Theorem 3.3: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  pairwise independence

  weak Pareto

  and if the social choices satisfy rationality conditions RC1 to RC4

- **Arrow's theorem (Vickrey variant)**
- **proof in two steps**
  1. **epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives**
  2. **the dictator: show that the fully decisive set can contain only one individual**

- **epidemic**
- **set D of individuals is semidecisive for a over b if $C(a, b) = \{a\}$ when aP_ib for everyone in D but bP_ia for everyone else**
- **assume D is semidecisive for a over b**

- **epidemic**
- **set** D **of individuals is semidecisive for** a **over** b **if** $C(a,b) = \{a\}$ **when** aP_ib **for everyone in** D **but** bP_ia **for everyone else**
- **assume** D **is semidecisive for** a **over** b
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **epidemic**
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- **unrestricted preferences make all these orders relevant**
- **pairwise independence means these orders can be considered in isolation from the rest of the preference orders, which may vary within each set of individuals**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **assume D is semidecisive for a over b**
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individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **I: $C(a,b) = \{a\}$ (semidecisive assumption); $C(b,c) = \{b\}$ (weak Pareto); $C(a,c) = \{a\}$ (RC1); D is semidecisive for a over c (definition)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **I:** $C(a,b) = \{a\}$ **(semidecisive assumption);** $C(b,c) = \{b\}$ **(weak Pareto);** $C(a,c) = \{a\}$ **(RC1);** D **is semidecisive for** a **over** c **(definition)**
- **II:** $C(a,b) = \{a\}$ **(semidecisive assumption);** $C(d,a) = \{d\}$ **(weak Pareto);** $C(d,b) = \{d\}$ **(RC1);** D **is semidecisive for** d **over** b **(definition)**

- **combining and generalizing examples I and II gives**
- **SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y**
- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**

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- SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y
- SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y
- combining and generalizing examples I and II gives
- SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b

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- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**
- **combining and generalizing examples I and II gives**
- **SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b**
- **SD4: if D is semidecisive for a over b, then SD2 implies that D is semidecisive for a over any alternative y**

- **combining and generalizing examples I and II gives**
- **SD1: if D is semidecisive for a over an alternative y, then D is semidecisive for any other alternative x over y**
- **SD2: if D is semidecisive for an alternative x over b, then D is semidecisive for x over any other alternative y**
- **combining and generalizing examples I and II gives**
- **SD3: if D is semidecisive for a over b, then SD1 implies that D is semidecisive for any alternative x over b**
- **SD4: if D is semidecisive for a over b, then SD2 implies that D is semidecisive for a over any alternative y**
- **SD5: if D is semidecisive for a over b, then SD3 and SD4 imply that D is semidecisive for any x over any y (except for b over a)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **III: $C(b,c) = \{b\}$ (SD5); $C(a,c) = \{c\}$ (weak Pareto); $C(a,b) = \{b\}$ (RC1); D is semidecisive for b over a (definition)**

- **assume D is semidecisive for a over b**
- **preferences demonstrating semidecisiveness epidemic:**

individuals	**I**	**II**	**III**
in D	aP_ibP_ic	dP_iaP_ib	bP_icP_ia
rest	bP_icP_ia	bP_idP_ia	cP_iaP_ib

- **III:** $C(b,c) = \{b\}$ **(SD5);** $C(a,c) = \{c\}$ **(weak Pareto);** $C(a,b) = \{b\}$ **(RC1);** D **is semidecisive for** b **over** a **(definition)**
- **if D is semidecisive for a over b, then D is semidecisive for any x over any y**

- **semidecisiveness implies decisiveness**
- D **is a decisive set of individuals if for any alternatives** x **and** y**,** $C(x, y) = \{x\}$ **whenever** xP_iy **for everyone in** D **whatever preferences others have**

- **semidecisiveness implies decisiveness**
- **assume D is semidecisive (i.e., for any x over any y)**

- **semidecisiveness implies decisiveness**
- **assume D is semidecisive (i.e., for any x over any y)**
- **preferences demonstrating decisiveness:**

  individuals

in D	dP_ieP_ic
in E	eP_idP_ic
in F	eP_icP_id
rest	eP_icI_id

- $C(d,e) = \{d\}$ **(semidecisiveness);** $C(c,e) = \{e\}$ **(weak Pareto);** $C(c,d) = \{d\}$ **(RC1);** D **is decisive for** d **over** c **(definition)**

- **extension to decisiveness over choices from sets larger than pairs**
- **assume** D **is a decisive set**
- **assume everyone in** D **has** aP_ib **for all other** $b \in T$
- **then** $C(a, b) = \{a\}$ **for all** $b \in T$

- **extension to decisiveness over choices from sets larger than pairs**
- **assume D is a decisive set**
- **assume everyone in D has aP_ib for all other $b \in T$**
- **then $C(a, b) = \{a\}$ for all $b \in T$**
- **let $b \neq a$ and $b \in T$ and suppose $b \in C(T)$; then $b \in C(a, b)$ (RC3), which contradicts $C(a, b) = \{a\}$; therefore $b \notin C(T)$; therefore $C(T) = \{a\}$**

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- **then** $C(a,b) = \{a\}$ **for all** $b \in T$
- **let** $b \neq a$ **and** $b \in T$ **and suppose** $b \in C(T)$**; then** $b \in C(a,b)$ **(RC3), which contradicts** $C(a,b) = \{a\}$**; therefore** $b \notin C(T)$**; therefore** $C(T) = \{a\}$
- $C(T) = \{a\}$ **matches the preference of the members of** D

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- **let** $b \neq a$ **and** $b \in T$ **and suppose** $b \in C(T)$**; then** $b \in C(a, b)$ **(RC3), which contradicts** $C(a, b) = \{a\}$**; therefore** $b \notin C(T)$**; therefore** $C(T) = \{a\}$
- $C(T) = \{a\}$ **matches the preference of the members of** D
- **a set** D **is fully decisive if** D **can ensure that the most preferred alternative of all individuals in** D **is the social choice**

- **Arrow's theorem (Vickrey variant)**
- **proof in two steps**
  1. **epidemic of decisiveness: show that semidecisiveness over any one pair of alternatives implies full decisiveness over all choices from every set of alternatives**
  2. **the dictator: show that the fully decisive set can contain only one individual**

- **the dictator**
- **the collective**

    i. **a collective is the smallest fully decisive set**

    ii. **all sets that include all members of the collective are fully decisive**

    iii. **no set that does not include all members of the collective is fully decisive**

    iv. **there is only one collective**

- **the collective**
- **intersecting fully decisive sets: D and E are fully decisive sets that have members in common**

in D and in E	bP_icP_ia
in D, not in E	aP_ibP_ic
in E, not in D	cP_iaP_ib
rest	aP_icP_ib

- **the collective**
- **intersecting fully decisive sets: D and E are fully decisive sets that have members in common**

in D and in E	bP_icP_ia
in D, not in E	aP_ibP_ic
in E, not in D	cP_iaP_ib
rest	aP_icP_ib

- $C(b,c) = \{b\}$ **(D decisive);** $C(a,c) = \{c\}$ **(E decisive);** $C(a,b) = \{b\}$ **(RC1);** $D \cap E$ **is semidecisive for** b **over** a**; therefore** $D \cap E$ **is a fully decisive set**

- **the dictator**
- **suppose the collective contains two or more members**
- **let some but not all collective members be in** E
- **these assumptions lead to a contradiction**

  in E $\quad aP_icP_ib$

  rest $\quad cP_ibP_ia$

- $C(a,b) = \{a,b\}$ **(neither subset is semidecisive);**
  $C(a,c) = \{a,c\}$ **(neither subset is semidecisive);**
  $C(b,c) = \{b,c\}$ **(RC2); but** $C(b,c) = \{b,c\}$ **violates weak Pareto**

- **the dictator**
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- $C(a, b) = \{a, b\}$ **(neither subset is semidecisive);** $C(a, c) = \{a, c\}$ **(neither subset is semidecisive);** $C(b, c) = \{b, c\}$ **(RC2); but** $C(b, c) = \{b, c\}$ **violates weak Pareto**
- **if only one subset contains the collective, there is no contradiction**
- **the collective is impossible to divide only if has only one member**

- **Arrow's theorem (as proven)**
- **Theorem 3.7: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  pairwise independence

  weak Pareto

  and if the social choices satisfy rationality conditions RC1, RC2 and RC3

- **manipulation**

- **manipulation**
- **a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences**

- manipulation
- a social choice rule is open to manipulation if an individual can obtain a more preferred collective choice by having the rule use preferences other than his own while using everyone else's true preferences
- a rule is non-manipulable if it is not open to manipulation by any individual on any set of alternatives

- **Gibbard-Satterthwaite theorem (variant)**
- **Theorem 5.4: There is a dictator if there are at least three alternatives and the social choice rule satisfies**

  choice from all subsets

  unrestricted preferences

  weak Pareto

  non-manipulability

  and if the social choices satisfy rationality conditions RC1 to RC4

- **pairwise independence rules out interpersonal cardinal utility comparisons**
- **von Neumann-Morgenstern utility**
  - **utility is invariant up to an affine transformation:** $v = a + bu$
  - **utility combines linearly with probability, with convexity:** $u(x) > u(y) > u(z)$ **implies there exists some probability** p **such that** $pu(x) + (1-p)u(z) = u(y)$ **and such that** $u(px + (1-p)z) = u(y)$ **and**
  - **utility can be determined by ascertaining all choices among lotteries: prefer** $px + (1-p)z$ **or** y **?**

- **Theorem 9.1: There is a dictator if a social choice rule defines complete and transitive social preferences and satisfies**

  unrestricted preferences

  weak Pareto

  independence of positive linear transformations

  independence

- **observing that the last two conditions imply pairwise independence, a proof proceeds along the lines of the original Arrow's Theorem**

- Theorem 9.1: There is a dictator if a social choice rule defines complete and transitive social preferences and satisfies

  unrestricted preferences

  weak Pareto

  independence of positive linear transformations

  independence

- observing that the last two conditions imply pairwise independence, a proof proceeds along the lines of the original Arrow's Theorem

- Arrow's Theorem can be averted if measurable and interpersonally comparable and additive utilities can be used. But it is widely disbelieved that such utilities can be found.

- Riker and Arrow (heresthetic and the general impossibility theorem)
- Arrow's Theorem (variant): There is a dictator if there are at least three alternatives and the social choice rule satisfies: choice from all subsets; unrestricted preferences; pairwise independence; weak Pareto; and if the social choices satisfy rationality conditions RC1, RC2 and RC3
- heresthetic is motivated by the basic principles: avoid dictatorship; ensure that collective choices reflect everyone's preferences; have collective choices be effective

- **Riker and Arrow (heresthetic and the general impossibility theorem)**
- **Arrow's Theorem (variant): There is a dictator if there are at least three alternatives and the social choice rule satisfies: choice from all subsets; unrestricted preferences; pairwise independence; weak Pareto; and if the social choices satisfy rationality conditions RC1, RC2 and RC3**
- **heresthetic is motivated by the basic principles: avoid dictatorship; ensure that collective choices reflect everyone's preferences; have collective choices be effective**
- **agenda control: follows from giving up transitivity**
- **dimension manipulation: follows from giving up independence**
- **strategic voting: follows from giving up independence, via nonmanipulability**

- **the spatial model**
  - **are preferences single peaked?**
  - **example: YES**

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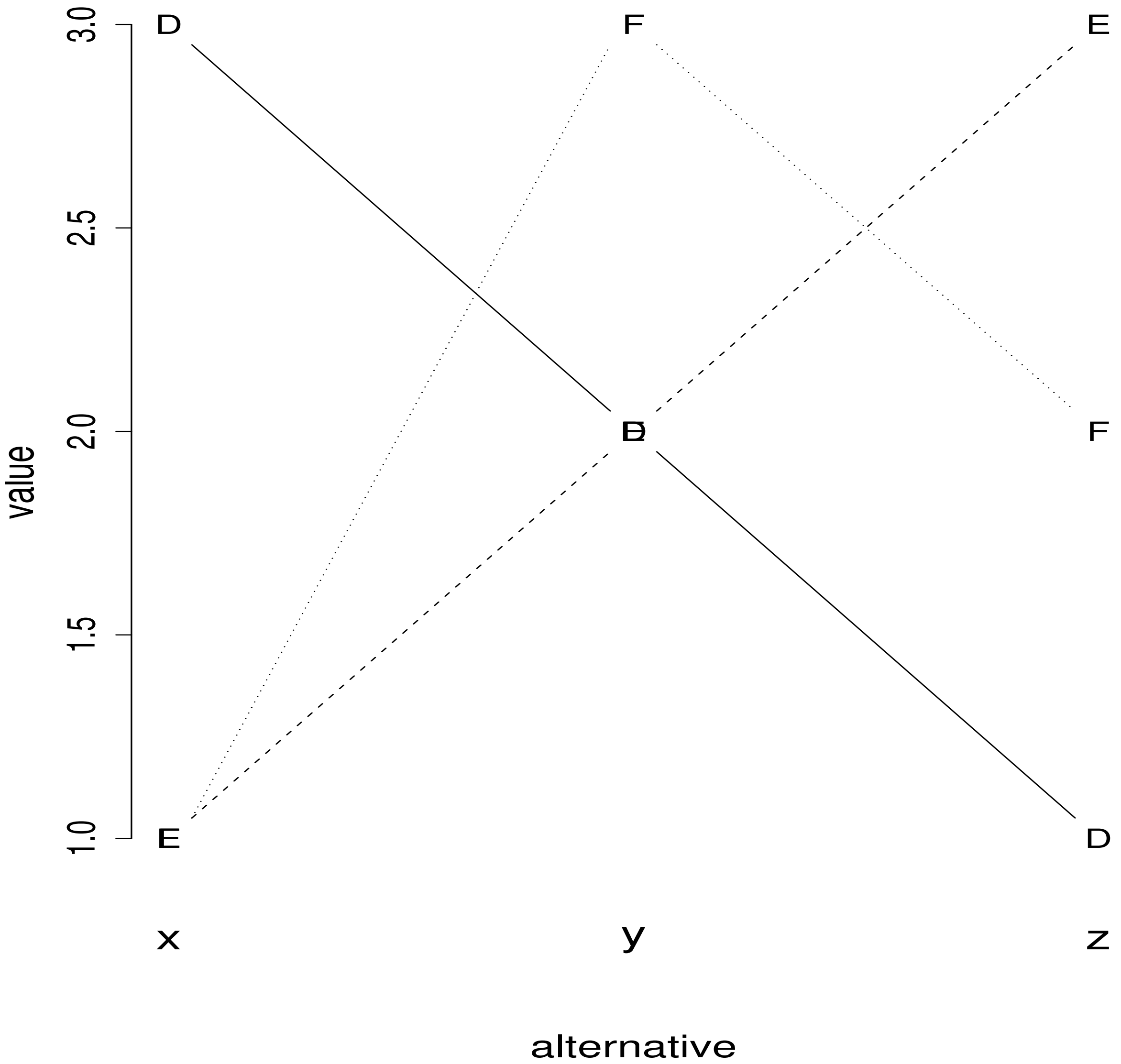
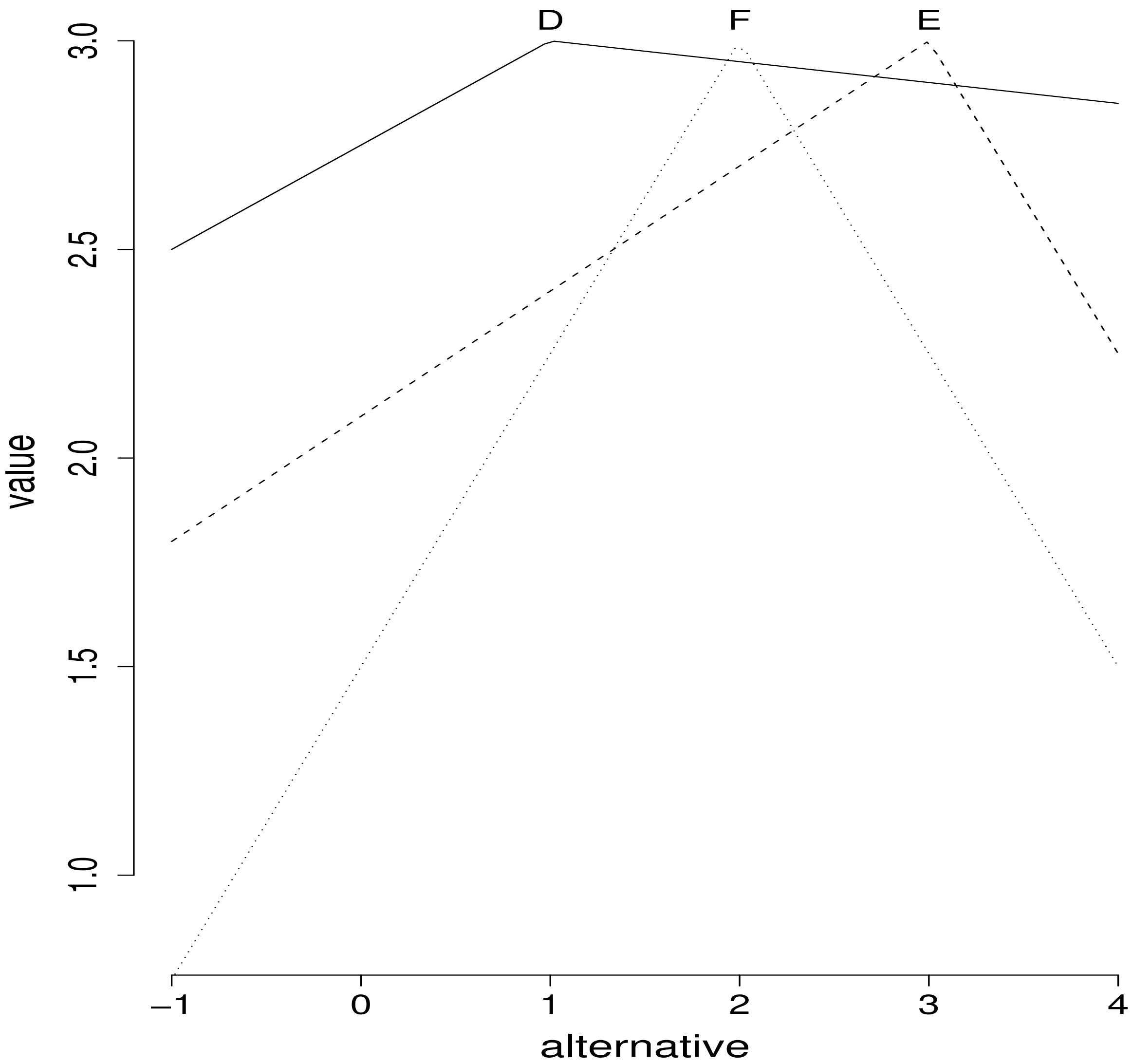
  - **example: YES**
  - **preferences:**
    - **person D:** xP_iyP_iz
    - **person E:** zP_iyP_ix
    - **person F:** yP_izP_ix

