Path Planning for Time-Optimal Information Collection

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Motivation
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Enhance autonomous information collection through:

1. Model the coupling between information collection and vehicle kinematics
2. Exploit this interaction to form optimal flight paths
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Related Work

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Problem

How can we improve the rate of information return from ISR missions?
Original Contributions

• Formulate the exploration problem as the communication of information over a noisy channel
• Identify characteristics for information optimal flight paths
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- Identify characteristics for information optimal flight paths
Outline

1 Motivation

2 UAV Exploration
   Models
   Problem Formulation
   Optimal Path Planning

3 Generalized Exploration

4 Information Constraints
   Isolated Objects
   Clustered Objects
   Application to Real-Time Controllers

5 Conclusions
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The collection of information about objects of interest with known locations, where information is understood in the classical sense of Shannon as *selection from a set*.

We do not seek to *classify* the object based on our acquired knowledge, merely *collect* information in a time-optimal way.
Communication and Exploration

Object of Interest →
Communication and Exploration

Object of Interest $\rightarrow$ Sensing Process $\rightarrow$
Communication and Exploration

Object of Interest → Sensing Process → Signal → Exploration can be viewed as a communication process where each object of interest is a transmitter, the explorer is the receiver, the sensing processes are noisy communication channels, and the sensor signals carry information that allows the explorer to identify the objects of interest.
Communication and Exploration

Object of Interest $\rightarrow$ Sensing Process $\rightarrow$ Signal $\rightarrow$ Sensor

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Communication and Exploration

Object of Interest $\rightarrow$ Sensing Process $\rightarrow$ Signal $\rightarrow$ Sensor
Transmitter $\rightarrow$ Noisy Channel $\rightarrow$ Information $\rightarrow$ Receiver

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Information Collection Model

The rate of information received by the sensor:

\[ i = W \log_2(1 + \text{SNR}) \]  

(1)
Information Collection Model

The rate of information received by the sensor:

\[ \dot{i} = W \log_2(1 + \text{SNR}) \]  

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The signal-to-noise ratio of a signal with an active sensor:

\[ \text{SNR} = \frac{K^4}{((X-A)^2+(Y-B)^2)^2} \]  

(2)
Information Collection Model

The rate of information received by the sensor:

\[ \dot{i} = W \log_2(1 + \text{SNR}) \]  (1)

The signal-to-noise ratio of a signal with an active sensor:

\[ \text{SNR} = \frac{K^4}{((X - A)^2 + (Y - B)^2)^2} \]  (2)

The rate of information received about a given object:

\[ \dot{i} = W \log_2(1 + \frac{K^4}{((X - A)^2 + (Y - B)^2)^2}) \]  (3)

where \((A, B)\) is the Cartesian location of the object of interest.
Problem Statement

Find a flight path that minimizes the total flight time for a UAV to collect a specified amount of information about each object of interest in a given area.

\[
\min_{\psi(\cdot)} t_f, \text{ subject to } I(t_f) \geq 1
\]
Necessary Conditions

States:
X Rate:

\[ \dot{x}_i = v \cos(\psi_i), \ 1 \leq i \leq n \]

Y Rate:

\[ \dot{y}_i = v \sin(\psi_i), \ 1 \leq i \leq n \]

Information Rate:

\[ I_j = \sum_{i=1}^{n} w \log_2 (1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), \ 1 \leq j \leq m \]

Optimality Condition:

\[ 0 = v \lambda_y \cos(\psi_i) - v \lambda_x \sin(\psi_i), \ 1 \leq i \leq n \]
Necessary Conditions

Costates:

X Costate:

\[ \dot{\lambda}_{x_i} = \sum_{j=1}^{m} \frac{4k_j^4w(x_i - a_j)\lambda_{l_j}}{\left((x_i - a_j)^2 + (y_i - b_j)^2\right)^3\Delta_j}, \quad 1 \leq i \leq n \]

Y Costate:

\[ \dot{\lambda}_{y_i} = \sum_{j=1}^{m} \frac{4k_j^4w(y_i - b_j)\lambda_{l_j}}{\left((x_i - a_j)^2 + (y_i - b_j)^2\right)^3\Delta_j}, \quad 1 \leq i \leq n \]

Information Costate:

\[ \dot{\lambda}_{l_j} = 0, \quad 1 \leq j \leq m \]

where \( \Delta_j = (1 + \frac{k_j^4}{((x_i - a_j)^2 + (y_i - b_j)^2)^2}), \quad 1 \leq j \leq m \)
Single UAV

Time-Optimal Flight Path

Assumptions:

- Constant Velocity
- Constant Altitude
- Instantaneous Turns
- Non-redundant additive information
Optimal Path Planning

**Proposition 1**
If the objects of interest are isolated, then the optimal flight paths consist of sequences of straight lines (far from the objects of interest) connected by short turns (near the objects of interest).

