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TELESCOPIC LIMITING MAGNITUDES

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ABSTRACT

The prediction of the magnitude of the faintest star visible through a telescope by a visual observer is a difficult problem in physiology. Many prediction formulae have been advanced over the years, but most do not even consider the magnification used. I have attacked the prediction algorithm problem with two complimentary approaches: (1) First, I have developed a theoretical algorithm based on physiological data for the sensitivity of the eye. This algorithm also accounts for the transmission of the atmosphere and the telescope, the brightness of the sky, the color of the star, the age of the observer, the aperture, and the magnification. (2) Second, I have collected 314 observed values for the limiting magnitude as a test of my formula. I find that the formula does accurately predict the average observed limiting magnitudes under all conditions. The differences between the observations and the model predictions are not correlated with any of the model input parameters or any parameter other than the observer's experience. When the dependence on experience is accounted for, the model error is typically 0.5 magnitude.

Key words: telescopes–limiting magnitudes

1. Introduction

Amateur astronomers have long discussed how faint an object will be visible with a given telescope. For example, perusal of Walter Scott Houston's long-running "Deep-Sky Wonders" column in *Sky and Telescope* magazine will show this debate in many different guises. The limiting-magnitude question is important to visual variable-star observers planning a night's program and to purchasers of telescopes. A complete prediction formula will allow the impact of light pollution to be evaluated and the optimal magnification to be calculated. Such a formula will allow the evaluation of various interesting historical observations, such as the visual recovery of Halley's comet in 1985 (O'Meara 1985). Finally, the difficult extension of such a formula to extended sources will allow for the prediction of the visibility of comets and possibly for an ab initio calculation of the aperture correction required for visual estimates of cometary brightness.

The utility of a formula for predicting limiting magnitudes has inspired many previously published formulae (Dmitroff and Baker 1945; Bowen 1947; Rosebrugh 1950; Kelly 1953; Brown 1953; Sidgwick 1971; Sinnott 1973; Steffey 1974; Kolman, Schoonveld, and Abels 1976). Most of these proposed criteria are of the form

$$m = N + 5\log(D) \ , \qquad (1)$$

where m is the limiting magnitude for a telescope with aperture D (in inches) and N is some normalization constant. The constant N is assigned a value ranging between 8.8 (Dmitroff and Baker 1945) and 10.7 (Sidgwick 1971). Any such formula will produce a value which is correct to within one or two magnitudes; however, for most applications this accuracy is too poor. This class of formulae is hopelessly naive because many effects are not included, of which the magnification and sky brightness effects are most important. Bowen (1947) and Kolman *et al.* (1976) both introduced formulae where the magnification was used, but other factors were ignored. All the above formulae are based on some chosen functional form (usually eq. (1)) where a normalization constant is deduced from observations whose number ranges from one to a dozen. The derived predictions differ from each other by typically up to three or four magnitudes for any given magnification.

In this paper I will derive a formula for predicting the limiting magnitude of a telescope based on physiological data of the sensitivity of the eye. The result will be a theoretical formula accounting for many significant effects with no adjustable parameters. I will test my formula against 314 observations that I have collected.

2. Formula

The sensitivity of the human eye to point sources has been measured exhaustively, but many of these studies (most notably Blackwell 1946) are not appropriate for use because the color temperatures of the light sources were not reported. An appropriate physiological study is that of

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Knoll, Tousey, and Hulburt (1946) for which Hecht (1947) gives a convenient summary. Their data can be represented as

$$I = C\,(1 + [KB]^{0.5})^2 \; , \qquad (2)$$

$$\log(C) = -9.80, \log(K) = -1.90 \quad \text{if} \log(B) < 3.17 \; ,$$

$$\log(C) = -8.35, \log(K) = -5.90 \quad \text{if} \log(B) > 3.17 \; ,$$

where B is the observed surface brightness of the background in units of millimicroLamberts (mμL) and I is the star brightness in footcandles for the conditions of the experiment. The two sets of constants correspond to day vision and night vision. Equation (2) is the primary equation on which my prediction algorithm is based. The experimental conditions were binocular vision, natural pupils, observer's choice fixation, and no atmospheric absorption. However, many corrections must be applied to relate I and B to the star and sky brightnesses in the V magnitude system.