- **the spatial model**
  - **are preferences single peaked?**
  - **example: YES**
  - **preferences:**
    - **person D:** xP_iyP_iz
    - **person E:** zP_iyP_ix
    - **person F:** yP_izP_ix
  - **each person has an IDEAL POINT**


value
3.0
2.5
2.0
1.5
1.0
D
F
E
B
F
E
D
x
y
z
alternative

- **the spatial model**
- **single-peaked preferences**

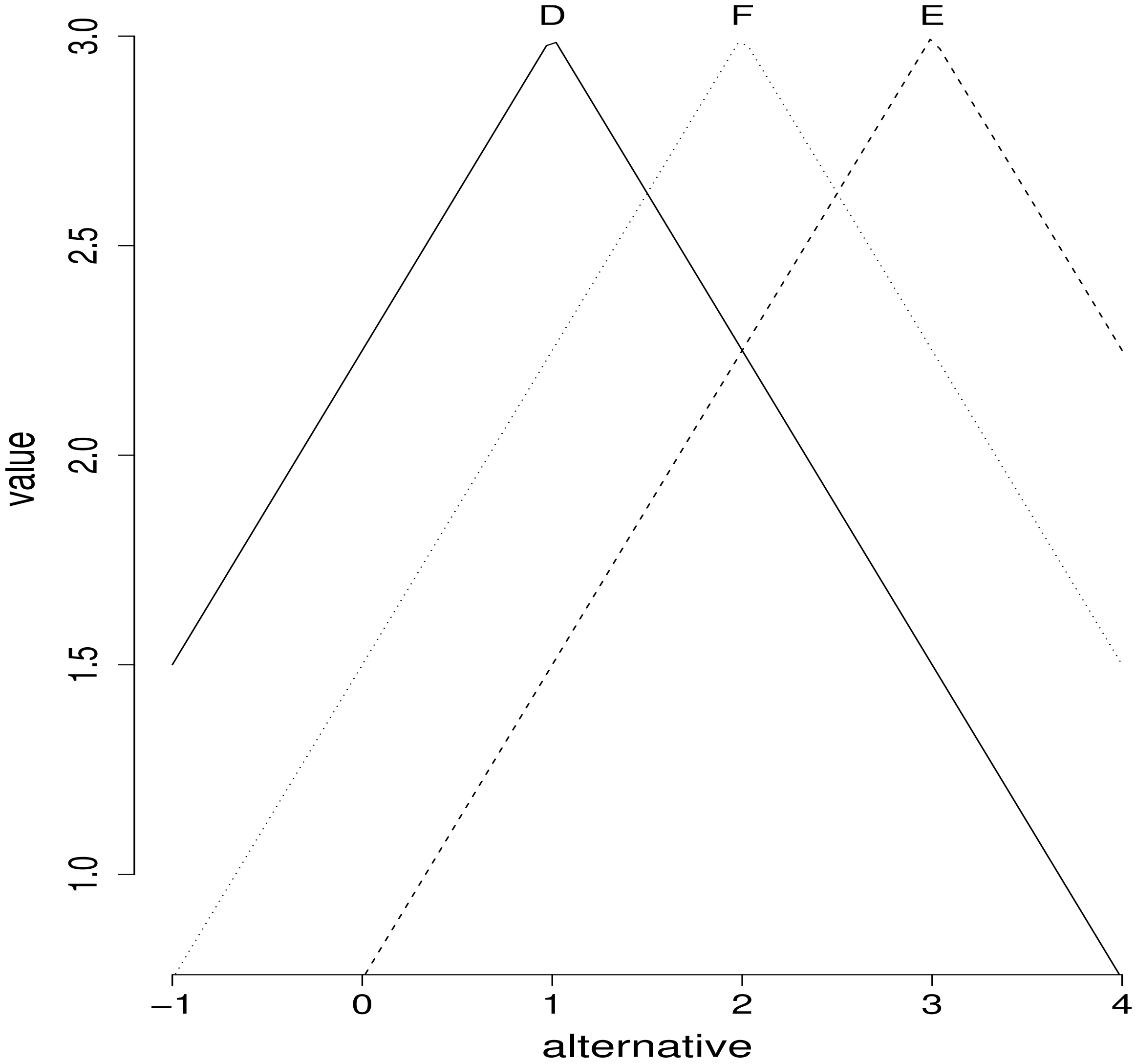
- **the spatial model**
- **single-peaked preferences**
- **which alternative wins in a series of pairwise votes?**



D
F
E
3.0
2.5
2.0
1.5
1.0
value
−1
0
1
2
3
4
alternative

- **the spatial model**
- **single-peaked preferences**
- **which alternative wins in a series of pairwise votes?**
- **with single-peaked preferences, the median is the Condorcet winner**

- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **example with symmetry and same distances for different people**
  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$


D
F
E
3.0
2.5
2.0
1.5
1.0
value
−1
0
1
2
3
4
alternative

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  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$
  - **apply an affine transformation:** $y_i = a + cx_i$

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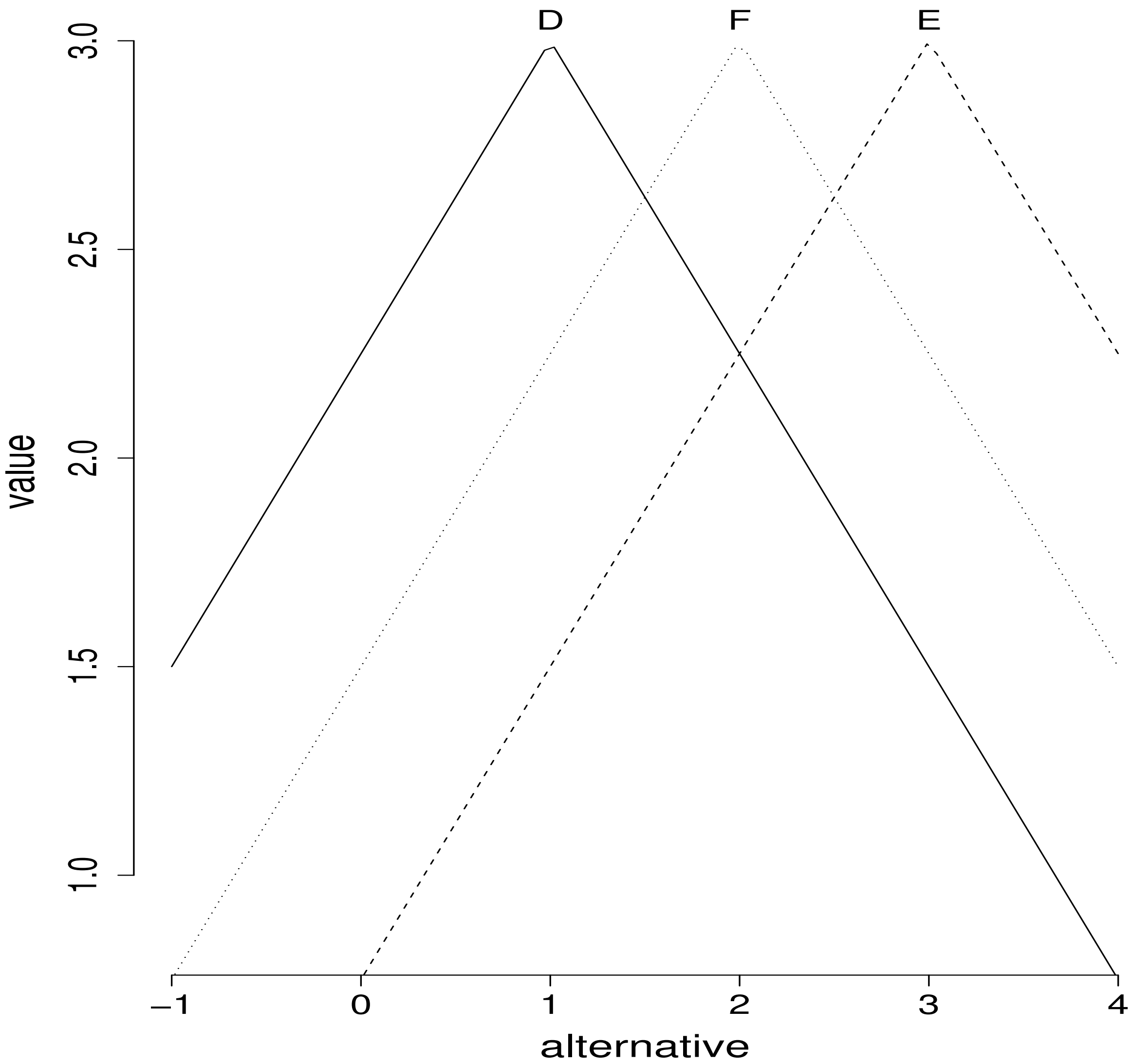
  - **example with symmetry and same distances for different people**
  - **tent metrics:** $u_{ik} = -b|x_k - x_i|$
  - **apply an affine transformation:** $y_i = a + cx_i$

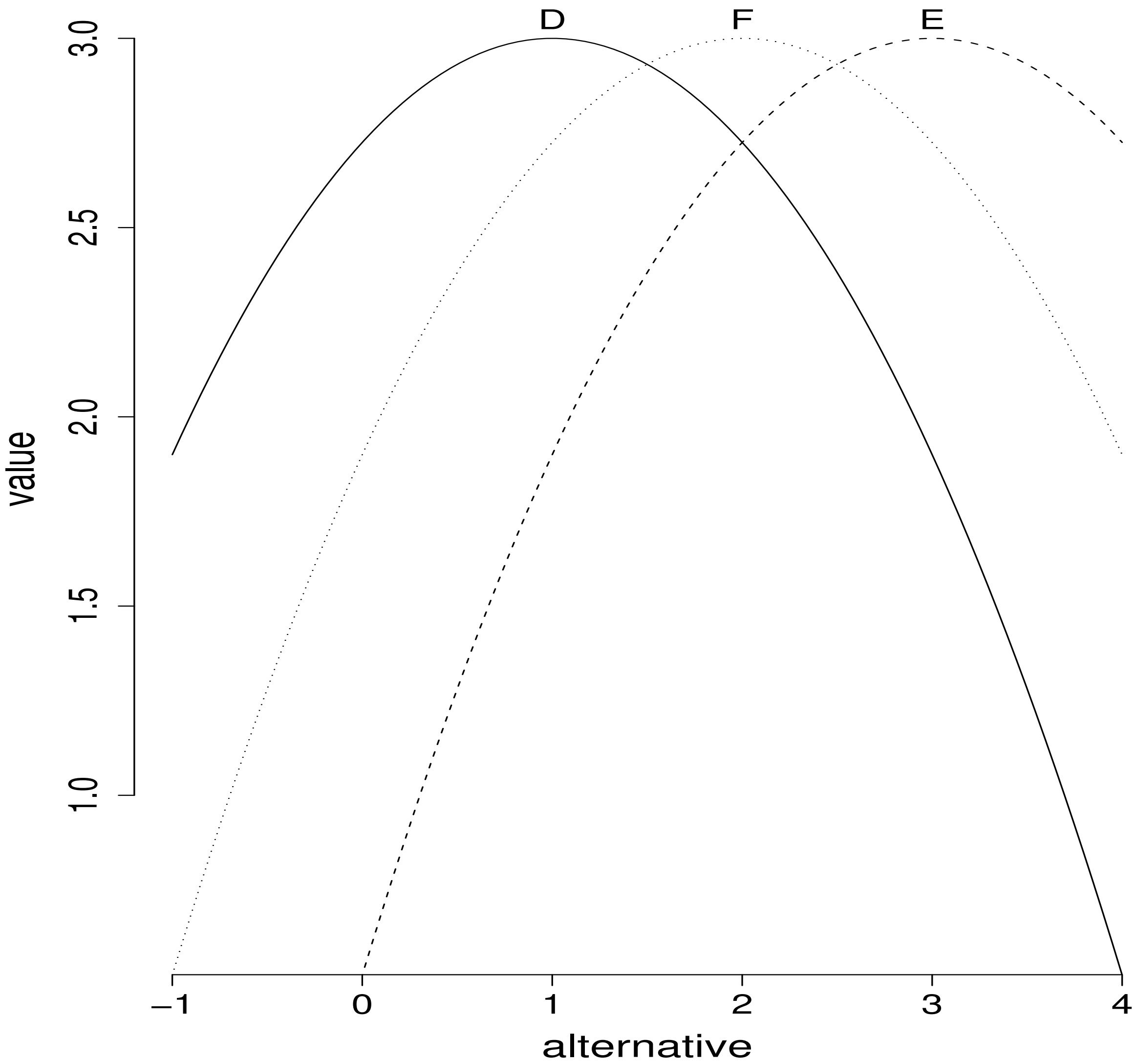
$$\begin{aligned} u_{ik} &= -b|y_k - y_i| \\ &= -b|a + cx_k - (a + cx_i)| \\ &= -bc|x_k - x_i| \end{aligned}$$

  - **the transformation changes the distance measure but not the shape of the preference curves: intensity doesn't matter**



D
F
E
value
3.0
2.5
2.0
1.5
1.0
−1
0
1
2
3
4
alternative

- **the spatial model**
- **single-peaked preferences and measures of distance**
  - **euclidean metrics:** $u_{ik} = -b[(x_k - x_i)^2]^{1/2}$
  - **symmetry**
- **the median remains the Condorcet winner**



D
F
E
3.0
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2.0
1.5
1.0
value
−1
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1
2
3
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alternative

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$$\begin{aligned} u_{ik} &= -b[(y_k - y_i)^2]^{1/2} \\ &= -b[(a + cx_k - (a + cx_i))^2]^{1/2} \\ &= -bc[(x_k - x_i)^2]^{1/2} \end{aligned}$$

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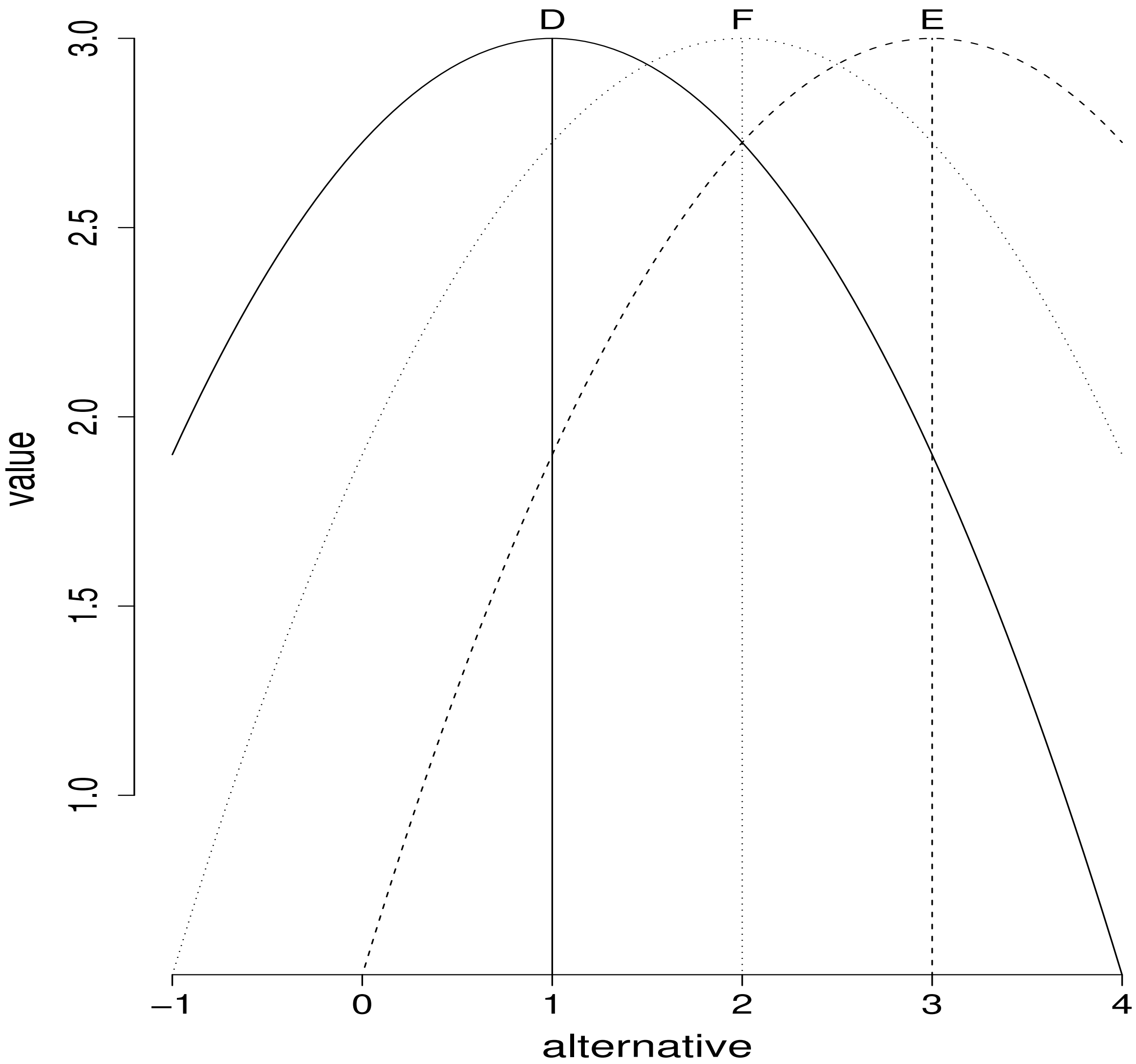
- **the spatial model**
- **with symmetric single-peaked preferences**
- **which is to say, in a pure one-dimensional spatial model...**

- the spatial model
- with symmetric single-peaked preferences
- which is to say, in a pure one-dimensional spatial model...
- all information about the chooser's preferences that is relevant for choice behavior is summarized by the chooser's ideal point



