Short-term, immediate pedagogical purpose: These problems will test your knowledge of 2-body elastic and inelastic collisions with particles having either equal or unequal masses. One of them (“the target”) may start off at rest, or be moving.

Long-term pedagogical purpose: These problems will build part of your foundation of understanding how radiation interacts with matter. The most relevant applications are in fission (decay, moderation, neutron scattering), fusion, nuclear materials, detector response and radiological sciences. These problems also serve to exercise your understanding of polar coordinates and Matlab, which are used extensively in particle-particle interactions, and in the rest of your undergraduate career.

Background material, follow up reading:
On-line Chapter 1 typeset notes is your best source, also,
MIT notes on coordinate systems: http://www.umich.edu/~ners311/CourseLibrary/MIT-CoordinateSystems.pdf
Wiki: http://en.wikipedia.org/wiki/Nuclear_fission

1. 2-body elastic collisions, equal masses

Refer to the picture below and familiarize yourself with the symbols and their meanings. The target is at rest, that is, \( \vec{v}_2 = 0 \). You may assume (this is completely general) that the entire process takes place in the \( xy \) plane. The projectile’s velocity, \( v_1 \) is known, as well as all the identical masses, \( m \).

(a) Show that \( \vec{v}_a \cdot \vec{v}_b = 0 \), and explain what that means. This is for review. Do not cut and paste
from the notes! This is a popular exam question (even on Graduate Written examinations!) and it is worth practicing, and rehearsing.

(b) Assume that you have obtained \( v_1 \) and \( \theta \) from a separate measurement. 
Find expressions for the unknowns, \( v_a \), \( v_b \), and \( \varphi \) in terms of \( v_1 \) and \( \theta \) only. 
Justify the sign you choose for \( \varphi \).

c) Show your work, and 
provide numerical answers for \( v_a \), \( v_b \), and \( \varphi \), given that \( v_1 = 1000 \text{ m/s}, \theta = 30.0^\circ \).

(d) Re-analyze the collision in the zero-momentum frame. 
Show that you obtain the same results as part a) once you transform back to the laboratory frame.

2. Inelastic processes

(a) Derive expression (1.35) in the on-line Chapter 1 typeset notes, Slide 32.

(b) Plot (using Matlab) the figure on Slide 33 of the typeset notes, and 
submit a printout of your Matlab code, as well as a copy of the plot.

(c) In the same typeset notes, refer to equations (1.23) and (1.24) on Slide 28. Then, using Matlab, 
plot \( v/v_0 \) vs. \( \cos \theta \) for a family of curves (on the same plot) for various values of \( n \), spanning the range \( 0 \leq n \leq \infty \). (At least 0.1, 0.2, 0.5, 1, 2, 5, 10). Note that only the positive sign is used when \( n > 1 \) and both signs apply when \( n < 1 \). 
Submit your Matlab plot as well as the Matlab code. You can check your results with the figure on Slide 29.

3. Nuclear decay

(a) A stationary (at rest, no velocity) nucleus of known mass \( M \) splits into two nuclei with unequal known masses \((m_1 \text{ and } m_2)\) accompanied by the release of a known amount of energy \( Q \) (i.e. an exothermic reaction). (You may assume that \( M = m_1 + m_2 \)). You have measured the direction of motion of one of the daughter particles, knowing \((\theta, \phi)\), in polar coordinates, measured from the \( z \)-axis. 
State (if already known), else find expressions for the kinetic energy, speed, momentum, and directions (all 3 components) of both resultant nuclei in Cartesian coordinates. Note: Remember that in the spherical-polar coordinate system, \( 0 \leq \theta \leq \pi \).

(b) Suppose that the parent nucleus was not at rest, but moving with known kinetic energy \( K_0 \), in the positive \( z \)-direction. After the split, the direction of the observed daughter nucleus (with subscript “1”) is in the positive \( z \)-direction. 
Find expressions for the kinetic energies of the two resultant nuclei, in terms of \( Q \), \( K_0 \), \( m_1 \), \( m_2 \), and \( M \). Find the direction (all three components, in Cartesian coordinates) of the direction of the second nucleus, in terms of the same quantities. Note 1: You can check your result by setting \( K_0 \), and then comparing with a). Note 2: Alternatively, you can change to the zero-momentum frame, use the result of part a), and then transform back to the laboratory frame.

(c) Now, we are considering numerical work related to the expressions calculated in part b). Use \( Q = 92.00 \pm 0.05 \text{ MeV} \) and \( K_0 = 40.0 \pm 0.1 \text{ MeV} \). One daughter nucleus is going in the \( +\hat{x} \)-direction (exactly). \( Q \) and \( K_0 \) are assumed to be uncorrelated. The parent nucleus has a mass of \( 8u \) (the “8” is exact), while the masses of the daughter nuclei are \( 5u \) (the “5” is exact) and \( 3u \) (the “3” is exact).

Obtain numerical results for two cases, one case is where the \( 5u \) daughter is going in the \( +\hat{x} \)-direction, and the other is where the \( 3u \) is going along the \( +\hat{x} \)-direction, and estimate the expected errors. Report your results with two digits in the error part, e.g. \( a = 0.09742 \pm 0.00027 \), or, more compactly \( a = 0.09742(27) \).

Note: You must propagate errors according to the rules given in Chapters 1.6 and 1.8.