One correction concerns the fact that equation (2) is based on observations made with both eyes, whereas most telescope observing is made with one eye. The correction from binocular to monocular sensitivity is discussed by Pirenne (1943), who concludes from both observations and theory that the difference in sensitivity is a factor of the square root of two (which corresponds to 0.38 magnitude). Hence, the brightnesses in equation (2) must be multiplied by a binocular correction factor, F_b, which equals 1.41.

The star brightness must be corrected for the absorption of light by the atmosphere of the Earth. This correction will depend on the extinction coefficient of the atmosphere (k) and on the zenith distance of the star, Z. The appropriate value of k will depend on local weather conditions and the effective wavelength of the eye. For day vision, the effective wavelength of the eye is 5550 Å, similar to the V magnitude system for which extinction coefficients (k_v) are frequently quoted. For night vision the effective wavelength is 5100 Å, for which the value of k is typically 20% larger. For good conditions, k_v might be 0.20 magnitude per air mass while more typical weather has a k_v value of 0.30 magnitude per air mass. The star brightness in equation (2) can be corrected for extinction by multiplication by a factor F_e where

$$2.5 \log(F_e) = q\, k_v \sec(Z) \; , \qquad (3)$$

$$q = 1.2 \quad \text{if} \log(B) < 3.17 \; ,$$

$$q = 1.0 \quad \text{if} \log(B) > 3.17 \; .$$

Although k_v might not be known to within 25%, the resultant uncertainty in the limiting magnitude is usually small.

Both the star brightness and the sky brightness must be corrected for the light lost within the telescope. For reflector telescopes with a central obstruction (such as the secondary mirror in a Newtonian telescope) of diameter D_s, the fraction of light lost will scale as $(D_s/D)^2$ when compared to a clear aperture. In addition, each optical surface of the telescope will transmit only a fraction of the incident light. The transmission by a single surface, t_1, can range from 0.96 for a clean uncoated lens surface to perhaps 0.70 for a dirty mirror surface. Special antireflection coatings may significantly improve these figures. This light loss will occur at each glass/air interface including those in the eyepiece. If we idealize all n surfaces to have the same transmission factor, then the correction factor (i.e., the inverse of the overall transmission of the telescope) will be

$$F_t = 1 / [t_1^n (1 - [D_s/D]^2)] \; . \qquad (4)$$

For example, a Newtonian reflector will have two reflections off the primary and secondary mirrors and perhaps four more surfaces within the eyepiece, for a total of six surfaces. Typically, a Newtonian reflector is constructed with D_s/D equal to 0.15. If the optics are freshly cleaned so that t_1 is 0.95 on average, then the total transmission of the telescope will be 72%.

Some of the light that leaves a telescope eyepiece may be lost before it enters the eye of an observer if the exit pupil of the telescope is larger than the pupil of the observer. The diameter of the exit pupil is D/M, where M is the magnification. The diameter of the pupil of the eye, D_e, will vary from person to person but will generally be a function of the observer's age. Kumnick (1954) and Kadlecova, Peleska, and Vasko (1958) present data for the average pupil diameter as a function of age which can be represented by the equation

$$D_e = 7 \text{ mm} \exp(-0.5[A/100]^2) \; , \qquad (5)$$

where A is the observer's age in years. The correction factor for the light loss outside the pupil is

$$F_p = (D/MD_e)^2 \quad \text{if } D_e < D/M \; , \qquad (6)$$

$$F_p = 1.0 \quad \text{if } D_e > D/M \; .$$

This correction factor will apply to both the star and sky brightness.

A telescope has a much greater light-collecting area than does the unaided eye. This implies a correction factor, F_a, for all brightnesses viewed through a telescope of $(D_e/D)^2$ which is the ratio of the collecting areas of the eye and the telescope. For extended sources such as the background sky brightness, the telescope not only collects extra light but also disperses the light by magnifying the image. The surface brightness of the sky will be reduced by a factor, F_m, of M^2 as a result of the light being presented to the observer with a magnification of M.