D
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1.5
1.0
value
−1
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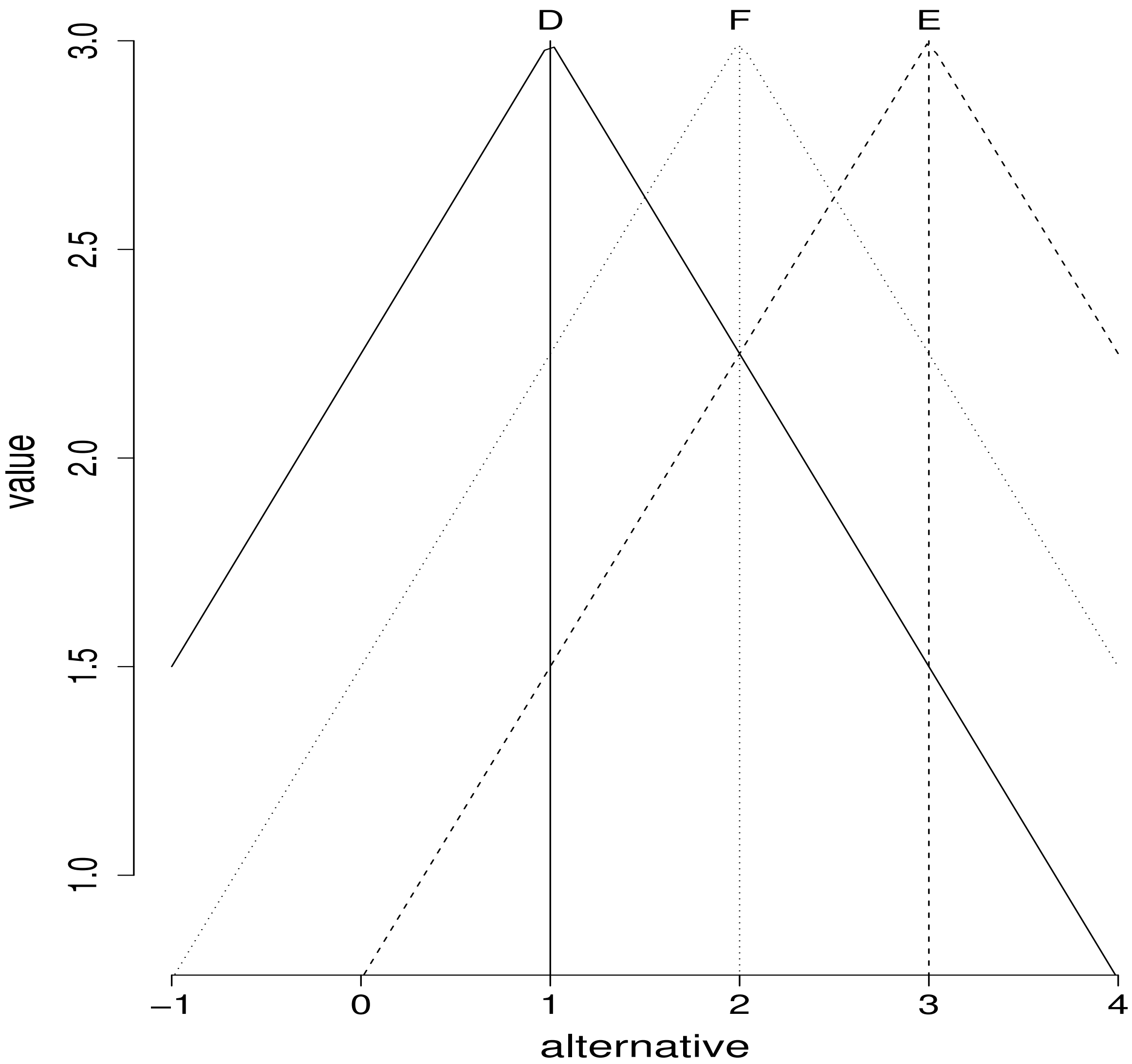
1
2
3
4
alternative


D
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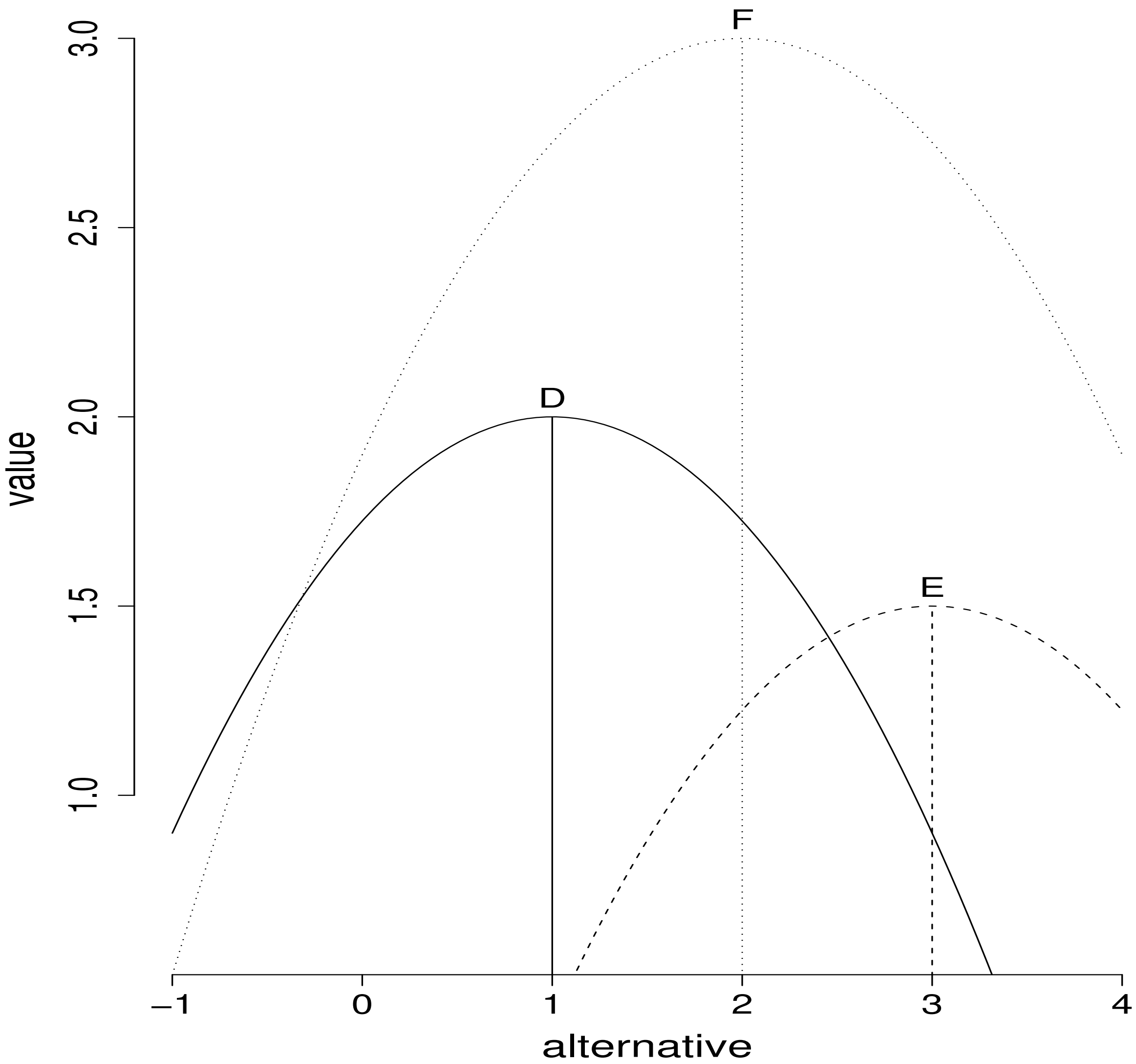
alternative



F
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value
3.0
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2.0
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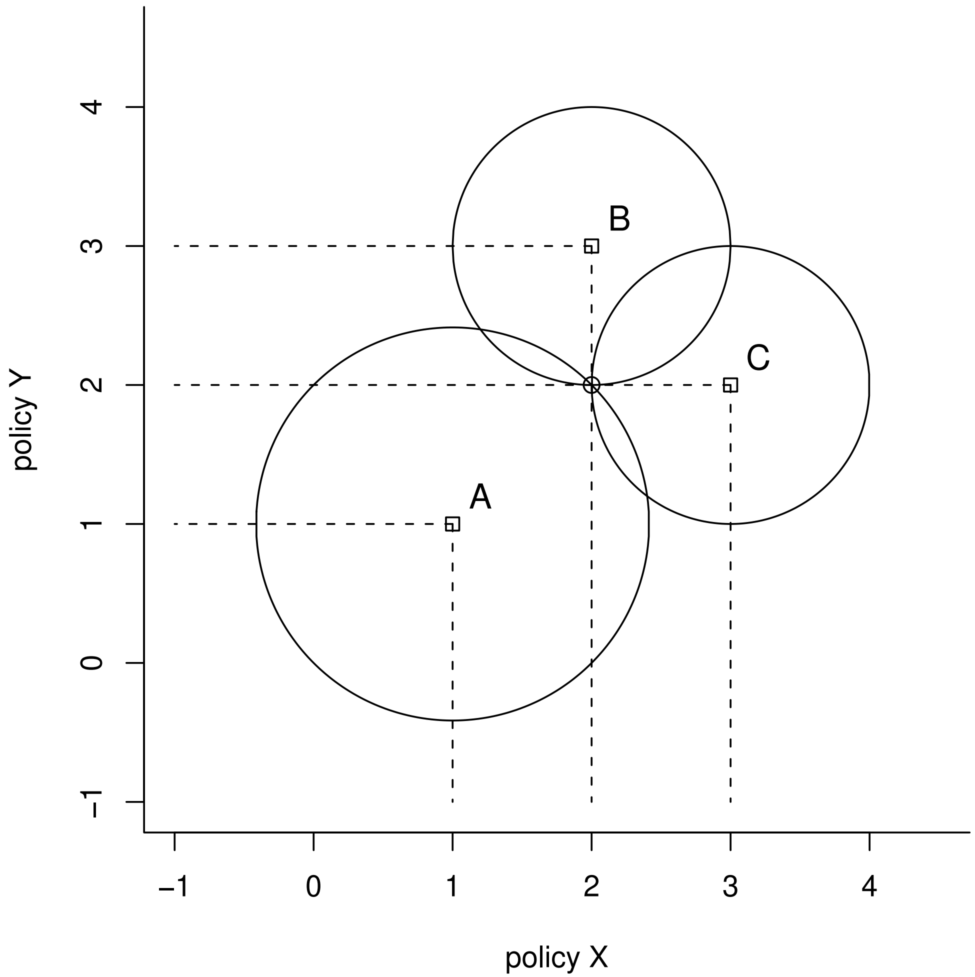
1.0
−1
0
1
2
3
4
alternative

- **a two-dimensional spatial model**
- **two-dimensional separable euclidean preferences:**

  $u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$
- **an example with three voters**

Three Voters with Separable Euclidean Preferences



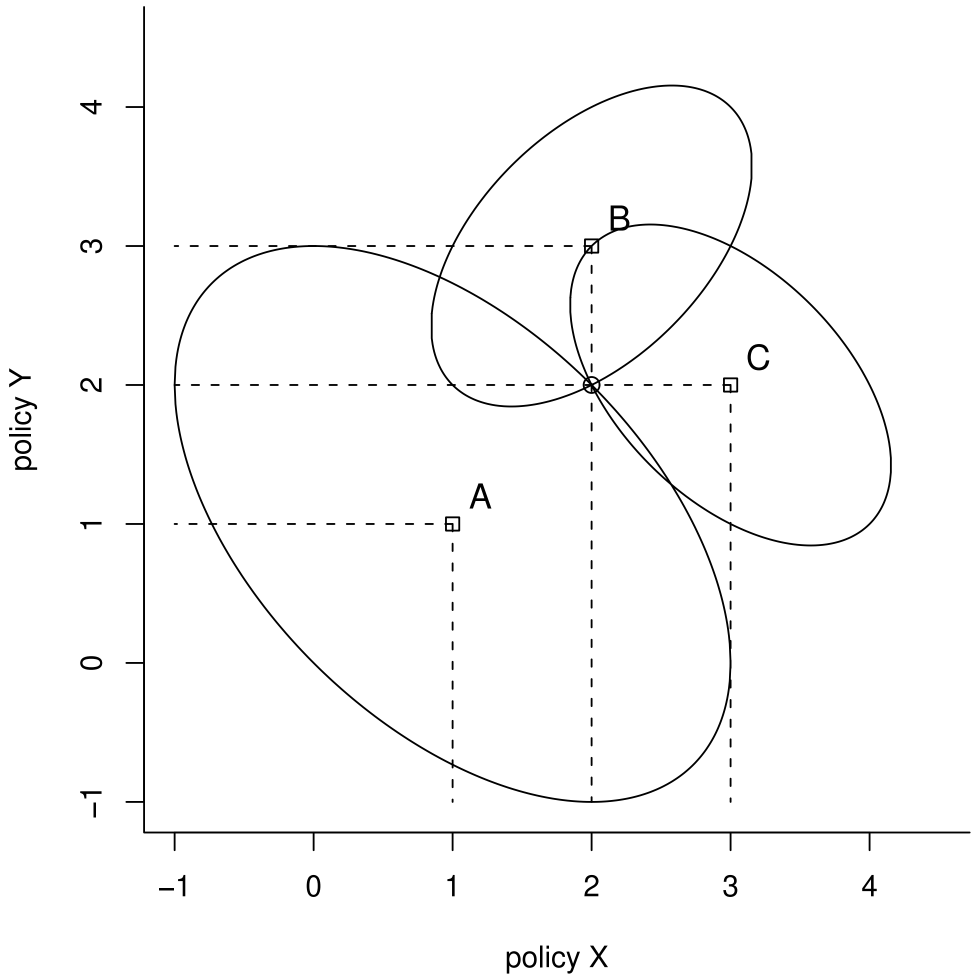
- **a two-dimensional spatial model**
- **two-dimensional separable euclidean preferences:**

  $u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2]^{1/2}$
- **with two-dimensional spatial preferences, in general the winset of any point x is not empty**
- **with separable preferences, the median on one dimension can be defeated by alternatives that shift along both dimensions**
- **with nonseparable preferences, a one-dimensional median is even more unstable**

- **two-dimensional nonseparable euclidean preferences:**

  $$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$$

Three Voters with Nonseparable Euclidean Preferences



- **two-dimensional nonseparable euclidean preferences:**

  $$u_{ik} = [(x_k - x_i)^2 + (y_k - y_i)^2 + b(x_k - x_i)(y_k - y_i)]^{1/2}$$

- **with two-dimensional spatial preferences, whether separable or nonseparable, in general the winset of any point x is not empty**

- **with nonseparable preferences, the median on one dimension can be defeated even by alternatives that shift along the same dimension**

- **instability with multidimensional spatial preferences**
- **the chaos theorems**

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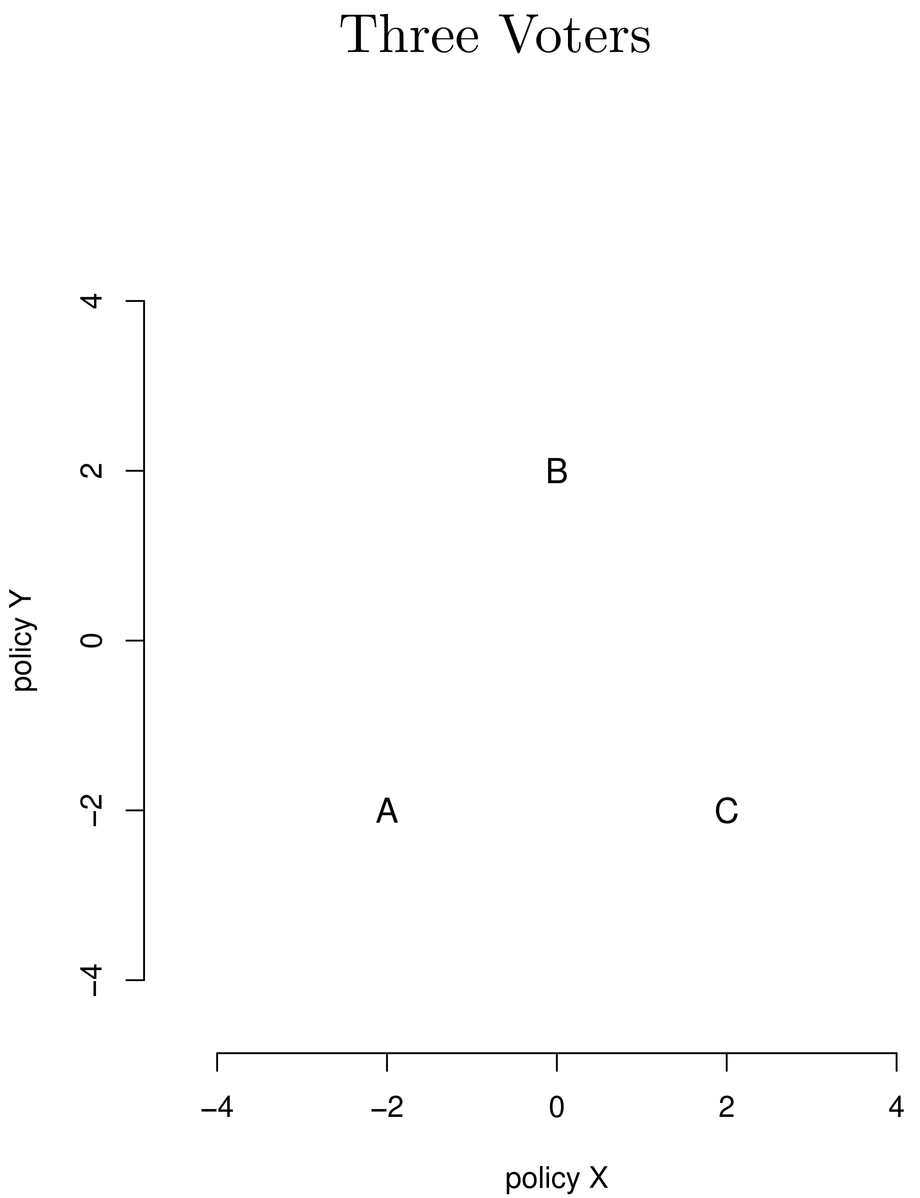

Three Voters
4
2
0
−2
−4
policy Y
B
A
C
−4
−2
0
2
4
policy X

- **Pareto set:**
  - **the set of all points that are not unanimously inferior to any other point**
- **each point in the Pareto set**
  1. **is not unanimously inferior to any other point**
  2. **is unanimously superior to at least one other point**