**Corollary 1.1**
If in addition to being isolated, the objects of interest are poorly visible, then the problem becomes a multi-vehicle TSP (MTSP).

**Proposition 2**
When the visibility of all the objects of interest approaches infinity, $t_f \rightarrow 0$ and the lengths of paths traveled by the UAVs approach zero.
Generalized Exploration

Kinematics:
\[ \dot{\chi}_i = f_i(u_i), 1 \leq i \leq n \]

Informatics:
\[ \dot{i}_j = \rho_j(\chi_1, ..., \chi_n, w_1, ..., w_n), 1 \leq j \leq m \]

where the functions \( \rho_j, 1 \leq j \leq m \) satisfy:
\[ \rho_j(\chi_1, ..., \chi_n, w_1, ..., w_n) = 0, \chi_1, ..., \chi_n \notin D_j, > 0, \text{ otherwise} \]
Generalized Exploration

States:
X Rate:
\[ \dot{\chi}_i = \bar{f}(u_i), \quad 1 \leq i \leq n \]

Information Rate:
\[ I_j = \rho_j(\bar{\chi}_1, \ldots, \bar{\chi}_n, w_1, \ldots, w_n), \quad 1 \leq j \leq m \]

Optimality Condition:
\[ 0 = \bar{\lambda} \chi_i \frac{\partial \bar{f}(u_i)}{\partial u_i}, \quad 1 \leq i \leq n \]
Generalized Exploration

Costates:

Position Costate:

\[
\dot{\lambda}_{x_i} = - \sum_{j=1}^{m} \lambda_{l_j} \frac{\partial \rho(\chi_1, \ldots, \chi_n, w_1, \ldots, w_n)}{\partial \chi_i}, \quad 1 \leq i \leq n
\]

Information Costate:

\[
\dot{\lambda}_{l_j} = 0, \quad 1 \leq j \leq m
\]
Generalized Exploration

Control Rate Magnitude:

\[
\dot{u}_i = \sum_{j=1}^{n} \rho_j(\chi_1, ..., \chi_n, w_1, ..., w_n) \frac{\partial \bar{f}(u_i)}{\partial u_i} \frac{1}{\sum_{i=1}^{n} \lambda_{\chi_i} \frac{d}{dt} \left( \frac{\partial \bar{f}(u_i)}{\partial u_i} \right)}
\]
Generalized Exploration

UAV Control Rate Magnitude:

\[
\dot{\psi}_i = \frac{-4k_j^4 \lambda_j \psi_i (\cos(\psi_i) (b_j - y_i) + \sin(\psi_i) (a_j - x_i)) \cos(\psi_i)}{\lambda_{x_i} r^6}, \quad 1 \leq i \leq n
\]
Information Constraints

- Two constraints are active
Information Constraints

- Two constraints are active
- These are critical objects
Information Constraints

- One active constraint
Information Constraints

- One active constraint
- One critical object
Information Constraints - Clustering

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- Two active constraints
Two active constraints
Two critical objects
Clustered Objects

Proposed Procedure:

1. Connect the spheres of influence with a greedy path

Evaluate information gain

Those items with an information gain equal to 1 bit are critical objects

Optimize path for critical objects
Clustered Objects

Proposed Procedure:

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2. Evaluate information gain
Clustered Objects

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3. Those items with an information gain equal to 1 bit are critical objects
Clustered Objects

Proposed Procedure:

1. Connect the spheres of influence with a greedy path
2. Evaluate information gain
3. Those items with an information gain equal to 1 bit are critical objects
4. Optimize path for critical objects
Parametric Study:

- Vary only $K_1$, the visibility of object 1
- Examine flight paths
Parametric Study

Radius of Closest Approach

Parametric Study:

- \( f(K) \propto K^3 \)
Conclusions

1. ISR missions seek information
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2. Autonomous exploration can be aided improved information return
3. The exploration problem can be expressed as the communication of information over a noisy channel
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1. ISR missions seek information
2. Autonomous exploration can be aided by improved information return
3. The exploration problem can be expressed as the communication of information over a noisy channel
4. Path-planning can be optimized to minimize the flight time required to obtain a specified amount of information
Conclusions

1. ISR missions seek information
2. Autonomous exploration can be aided improved information return
3. The exploration problem can be expressed as the communication of information over a noisy channel
4. Path-planning can be optimized to minimize the flight time required to obtain a specified amount of information

Conclusion

Autonomous exploration can be enhanced through optimal path planning
Future Work

- Investigate Exploration vs Exploitation
- Expand exploration formulation for cooperative vehicles
Thank you