Normally, the F_m factor should only be applied to the background brightness because a point source under magnification will still appear as a point source. However, if too high a magnification is used, then a star image can be blown up to where the sensitivity of the eye is like an extended source. The critical size will vary somewhat with the background brightness, but for practical cases the critical size is roughly 15 arc minutes or 900″ (Blackwell 1946; Brown *et al.* 1953; Cornsweet 1970). If the radius of the seeing disk is θ then the apparent size of the image will be $2\theta M$. So, for example, if the seeing is poor, say θ is 3″, and a magnification of greater than 150 is used, then a star will appear as an extended source so that equation (2) will no longer apply. The data in Cornsweet (1970) allow an approximate correction factor to equation (2) as

$$F_r = (2\theta M/900'')^{0.5} \quad \text{if } 2\theta M > 900'' \ , \tag{7}$$
$$F_r = 1.0 \quad \text{if } 2\theta M < 900'' \ .$$

Generally, the limiting magnitude of a telescope improves as the magnification increases. However, this will break down when the apparent seeing disk becomes resolved. The resolution of the eye will depend on both the object and background brightness. For bright sources the eye has a resolution that is better than the critical visual angle. Thus, when we look at the image of a bright star, we might be able to see airy rings. But it is impossible to resolve a double star whose separation is smaller than the critical visual angle when both stars are at the threshold of visibility.

The light that enters the eye near the outer edge of the pupil will be less efficiently used than light that enters near the middle of the pupil. This Stiles-Crawford effect is caused by the photon-detection efficiency falling with distance from the center of the eye, r, as

$$E(r) = \exp(-0.105\, r^2) \quad \text{if } \log(B) > 3.17 \ , \tag{8}$$
$$E(r) = 1 - 0.002\, r^4 \quad \text{if } \log(B) < 3.17 \ .$$

The case for bright background is the well-known photopic Stiles-Crawford effect where the equation for the efficiency is equation (5a) from Moon and Spencer (1944). The less-well-known scotopic Stiles-Crawford effect is much weaker, with the formulation in equation (8) based on data from Van Loo and Enoch (1975). So, if a magnification is used where the exit pupil is large (yet smaller than the D_e), the source will appear fainter than if high magnification is used. Since equation (2) is valid for observations made with a natural pupil, a telescope that concentrates the same amount of light nearer to the center of the eye will allow slightly fainter objects to be seen. The correction factor is equal to the ratio of efficiencies averaged over the utilized part of the eye,

$$F_{SC} = (D_e M/D)[1 - \exp(-0.026\,\{D/M\}^2)] \,/\, [1 - \exp(-0.026\, D_e^2)] \tag{9}$$
$$\text{if } D_e > D/M \text{ and if } \log(B) > 3.17 \ ,$$
$$F_{SC} = [1 - (D/12.4\, M)^4] \,/\, [1 - (D_e/12.4)^4]$$
$$\text{if } D_e > D/M \text{ and if } \log(B) < 3.17 \ .$$

This correction factor applies to both star and sky light.

For night vision an additional correction is needed because Knoll *et al.* (1946) report their photometry in millimicroLamberts, which is tied to the photopic sensitivity curve, whereas the scotopic curve is relevant for night vision. This would be no problem if the stars and the night sky had the same spectrum as the experimental light source. However, the experimental light source had a color temperature of 2360 K while the moonless night sky has a color temperature of 5500 K (corresponding to a color index of 0.7) and stars have a color temperature typically ranging from 3000 K to 20,000 K. The total number of photons detected by an instrument can be idealized as the integral over wavelength of a blackbody curve (with some appropriate color temperature T) weighted by the sensitivity curve. If the source brightness is $IN_\lambda(T)$, and the night vision (scotopic) sensitivity curve is S_n, then the total number of detected photons will be

$$N_n(2360\text{ K}) = \int IN_\lambda(2360\text{ K})\, S_n\, d\lambda \ . \tag{10}$$

A similar equation can be constructed to calculate $N_d(2360\text{ K})$, which would be the number of photons registered by a detector with a day vision (photopic) sensitivity curve. In equation (2), the quoted intensities are as measured by a photometer with a day vision curve, so I is proportional to $N_d(2360\text{ K})$. However, what we really want in this equation is an intensity that relates to how many photons are detected by the eye. The need for such a correction is apparent if we consider the extreme case where the experimental light source is, say, very red so that the light source will have to be made brighter so that the observer can detect the source, despite the photometer detecting many photons. The correction factor to convert I to a quantity proportional to the number of photons detected by the observer is

$$F_o = N_n(2360\text{ K})/N_d(2360\text{ K}) \quad \text{if } \log(B) < 3.17 \ , \tag{11}$$
$$F_o = 1.0 \quad \text{if } \log(B) > 3.17 \ .$$

The numerical value of F_o will depend on the normalization factors of the sensitivity curves.