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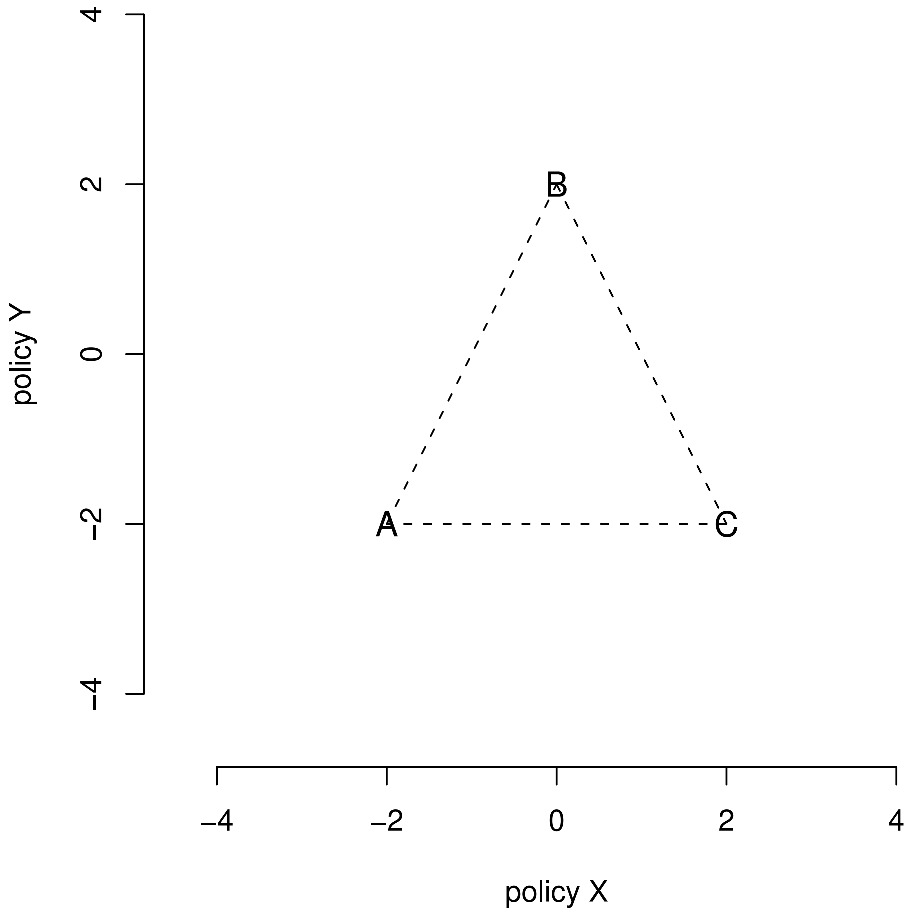
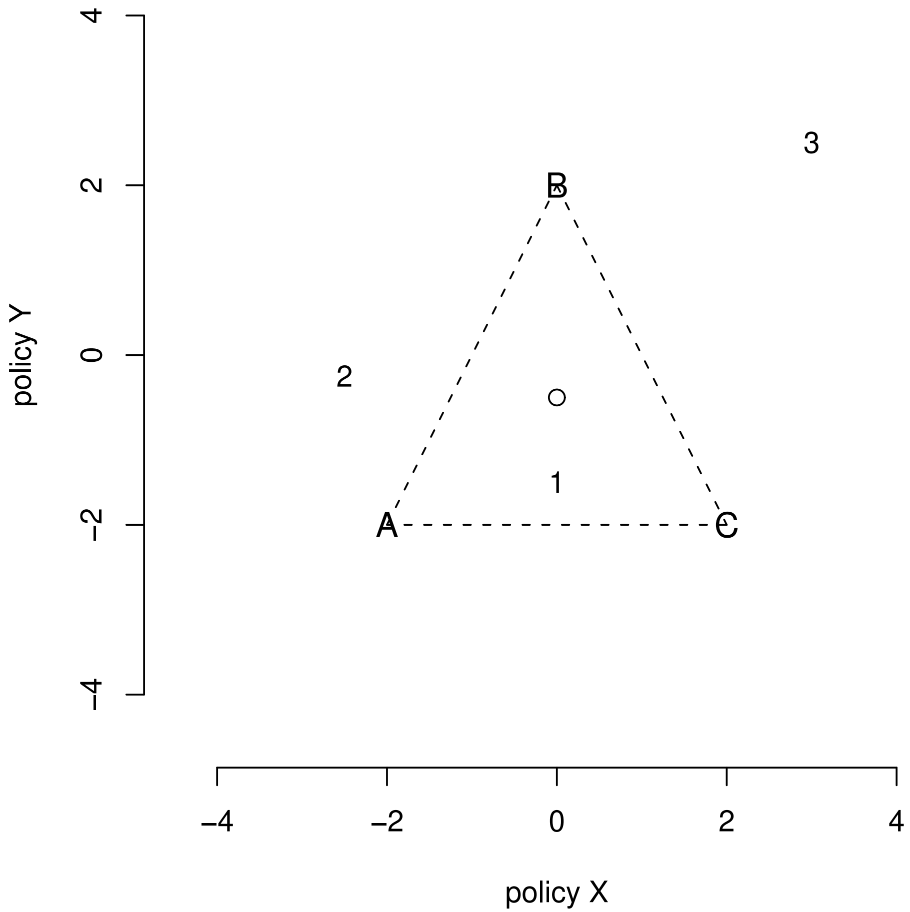
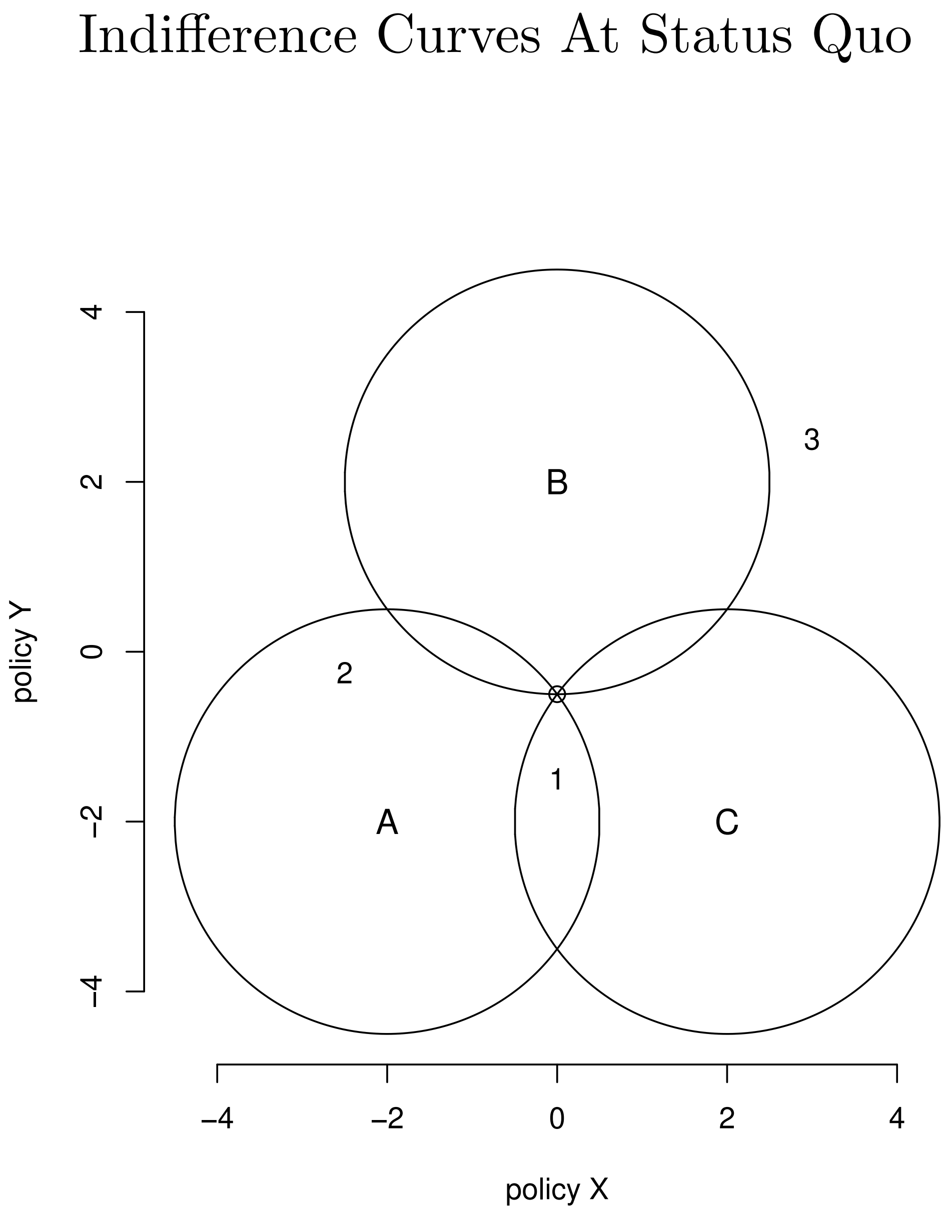
  - **the set of all points that are not unanimously inferior to any other point**
- **each point in the Pareto set**
  1. **is not unanimously inferior to any other point**
  2. **is unanimously superior to at least one other point**
- **with three voters, the boundaries of the Pareto set coincide with the median lines**
- **a median line connects two voters such that half (or half plus one for an odd number of voters) of the remaining voters are on either side of the line**

Three Voters with Pareto Set

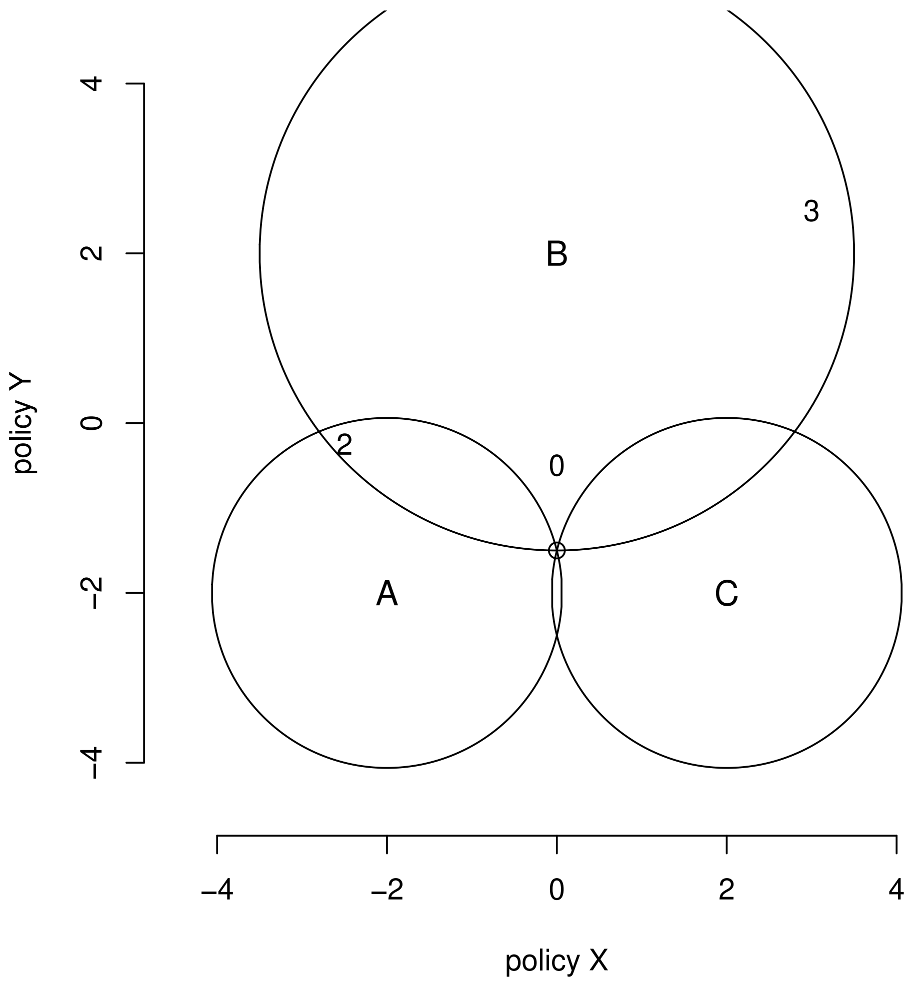


Three Voters Facing an Agenda

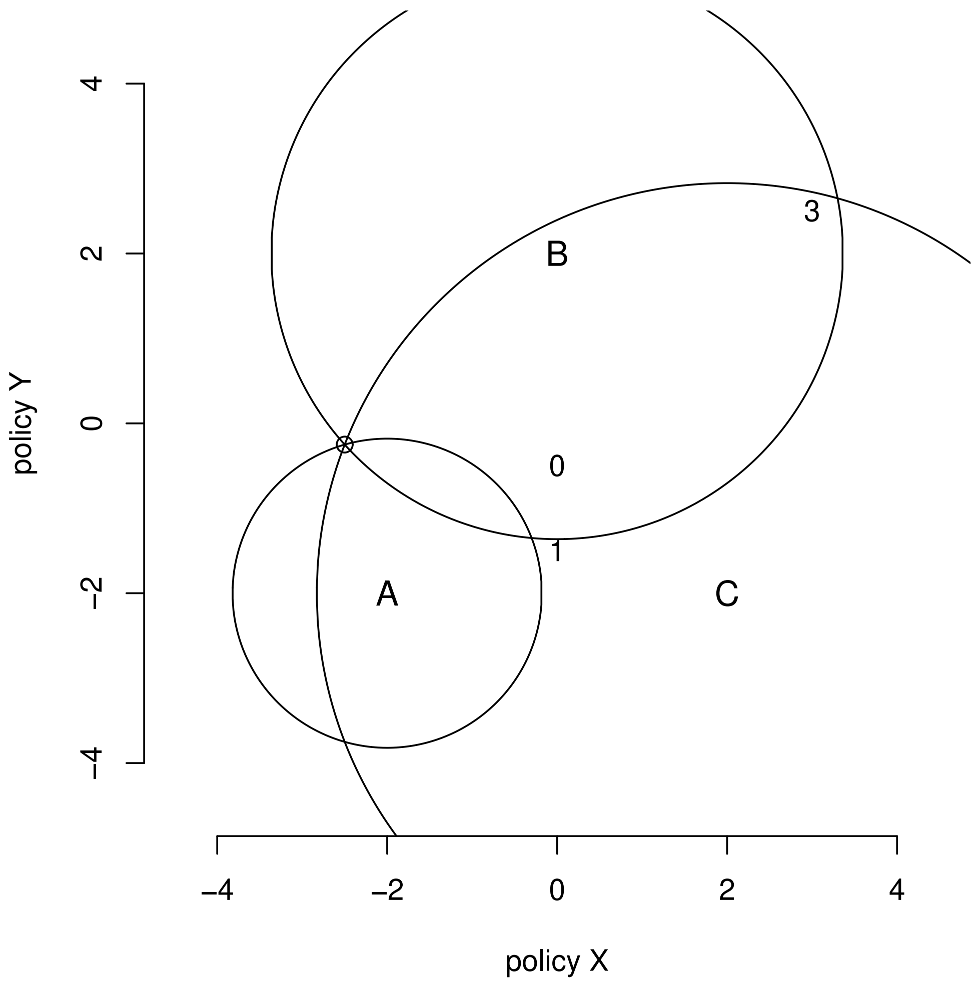



Indifference Curves At Status Quo
B
3
2
1
A
C
policy Y
policy X
4
2
0
−2
−4
−4
−2
0
2
4

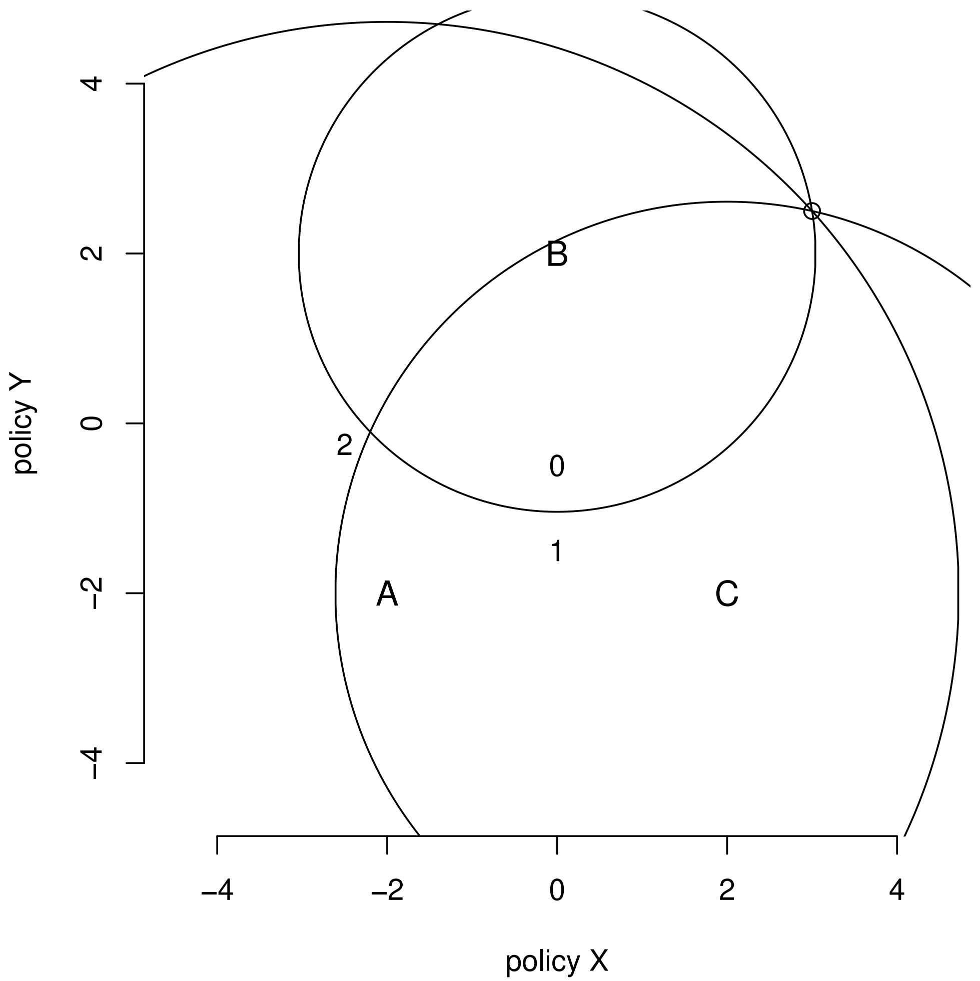
Indifference Curves At First Alternative



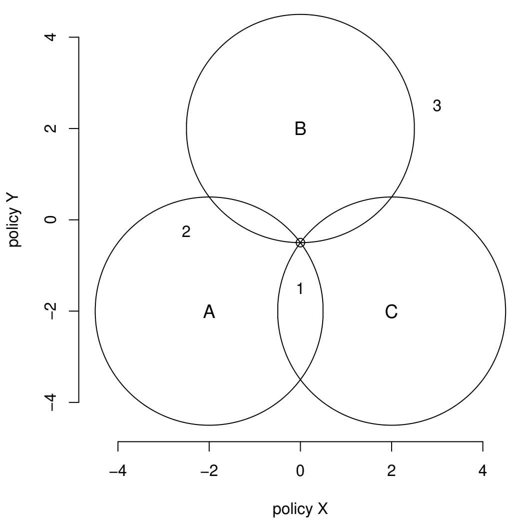
Indifference Curves At Second Alternative



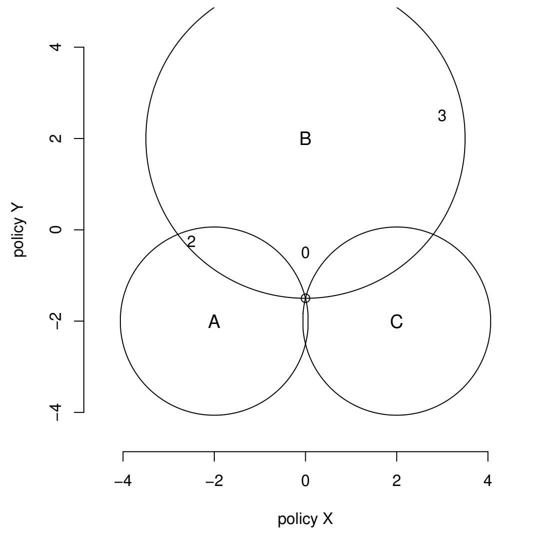
Indifference Curves At Third Alternative



