Parts of the solution may be derived in class. However, solve the solution starting from the solution to the one-dimensional time-independent Schrödinger equation for all space, and be sure to show the steps you used to find the solution.

Consider a wave of the form \( A \exp(ik_1x) \) incident on a potential of the form:

\[
V = \begin{cases} 
0 & \text{for } x < 0 \text{ or } x > a \\
V_0 & \text{for } 0 \leq x \leq a 
\end{cases}
\]

where \( k_1 = \sqrt{2mE/\hbar^2} \) relates the wavenumber of the incident particle to its energy. \( V_0 \) is real, but can have any sign, or magnitude. You may assume that \( A \) is known, and that it’s value correctly normalizes the wavefunction, if the other amplitudes are are expressed in terms of \( A \).

There is no wave incident on the barrier/well from the right.

1. [20% Extra Credit] Derive the solution to the one-dimensional time-independent Schrödinger equation for all space. (The results are given below)
2. Find the transmitted and reflected probability currents in terms of \( A \).
3. Find the reflection (\( R \)) and transmission (\( T \)) coefficients.
4. Verify that \( T + R = 1 \).
5. Plot the real and imaginary parts of the incoming, reflected and transmitted waves, everywhere in the special case, \( V_0 = E/2 \). Additionally, plot the probability density of the wavefunction everywhere.
6. Plot the real and imaginary parts of the incoming, reflected and transmitted waves, everywhere in the special case, \( V_0 = -2E \). Additionally, plot the probability density of the wavefunction everywhere.
7. Plot the transmission coefficient as a function of \( V_0/E \).
8. Plot the reflection coefficient as a function of \( V_0/E \).

Some hints:

The incident wave has the form, \( u_i(x) = A \exp(ik_1x) \), where \( A \) is its amplitude, the reflected wave has the form, \( u_r(x) = B \exp(-ik_1x) \), and the transmitted wave has the form \( u_t(x) = F \exp(ik_1x) \). Additionally, the solution in the region \( 0 < x < a \) has the form, \( C \exp(ik_2x) + D \exp(-ik_2x) \), where \( k_2 = \sqrt{2m(E-V_0)/\hbar^2} \). \( k_2 \) can be complex. The solution to the first question is:
\[
\begin{align*}
F &= \exp(-ik_1a) \\
\overline{A} &= \cos(k_2a) - \frac{i}{2} \left( r + \frac{1}{r} \right) \sin(k_2a) \\
\overline{B} &= \frac{i}{2} \left( r - \frac{1}{r} \right) \sin(k_2a) \exp(ik_1a) \\
\overline{C} &= \frac{1}{2} \left( 1 + \frac{1}{r} \right) \exp(-ik_2a) \exp(ik_1a) \\
\overline{D} &= \frac{1}{2} \left( 1 - \frac{1}{r} \right) \exp(ik_2a) \exp(ik_1a)
\end{align*}
\]

where \( r = k_2/k_1 \).

[20% More Extra Credit]:

Let \( V_0 \) have both real and complex parts. Show that probability generation or destruction can occur, and provide a complete discussion, accompanied by graphs.