Another similar correction factor is needed for night vision because the reported magnitudes of stars are in the V magnitude system which has a similar spectral response as the day vision sensitivity curve. The need for such a correction is apparent if we consider the case of two stars with equal V magnitude but different color. An observer using day vision would pronounce the two stars to be of equal brightness, whereas if night vision were being used the redder of the two stars would appear fainter. The necessary correction factor for a star with a color temperature of T is

$$F_v = N_d(T)\,/\,N_n(T) \quad \text{if } \log(B) < 3.17 \ , \tag{12}$$
$$F_v = 1.0 \quad \text{if } \log(B) > 3.17 \ .$$

The two correction factors F_o and F_v can be combined into one color-correction factor which I will call F_c. This combination has the advantage that the normalization constants cancel out. I have evaluated the necessary integrals by numerical integration and have related the color temperature to the color index (that is, the $(B-V)$) of the star in question. The value of F_c is adequately represented by the equation

$$-2.5\log(F_c) = 1 - (B-V)/2 \quad \text{if} \log(B) < 3.17 \ , \qquad (13)$$
$$-2.5\log(F_c) = 0 \quad \text{if} \log(B) > 3.17 \ .$$

This color correction must be applied to both the star and sky brightness.

One final correction may be needed if the observer has high or low sensitivity for the detection of point sources compared to the average observer. Let this correction factor be F_s, which will be less than unity for an observer of high acuity.

Now these corrections can be applied to equation (2). The value of the star brightness ($I*$) will be related to the star brightness as perceived by a telescopic observer as

$$I* = IF_b F_e F_t F_p F_a F_r F_{SC} F_c F_s \ . \qquad (14)$$

The V brightness of the sky (B_s) will be related to the background brightness as viewed by a telescopic observer (B) as

$$B_s = B F_b F_t F_p F_a F_{SC} F_m F_c \ . \qquad (15)$$

From this point on I will make the assumption that the sky brightness is sufficiently dark that night vision is involved. This assumption is valid for all observations to be discussed below. For cases where this assumption is not valid (such as observations of the crescent moon in twilight), the application of the above equations is easy.

The brightness of the star must be related to the V magnitude system. The data in Allen (1973) yield

$$m = -16.57 - 2.5\log(I*) \ , \qquad (16)$$

where m is the V magnitude.

The only remaining task is to evaluate B_s. This can be achieved by either of two methods: The first method is to estimate the sky brightness based on experience of local conditions or on actual measurements. Experience can be gained by tabulations of data such as in Koomen *et al.* (1952), Walker (1970, 1973), Garstang (1989), and Pilachowski *et al.* (1989). Typically, the best sites in the world have sky brightnesses of 21.8 magnitudes per square arc second in the V while more normal sites may have a brightness of 21.0 magnitudes per square arc second. The relation between the V sky brightness in magnitudes per square arc second (B_s') and millimicroLamberts (B_s, as used in eq. (2)) is given by equation (27) of Garstang (1989) to be

$$B_s = 34.08\exp(20.7233 - 0.92104\,B_s') \ . \qquad (17)$$

So, a typical site might have a sky brightness of 136 mμL and the best skies in the world might have 65 mμL. For many practical cases the value of B_s does not need to be known with any great accuracy since the KB term in equation (2) will become insignificant with magnification.

The second method for evaluating B_s uses an observation of the faintest star at zenith visible to the unaided eye. For such an observation, $F_b = 1$, $F_t = 1$, $F_p = 1$, $F_a = 1$, $F_r = 1$, $F_{SC} = 1$, $F_m = 1$, and $F_c = 0.5$ on average for an ensemble of naked-eye stars. So, for this observation, equations (2), (3), and (14)–(16) become

$$m_z = 8.68 - 2.5\log(F_s) - 1.2\,k_v - 5\log(1 + 0.158\,B_s^{\,0.5}) \ , \qquad (18)$$