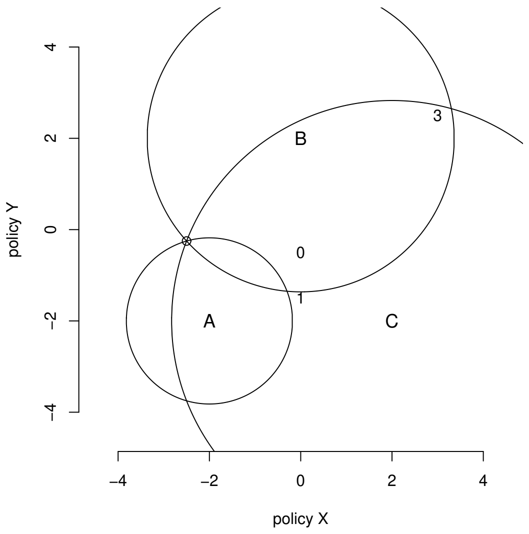
At Status Quo



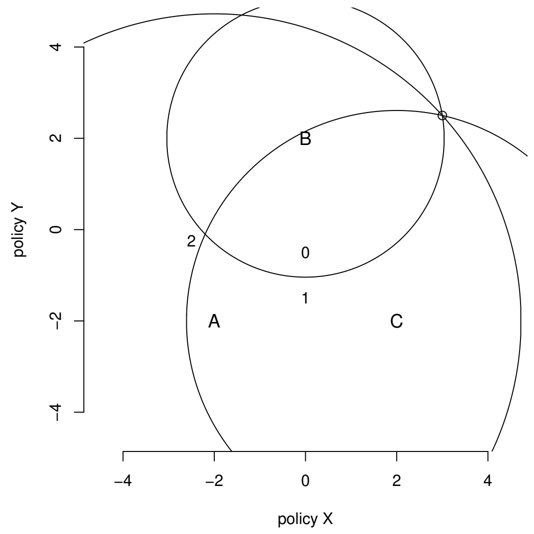
At First Alternative



At Second Alternative



At Final Alternative



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- **the chaos theorems**
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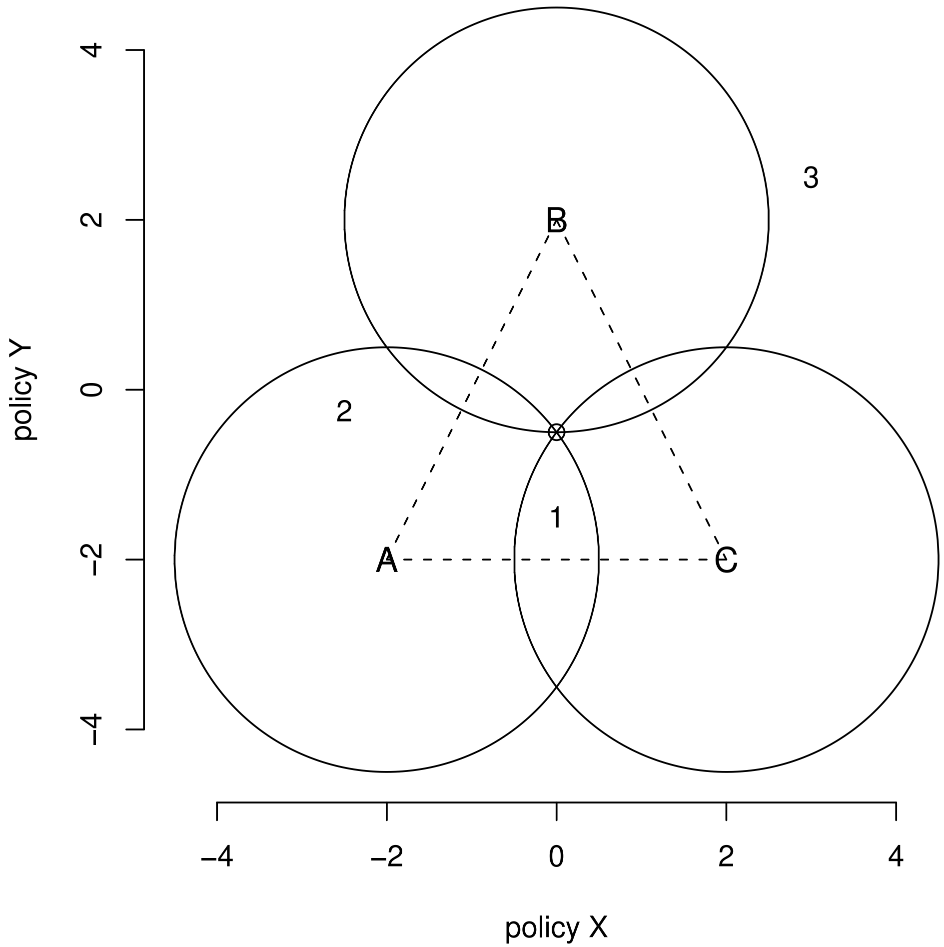
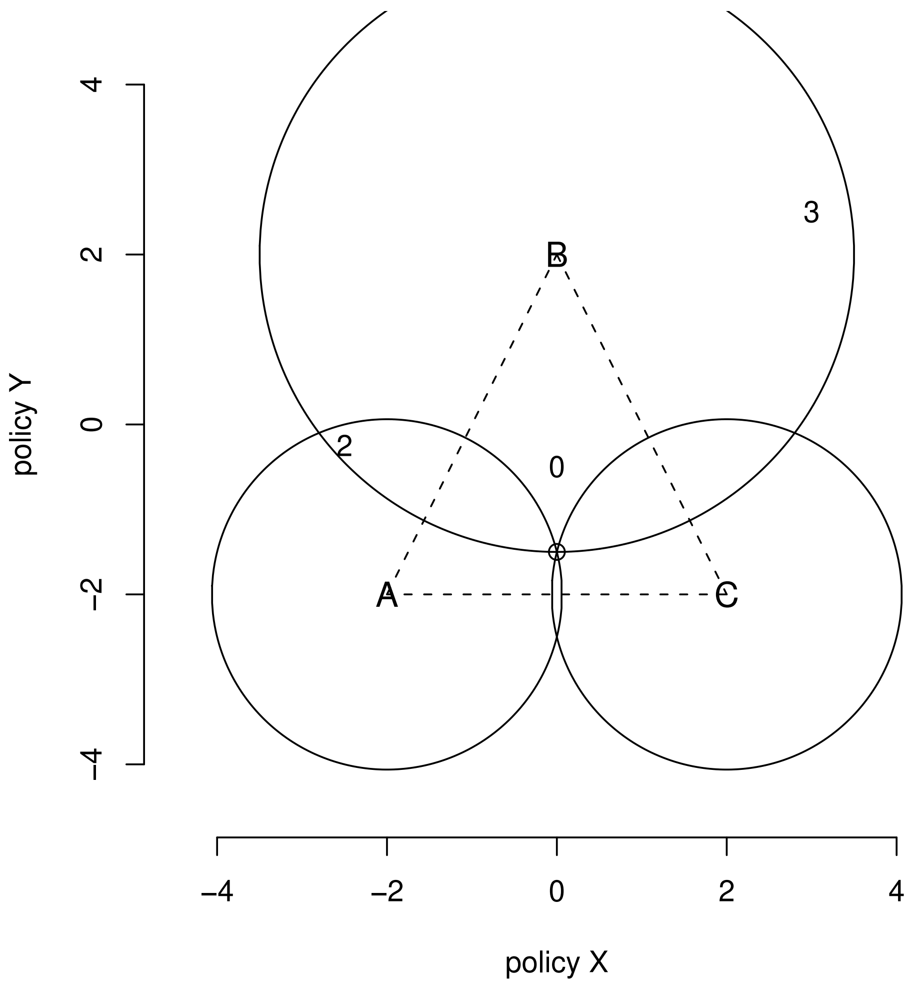
- **the chaos theorems**
  - **there is a finite path of pairwise majority votes from any alternative to any other alternative, and back**
  - **power of the agenda setter given sincere voters**
- **chaos is not possible only if there are Condorcet winners, in other words, only if the core (the set of Condorcet winners) is not empty**
- **under certain conditions on the type of agenda, strategic voters can keep outcomes inside the Pareto set**

Indifference Curves At Status Quo, with Pareto Set



Indifference Curves At First Alternative, with Pareto Set



- **instability with multidimensional spatial preferences**
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  - **technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"**

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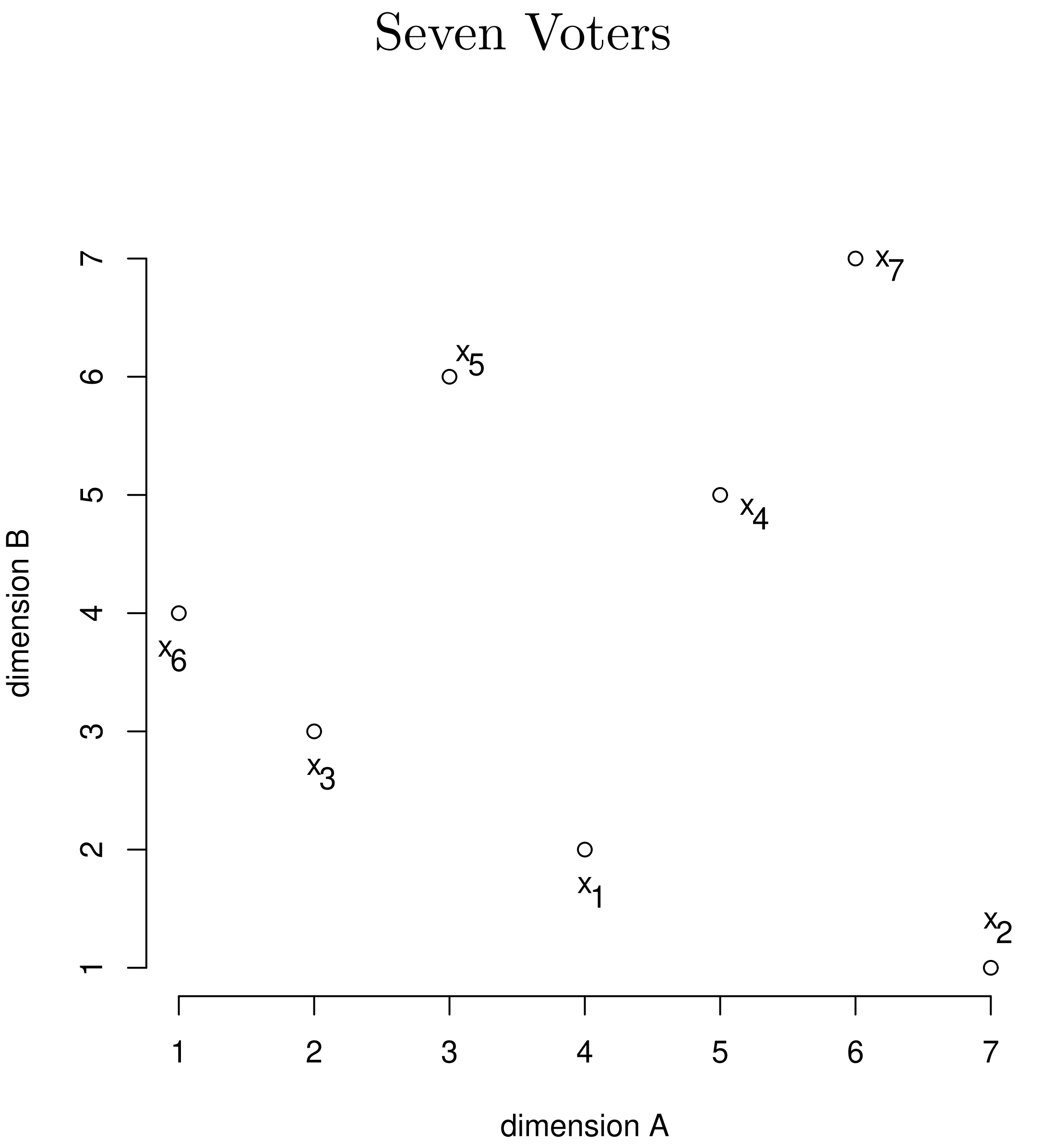
- **strategic voters can keep outcomes inside the Pareto set**
  - **technically, strategic voters can keep outcomes inside a smaller set, the "uncovered set"**
- **if the agenda is endogenous, which means the voters propose alternatives, then strategic voting can keep outcomes inside the uncovered set**

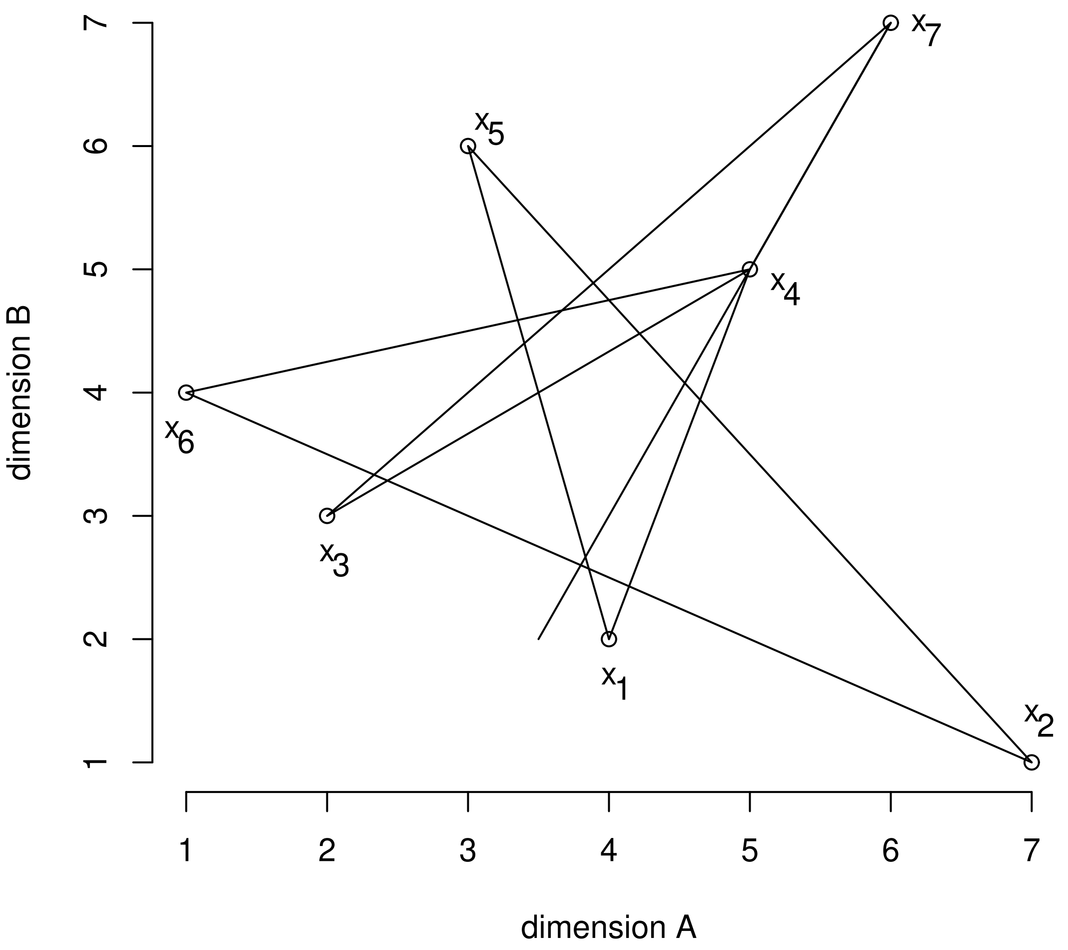
- **the uncovered set**
- **McKelvey, Richard D. 1986. "Covering, Dominance, and Institution-Free Properties of Social Choice." American Journal of Political Science 30: 283–314.**
- x **covers** y**, denoted** xCy**, if** xPy **and for all** z **such that** zPx**,** zPy
- **the uncovered set is the set of all alternatives** w **such** xCw **is false for all** x

- **the uncovered set and the yolk (generalized median set)**
- **the generalized median set (or the yolk) is the smallest ball that intersects all median lines**

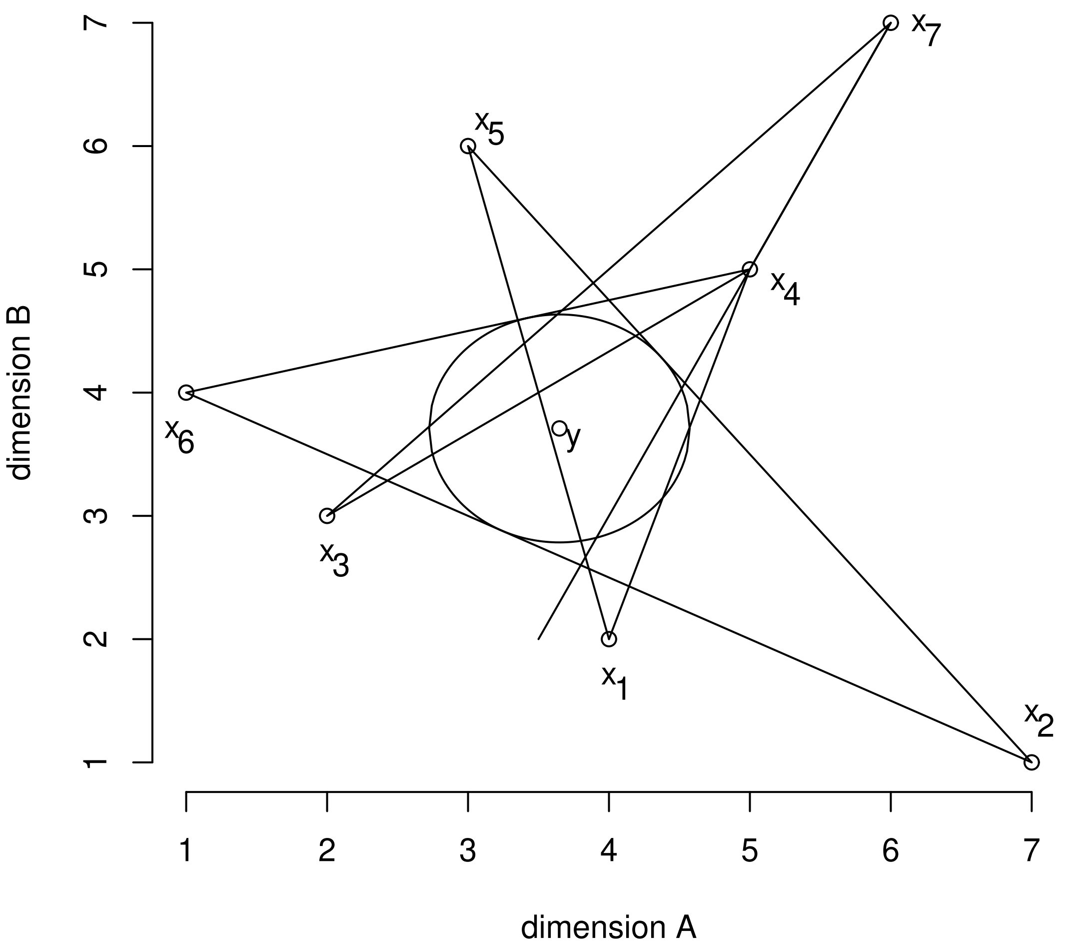
- **the uncovered set and the yolk (generalized median set)**
- **the generalized median set (or the yolk) is the smallest ball that intersects all median lines**
- **the generalized median is the center point of the yolk**
- **the uncovered set is contained in the a region four times the yolk's radius**
- **in general, the yolk shrinks as the number of voters increases (assuming sufficient dispersion)**

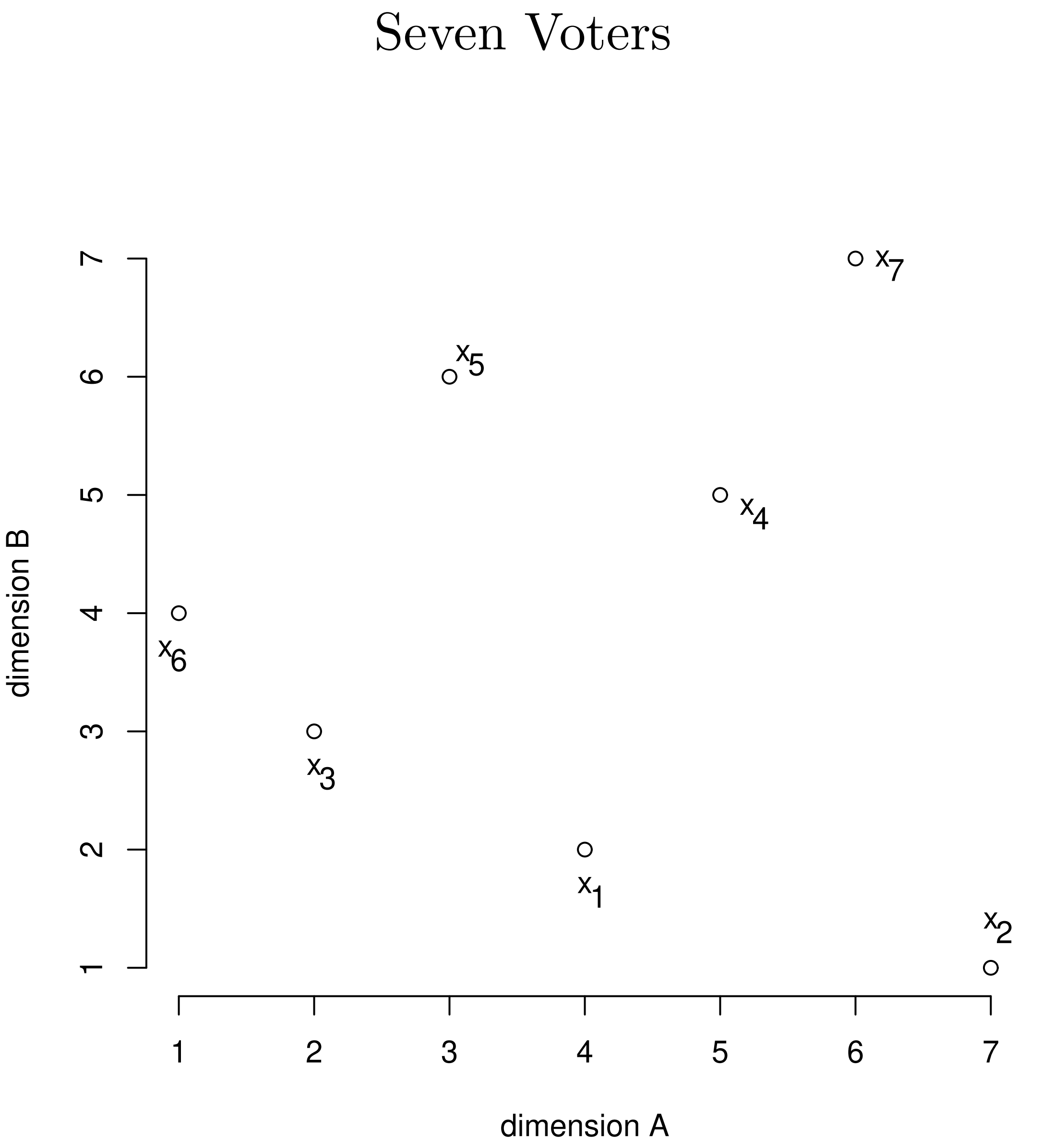

Seven Voters
dimension A
dimension B
1
2
3
4
5
6
7
x_1
x_2
x_3
x_4
x_5
x_6
x_7

Seven Voters with Median Lines

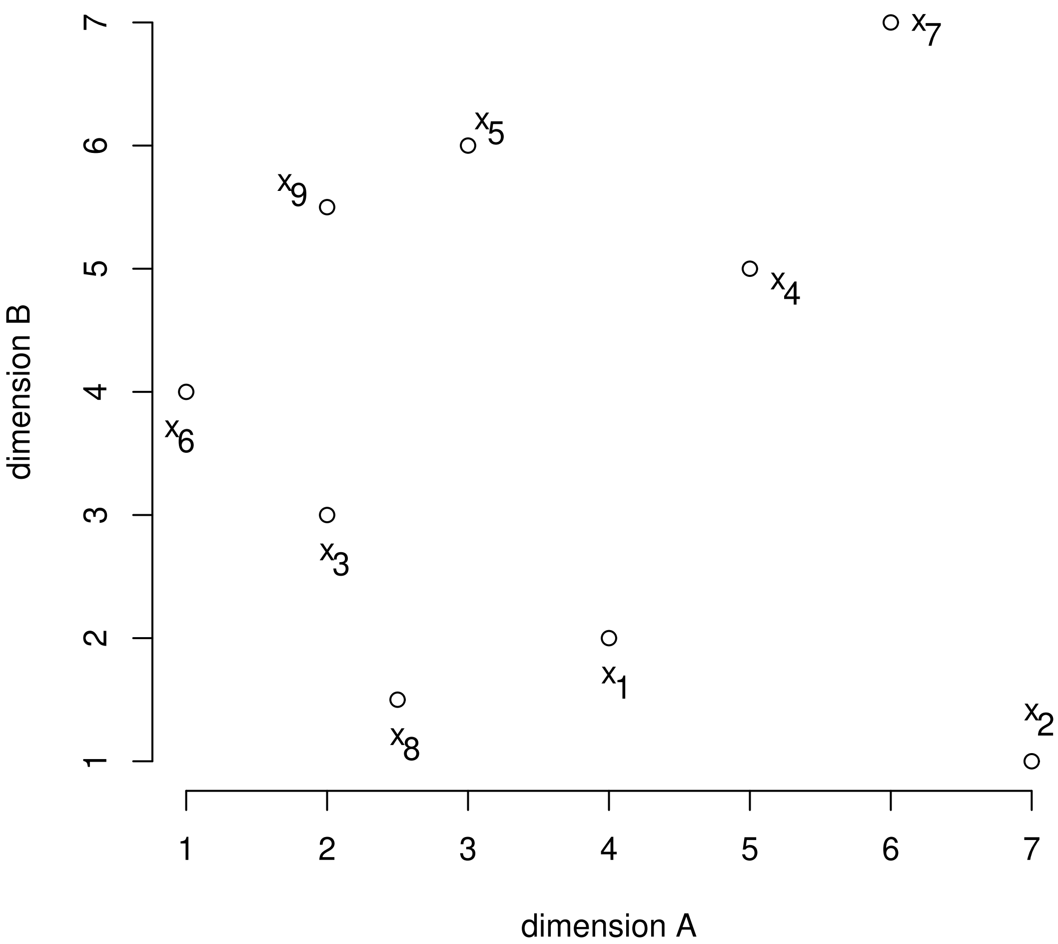


Seven Voters with the Yolk



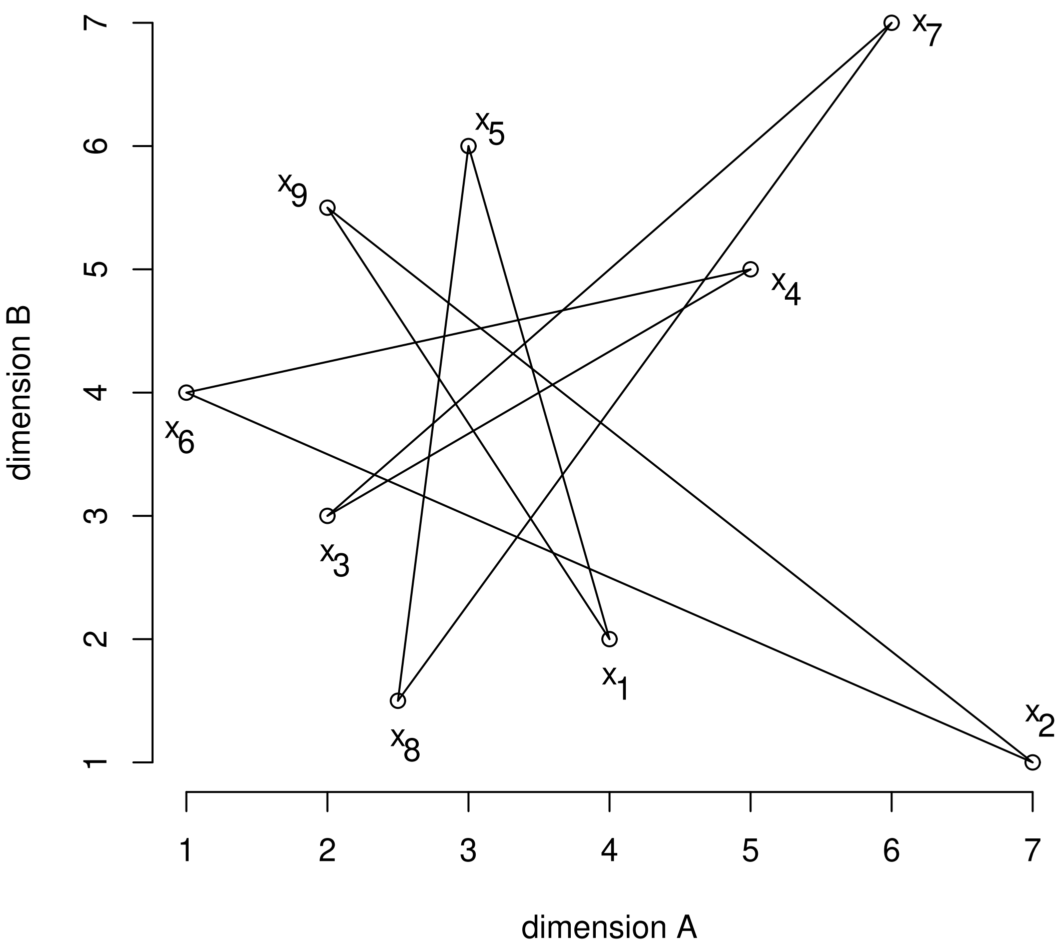

Seven Voters
dimension A
dimension B
1
2
3
4
5
6
7
x_1
x_2
x_3
x_4
x_5
x_6
x_7

Nine Voters

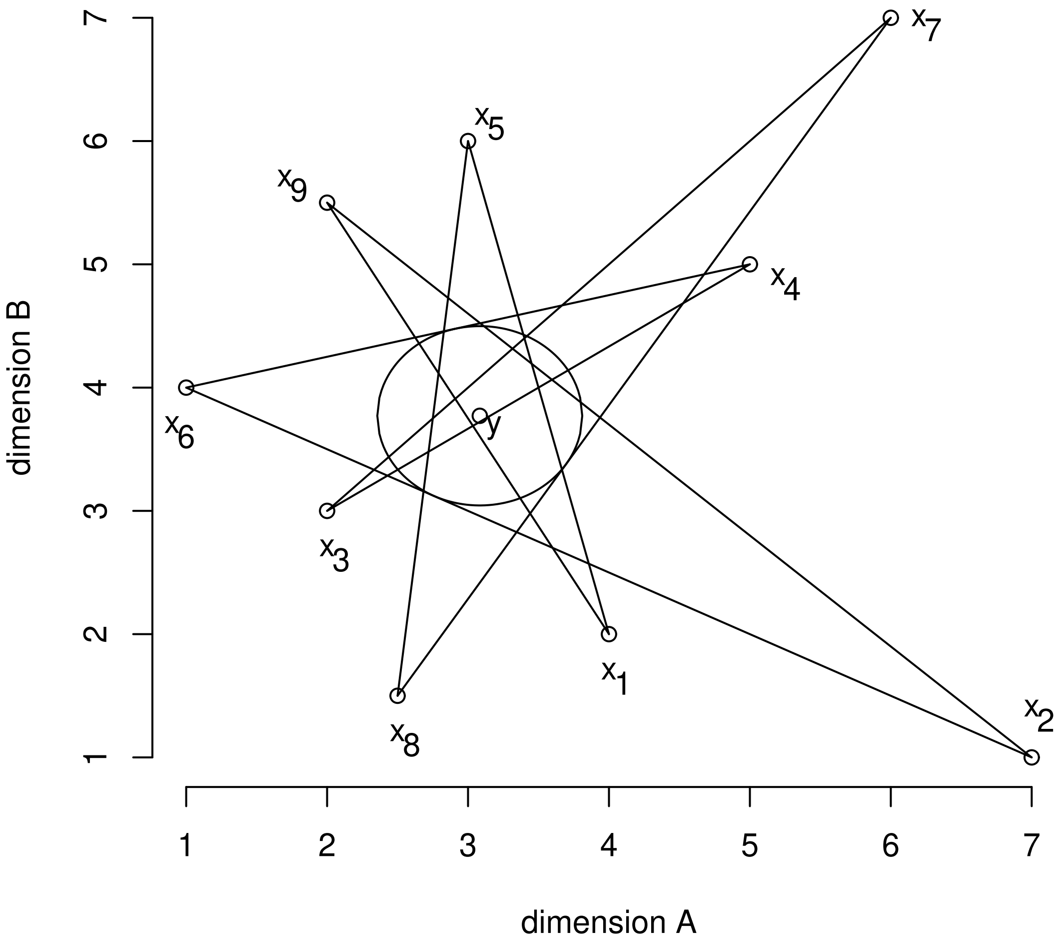


7
6
5
4
3
2
1
dimension B
1
2
3
4
5
6
7
dimension A
x7
x5
x9
x4
x6
x3
x1
x8
x2

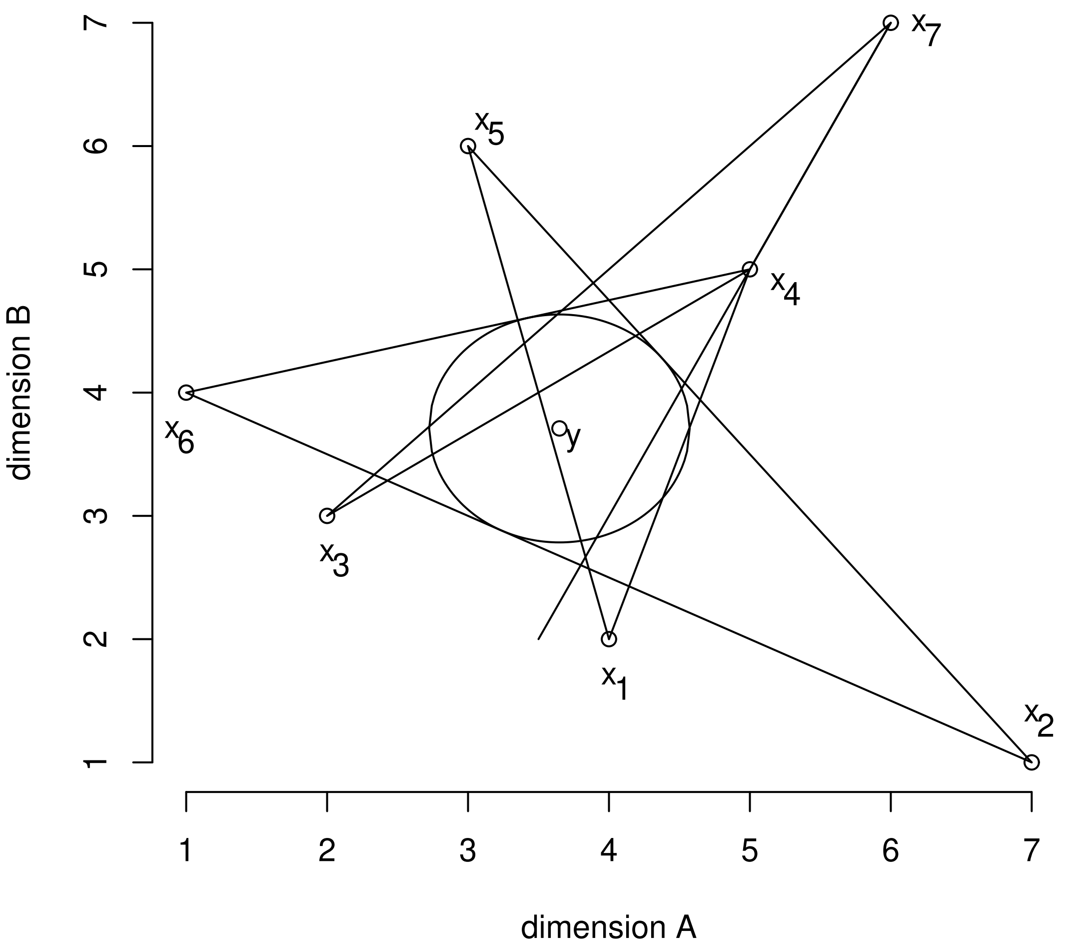
Nine Voters with Median Lines



Nine Voters with the Yolk

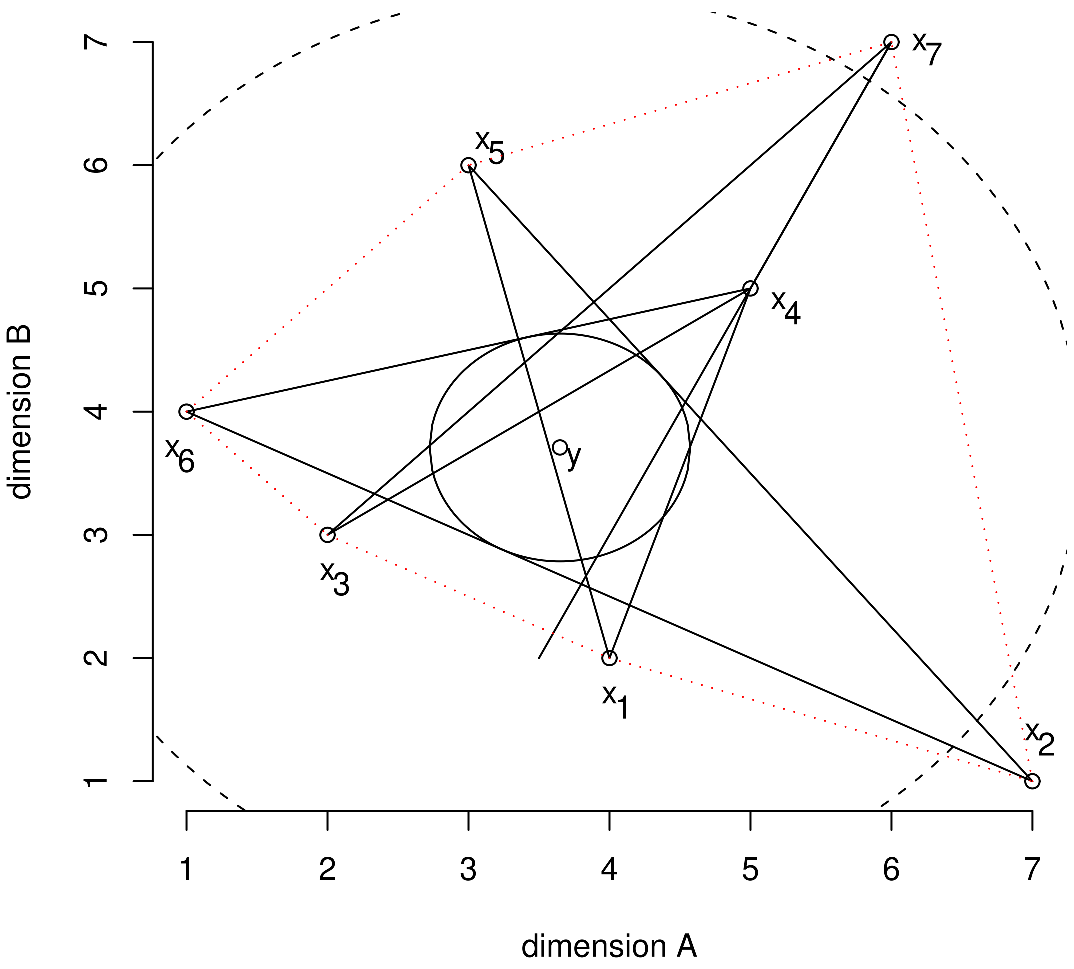


Seven Voters with the Yolk



- **the Pareto set and four times the yolk**
- **the uncovered set is contained in the intersection of these two sets**