where m_z is the magnitude of the faintest star near the zenith that is visible to the unaided eye. Here there is a basic dilemma that, to use m_z to evaluate B_s, the observer's acuity must be known. In most cases it is reasonable to set F_s to unity. For the collection of observations reported in the next section, the average F_s value will certainly be near unity, even though some observers have much greater or lesser acuity than average. To a large degree the variations in acuity will be accounted for with the second method, since a keen-eyed observer will see faint stars at zenith and the deduced sky brightness will be dimmer by the right amount to compensate for the error in F_s. For a typical observer ($F_s = 1$) and for a typical sky ($B_s = 136$ mμL and $k_v = 0.3$), the zenith limiting magnitude is then 6.05, in excellent agreement with common lore. Russel (1917), Green (1985), and this paper (see Section 4) report on observations with m_z as faint as 8.9. This would imply impossibly dark skies if F_s were unity; however, it is known that the observers had much greater than average acuity. If the sky brightness was 65 mμL, then the sensitivity would be as good as F_s equaling 0.12.

The sky brightness at the zenith is the dimmest part of the night sky. The typical variation with zenith distance can be quantified as

$$B_s(Z) = B_s(Z=0)\,(1 + Z^2/2) \qquad (19)$$

over the relevant range of Z in radians (cf. Pilachowski *et al.* (1989) or Garstang (1989)). This correction should be applied when evaluating the sky brightness from zenith limiting magnitudes.

The long set of equations leading to a prediction of the visual limiting magnitude of a telescope is best implemented as a computer program. Such a program is presented in Schaefer (1989*b*). Figure 1 illustrates the predictions for a variety of standard conditions.

3. Observations

Any theory should be tested against observations. I have located 53 observations in the literature (Kelly 1953; Rosebrugh 1950; Kolman *et al.* 1976; Bowen 1947; Green 1985). This has been supplemented by eight observations from my own observing logs and nine observations contributed by J. Bergeron and K. Krisciunas. I have also published a questionnaire (Schaefer 1989*a*) in *Sky and Telescope* requesting observations. I received 250 individual observations. Six of these reports were rejected because the observer gave grossly inconsistent data or specifically stated that his eyes were adapted to bright lights. Hence, I have 314 observations to test my theoretical model.

These observations are tabulated in Table 1 where they are ordered first by aperture and then by magnification. The first column gives a running number for use in identifying a specific observation. The second column gives the last name of the observer while the next column gives the observer's age in years. The fourth column indicates the type of telescope, with "bino" being a pair of binoculars, "r" being a refractor, "n" being a Newtonian reflector, "sc" being a Schmidt-Cassegrain system, and "mak" being a Maksutov system. The fifth through seventh columns give the telescope aperture in inches, the *f*-ratio, and the magnification, respectively. The eighth column gives the experience of the observer as described below. The ninth column lists the magnitude of the faintest star near the zenith that the observer could see with the unaided eye. The next column gives the source of the magnitude sequence used, with "S" being Schaefer (1989*a*), "A" being an AAVSO chart, "E" being Everhart (984), and "O" being some other sequence. The eleventh and twelfth columns list the transmission of the telescope and the light loss in the atmosphere as discussed below. The thirteenth column gives the zenith distance of the star in degrees. The fourteenth column lists the magnitude of the "faintest star seen for sure", which would correspond roughly to a 90% probability of detection. The next column lists the magnitude of the "faintest star suspected", which would correspond roughly to a 10% probability of detection. The sixteenth column lists the theoretical prediction of the faintest limiting magnitude as derived in Section 2 of this paper. The last column lists the observed minus the predicted (the average of the fourteenth and fifteenth columns minus the sixteenth column).

The observers in the *Sky and Telescope* questionnaire were asked to evaluate their experience on a scale from 1 to 9 with 9 being very experienced. Unfortunately, the scale was ill-defined and the answers had many anomalies. To overcome this lack of realistic data I telephoned many observers and asked a series of five questions designed to evaluate their experience. This and other information available to me was used to evaluate observer experience as shown in the eighth column of Table 1.