Seven Voters with Uncovered Set Bounds



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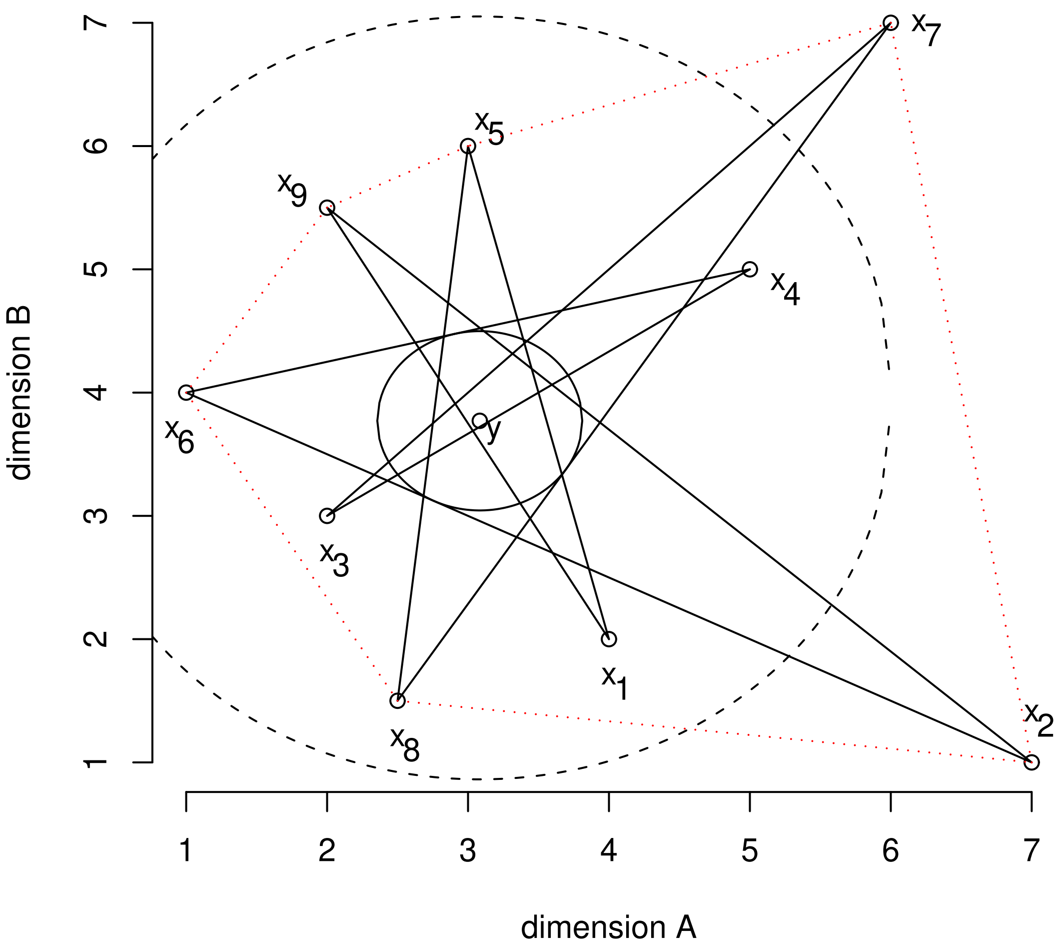
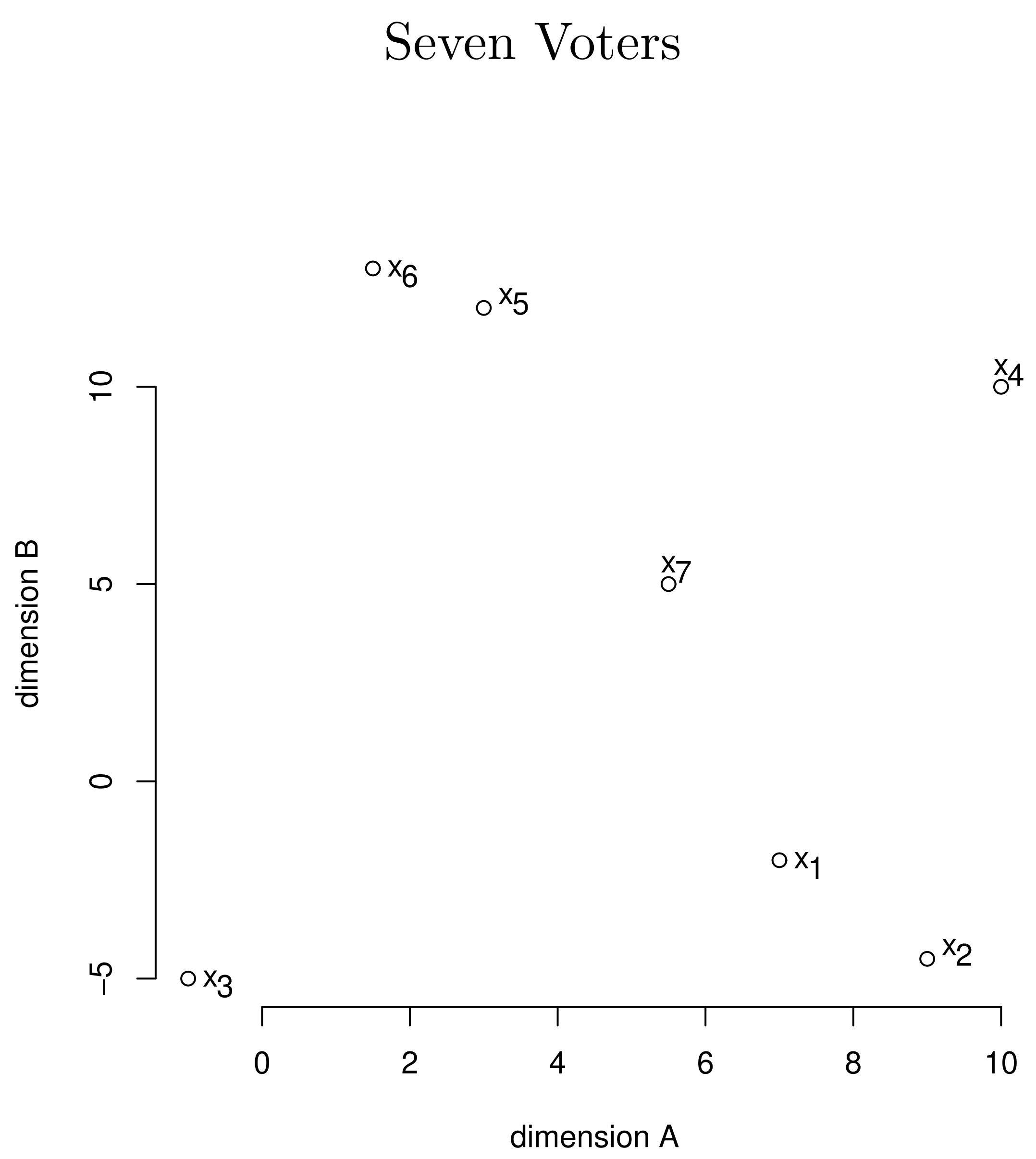
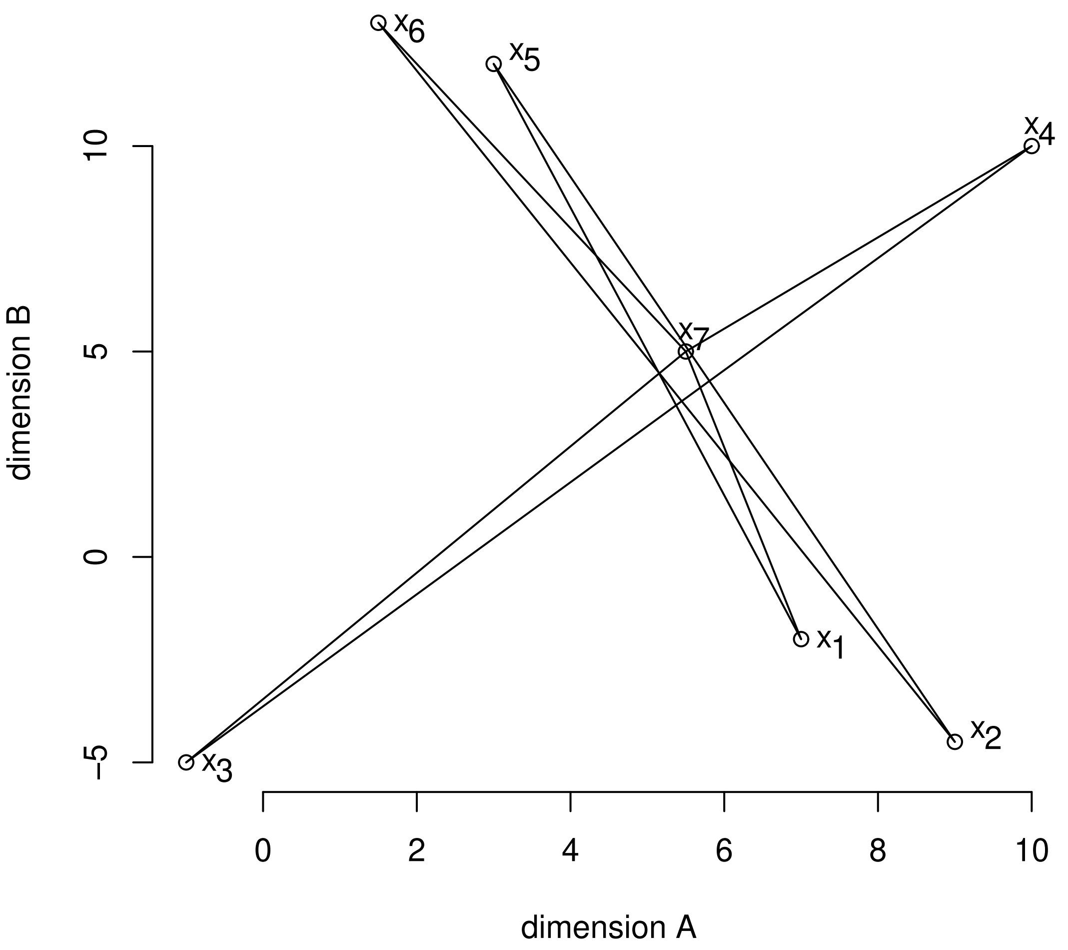


- **the yolk is small when a Condorcet winner almost exists**
- **in this case, strategic, open-agenda outcomes are near where the Condorcet winner would be (if it existed)**

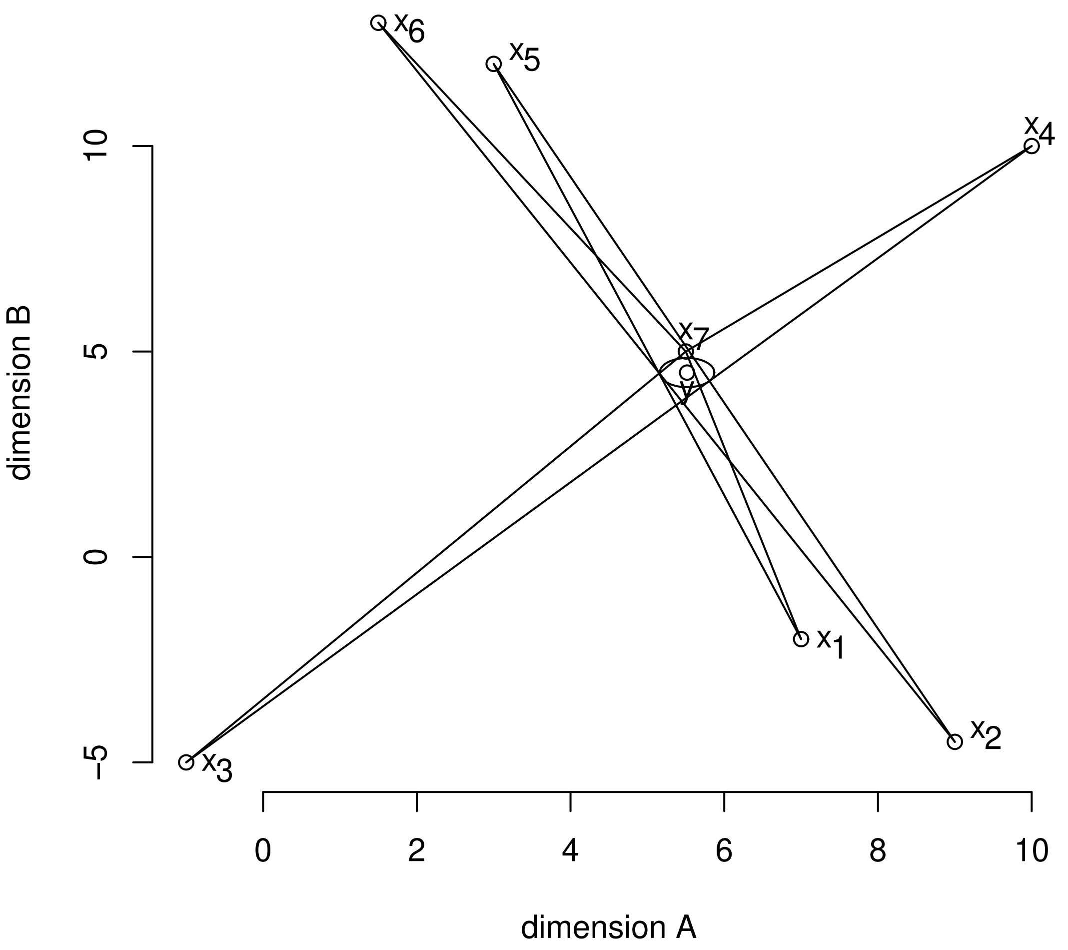


Seven Voters
dimension A
dimension B
0
2
4
6
8
10
−5
0
5
10
x_1
x_2
x_3
x_4
x_5
x_6
x_7

Seven Voters with Median Lines



Seven Voters with Yolk



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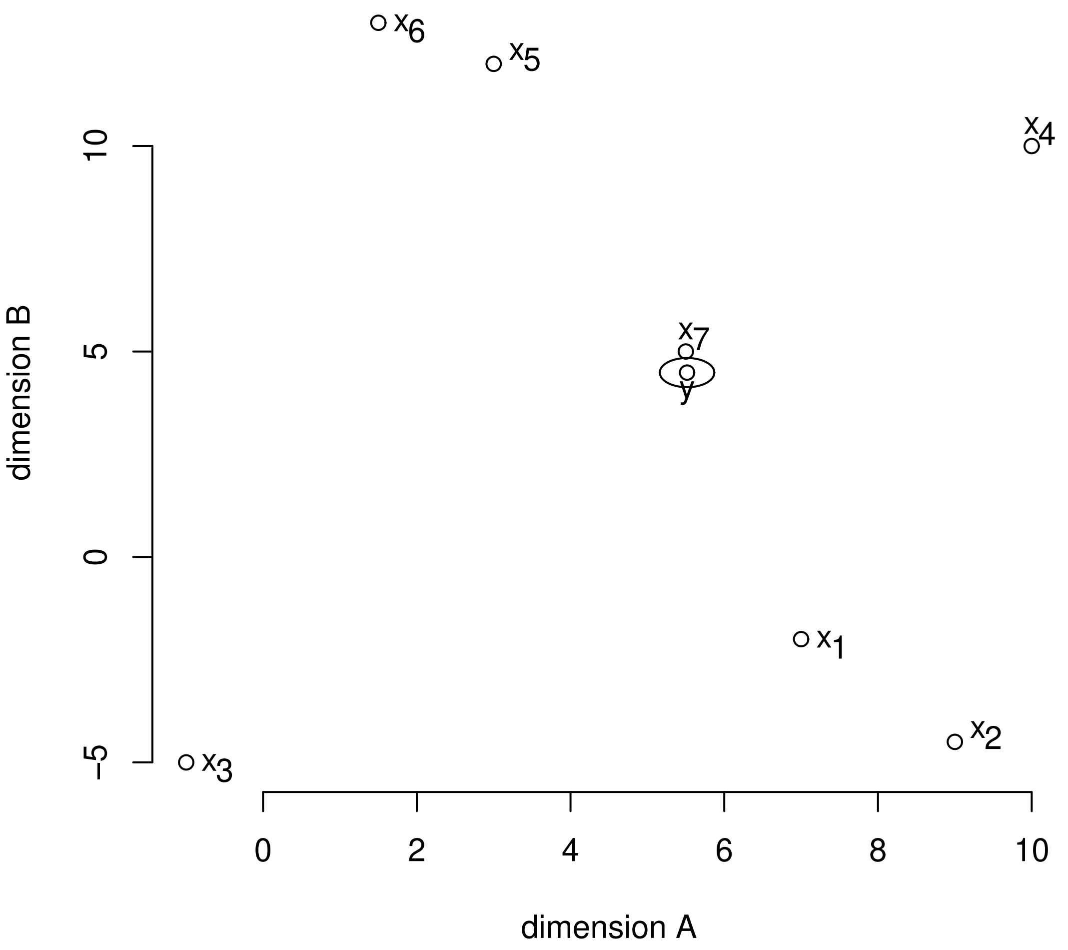
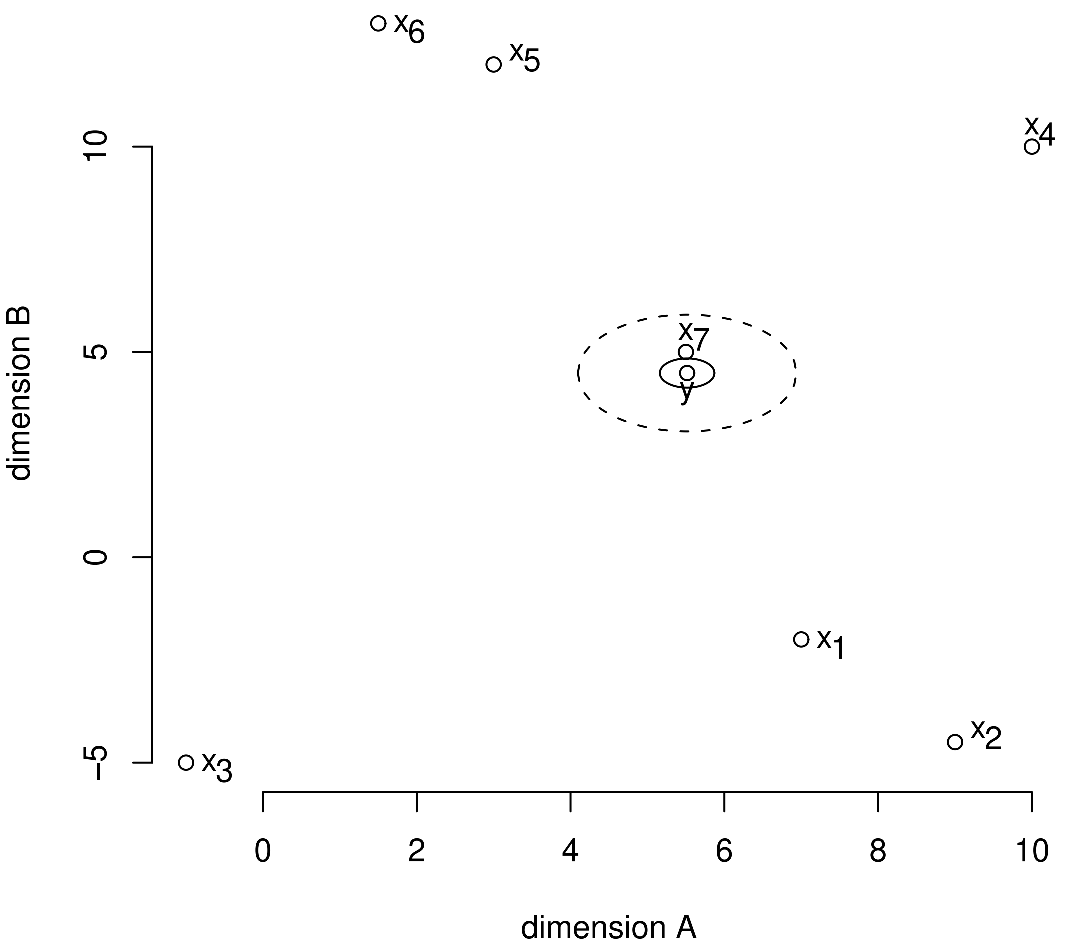
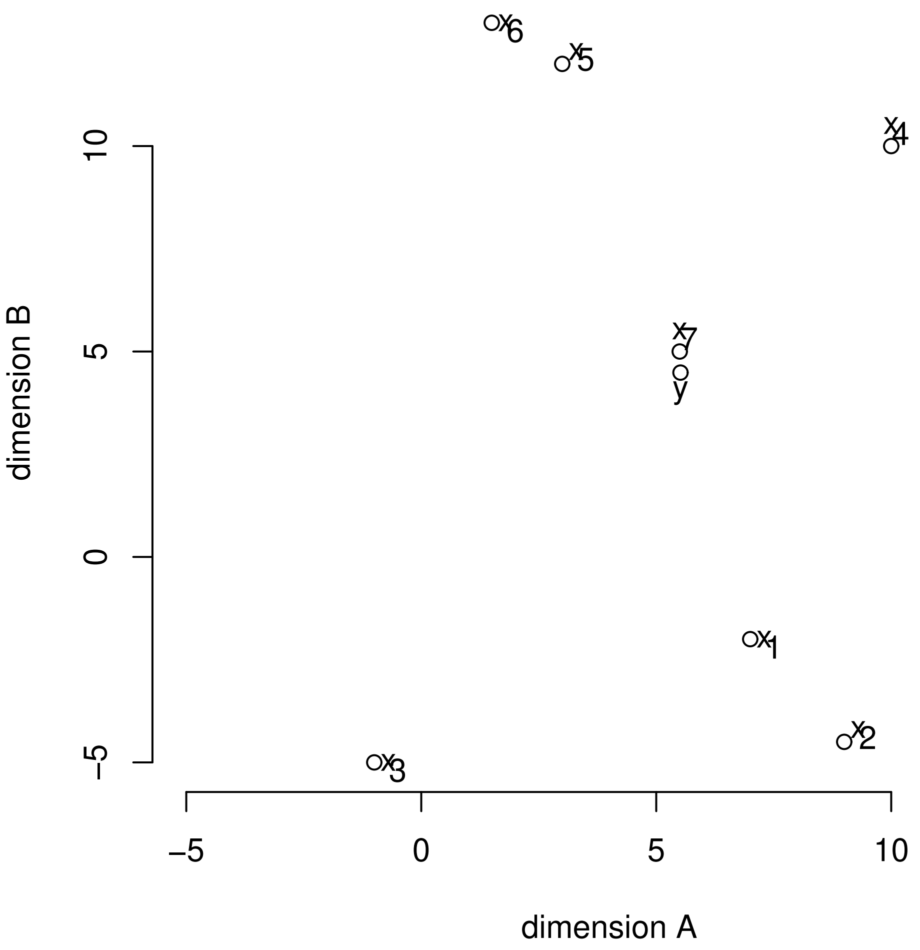


Seven Voters with Uncovered Set Range

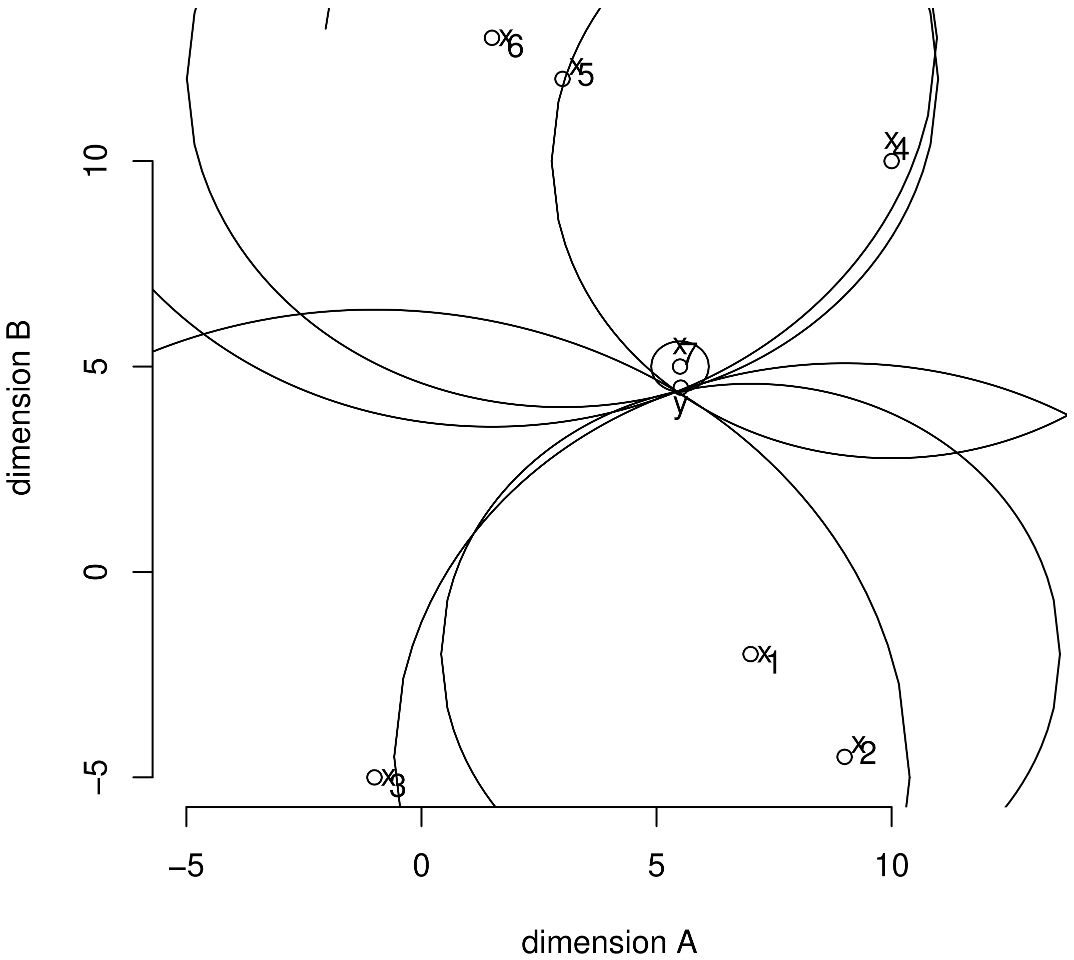


- **the generalized median and indifference curves**

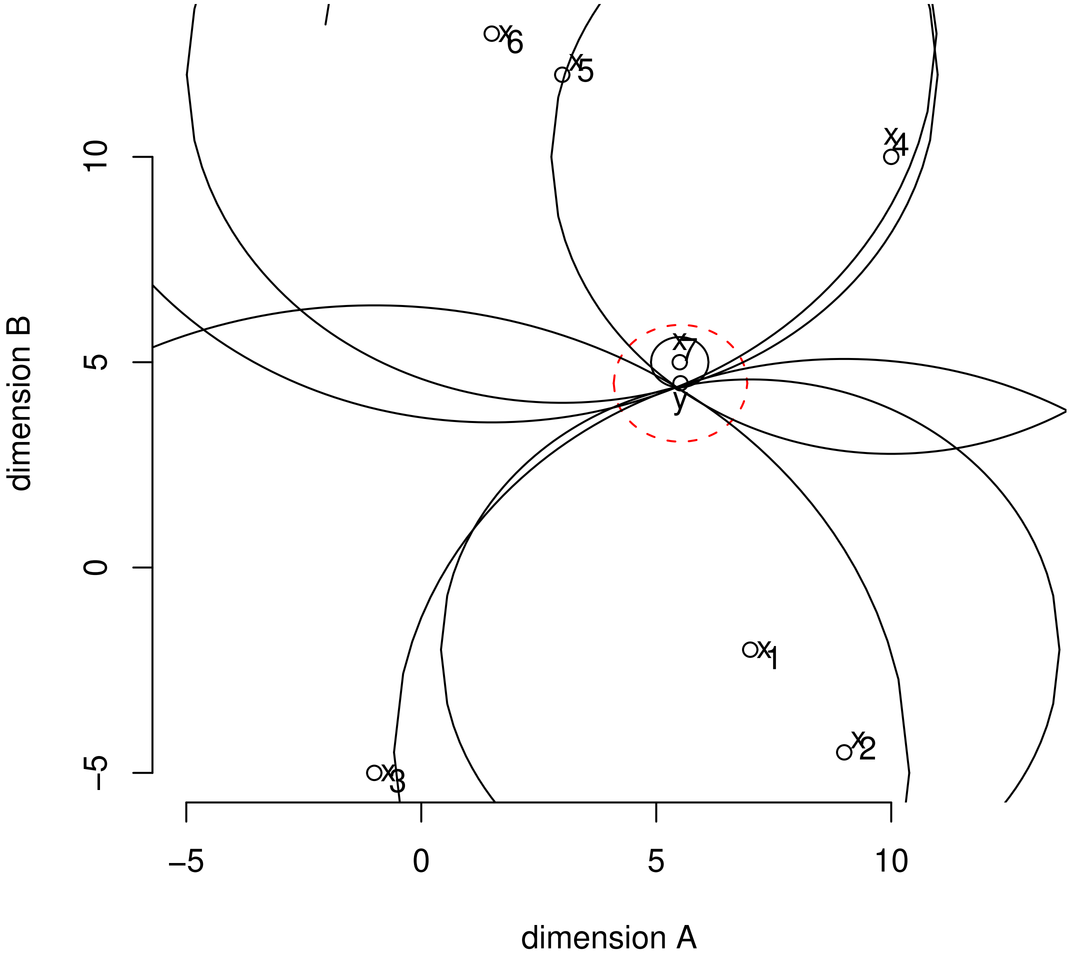
Voters with Generalized Median



Indifference Curves to Generalized Median



Indifference Curves and Uncovered Set Range



- **instability when there is no Condorcet winner**
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  - an agenda is an amendment agenda iff it is symmetric and continuous
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- **instability when there is no Condorcet winner**
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- **strategic voters can keep outcomes inside the Pareto set—specifically, inside the uncovered set—if the agenda is an amendment agenda**
  - **an agenda is an amendment agenda iff it is symmetric and continuous**
  - **a symmetric agenda is nonrepetitive, complete and uniform**
- **for other agendas, strategic voting can confine outcomes to alternatives that are dominant or are in the dominant cycle**

- “But agenda power has two sources: the ability to choose a procedure for selecting from fixed set of alternatives and the ability to choose that set.” (Ordeshook and Schwartz 1987, 194)

- **heresthetic and the critique of democracy: instability and meaninglessness**

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- **possible answers to the critique**
  - **transaction costs induce stability**
  - **similarity of preferences make cycles rare**
  - **similarity of preferences make it possible for people to determine others' preferences**
  - **does sympathy, or affinity, allow one to pin down b interpersonally in $v = a + bu$?**

- heresthetic and the critique of democracy: instability and meaninglessness
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  - does sympathy, or affinity, allow one to pin down b interpersonally in $v = a + bu$?
- reject Arrow?
  - Saari: design institutions for rational people
  - leads to favoring the Borda Count

BoS

	$Bach$	$Cage$
$Bach$	$2, 1$	$0, 0$
$Cage$	$0, 0$	$1, 2$

BoS

	$Bach$	$Cage$
$Bach$	$2, 1$	$0, 0$
$Cage$	$0, 0$	$1, 2$

Coordination

	$Melissa$	$Mariah$
$Melissa$	$2, 2$	$0, 0$
$Mariah$	$0, 0$	$1, 1$

PD

	$Don't\ Confess$	$Confess$
$Don't\ Confess$	$3,3$	$0,4$
$Confess$	$4,0$	$1,1$

PD

	Don't Confess	*Confess*
Don't Confess	3, 3	0, 4
Confess	4, 0	1, 1

Hawk-Dove

	Dove	*Hawk*
Dove	3, 3	1, 4
Hawk	4, 1	0, 0

- **collective action problem (Aldrich Table 2.1)**

	Bill		
Legislator	**X**	**Y**	**Z**
A	4	3	-9
B	3	-9	4
C	-9	4	3

- **collective action problem (Aldrich Table 2.3)**

	Bill		
Legislator	**X**	**Y**	**Z**
A	3	-1	-1
B	-1	3	-1
C	-1	-1	3

- **strategic voting in electoral systems**
- **examples of electoral system features**
  - **districts and district magnitude (M)**
  - **presidents versus parliaments**
  - **proportional representation (PR)**
    * **with quotas (e.g., Hare: $Q = V/M$)**
    * **with averages (divisor methods) (e.g., d'Hondt: $a_i(t) = v_i/(s_i(t)+1)$)**
  - **party lists**
  - **runoffs (dual ballots)**

- **examples of voting rules**
  - **majority rule**
  - **plurality voting with one winner (winner take all, first past the post)**
  - **plurality voting with** $M > 1$
  - **approval voting**
  - **Borda count**
  - **utilitarian systems**

- examples of voting rules
  - majority rule
  - plurality voting with one winner (winner take all, first past the post)
  - plurality voting with $M > 1$
  - approval voting
  - Borda count
  - utilitarian systems
- there is extensive theory for single districts, little for whole systems

- **Duverger's Law: electoral systems cause party systems**
  - **plurality voting (with $M = 1$) favors a two-party system**
  - **PR favors a multiparty system**
- **motivated by strategic voting (voters) and coalition formation (elites)**

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  - **plurality voting (with $M = 1$) favors a two-party system**
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- **generalizing Duverger's Law for $M > 1$**
  - **the $M + 1$ rule**
  - **non-Duvergerian outcomes: parties finishing below the $M + 1$-th place get equal numbers of votes greater than zero**
  - **leads to the bimodality hypothesis, which Cox tests using the SF ratio: $v_{M+2}/v_{M+1} = 0$ or , $v_{M+2}/v_{M+1} = 1$**

- **strategic voting under plurality voting (Cox)**
- **preferences: von Neumann-Morgenstern utilities** $u_i = (u_{i1}, \ldots, u_{iK})'$, $\max(u_i) = 1$, $\min(u_i) = 0$
- **beliefs:** $F_i = F$ **(common knowledge)**
- **expectations: proportions** $\pi_i = (\pi_{i1}, \ldots, \pi_{iK})'$
- **publicly generated expectations:** $\pi_i = \pi$
- **vote to maximize expected utility** $\sum_{k=1}^{K} \pi_{ik} u_{ik}$

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- **Theorem 1: If** $0 < \pi_j < \pi_{M+1}$ **for** $j > M + 1$**, then** π **is not a limit of rational expectations**
- **Corollary: If** π **is a limit of rational expectations, then** $\pi_j \in \{0, \pi_{M+1}\}$ **for all** $j > M + 1$

- **Duverger's Law, the $M+1$ rule and other strategic voting: examples (using $>$ to denote P_i), find the rational expectations Nash equilibrium vote distributions**

$$1/8 \quad A > B > C$$

$$4/8 \quad B > C > A$$

$$3/8 \quad A > C > B$$

- **simple plurality**
  - **(A, $1/2$; B, $1/2$; C, 0)**
  - **(A, $1/2$; B, 0; C, $1/2$)**
- **plurality with $M = 2$**
- **two-stage with plurality $M = 2$ in first stage**
- **Borda count**
  - **consider coalition-proof refinement to Nash equilibrium**

- **Duverger's Law, the $M+1$ rule and other strategic voting: examples (using $>$ to denote P_i)**

  $1/8 \quad A > B > C > D$

  $2/8 \quad B > C > D > A$

  $2/8 \quad D > B > C > A$

  $1/8 \quad A > C > D > B$

  $2/8 \quad D > A > C > B$

- **simple plurality**
  - **(**A**,** $1/2$**;** B**,** $1/2$**;** C**,** 0**;** D**,** 0**)**
  - **(**A**,** 0**;** B**,** 0**;** C**,** $1/2$**;** D**,** $1/2$**)**
- **plurality with $M = 2$**

- **why parties?**
- **parties may help solve coordination dilemmas**
  - **electoral (party-in-elections)**
  - **legislative (party-in-government)**
  - **one role for activists: high intensity or high resource people who work or spend to promote focal points**
  - **party labels**

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- **parties may help deploy resources efficiently**
  - **information about policies and candidates (cheap for voters and candidates)**

- **some Voting Right Act concepts**
- **racially polarized voting**
- **minority vote dilution**
  - **gerrymander**
  - **at-large election plans**
  - **majority-runoff requirements**
  - **anti-single-shot devices (defeats “bullet votes”)**

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- **Section 5 preclearance**
- **Section 2, Mobile v. Bolden, and the 1982 VRA extension**
- **1975 VRA extension and language minority provisions**

- **electoral systems (Grofman and Davidson, 7)**
  - **at-large**
  - **single-member districts**
  - **mixed systems**

- **electoral systems (Grofman and Davidson, 7)**
- **at-large**
  - **"In an at-large system, all the contested seats on a governmental body, such as a city council, county commission, or school board, are filled by voters in the jurisdiction at large. If there are eight seats to be filled, all voters have eight votes and theoretically have a chance to influence who gets elected to all eight seats."**

- **electoral systems (Grofman and Davidson, 7)**
- **single-member districts**
  - **"the city is divided into geographical districts, and voters in each district, like voters in congressional elections, are limited to a vote for a single candidate running to represent their district"**
- **mixed systems**
  - **"some of the seats are voted on at large, and some by district"**

- **cube law**

$$\frac{S}{1-S} = \left(\frac{V}{1-V}\right)^3$$

- is election fraud a kind of heresthetic?
  - dimension manipulation (maybe)
  - strategic voting (maybe)
  - agenda control (yes)
- does Arrow's Theorem legitimate election fraud?