The transmission of the telescope was estimated with the following assumptions: The light loss is 1% at each coated air/glass interface. The light loss is 4% at each uncoated air/glass interface. All eyepieces are coated and

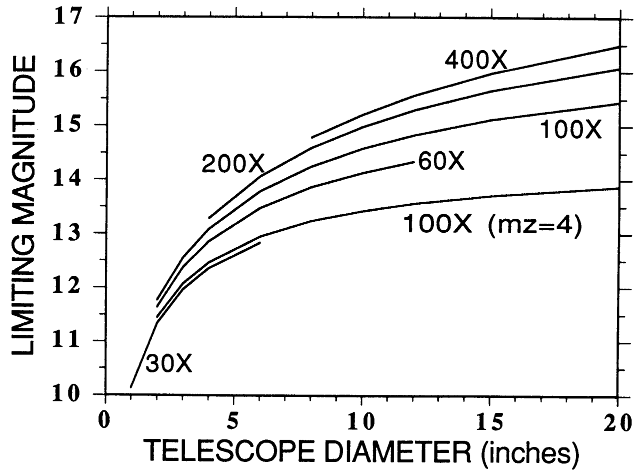

FIG. 1–Theoretical telescopic limiting magnitude. This graph plots the theoretical limiting magnitude as derived in Section 2 as a function of the telescope aperture and magnification. In general, as either the telescope aperture or the magnification is increased, a visual observer will see fainter stars. The plotted curves are for $m_z = 6$, $A = 30$, a telescope throughput of 80%, and an atmospheric throughput of 70%. Also plotted is a curve for 100 power with an $m_z = 4$.

all pairs of glasses are not coated. Both star diagonals and pairs of glasses have two air/glass interfaces. Most eyepieces have four air/glass interfaces, although the erfle type has six interfaces and the Nagler type I has eight interfaces. The reflectivity of an uncoated mirror is 88%. The reflectivity of a coated mirror is 95%. The ratio of D_s/D is 15% for any telescope type with an obstructing secondary mirror. From the preceding assumptions, the total transmission of a clean telescope can be estimated. The observers were asked to rate the cleanliness of their optics on a scale from 1 to 9 (9 being freshly cleaned). I roughly corrected for the dirtiness of the optics by subtracting 1% off the transmission for every unit below 9 in the estimated cleanliness.

The total light loss by extinction in the atmosphere requires knowledge of both Z and k_v. Z was calculated from the observer's latitude, date, and time of observation. The extinction coefficient was estimated from the observer's latitude, relative humidity, time of year, time of day, altitude, and proximity to major pollution sources. The details of similar calculations are presented for one location in Schaefer (1990). In addition, the extinction coefficient was also estimated on the basis of monthly data for 231 sites which I have collected from the literature. These sites are distributed geographically such that all observers were within 200 km of a site for which I have monthly extinction data. The adopted extinction coefficient is the average of my theoretical and empirical estimates.

Blackwell (1946) demonstrated that the limiting magnitude is actually a statistical concept. Since the photon

TABLE 1

Observed Limiting Magnitudes

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
1	Kelly		r	0.3		6		6.00	A	0.88	0.35	30	6.70	6.70	7.33	-0.63
2	Bowen		r	0.3		4		6.00	O	0.88	0.15	30	6.70	6.70	7.27	-0.57
3	Bowen		r	0.3		8		6.00	O	0.88	0.15	30	7.70	7.70	7.66	0.04
4	Kelly		r	0.4		6		6.00	A	0.88	0.35	30	7.40	7.40	7.80	-0.40
5	Kelly		r	0.5		10		6.00	A	0.88	0.35	30	8.60	8.60	8.44	0.16
6	Locher		r	0.8		5		6.60	A	0.88	0.30	30	8.40	8.40	9.06	-0.66
7	Kelly		r	1.0		16		6.00	A	0.88	0.35	30	9.80	9.80	9.83	-0.03
8	Kelly		r	2.0		5		6.00	A	0.88	0.35	30	9.80	9.80	9.11	0.69
9	Krisciunas	35	bino	2.0		7	9	6.40	A	0.90	0.15	30	9.80	9.80	10.66	-0.86
10	Kolman		r	2.0		7		7.50	A	0.88	0.30	30	9.80	9.80	10.85	-1.05
11	Locher		r	2.0		10		6.60	A	0.88	0.30	30	10.30	10.30	10.86	-0.56
12	Bortle		bino	2.0		16	9	7.00	A	0.88	0.30	30	12.00	12.00	11.88	0.12
13	Kelly		r	2.0		32		6.00	A	0.88	0.35	30	11.40	11.40	11.34	0.06
14	Tirado	19	r	2.0	14.0	58	6	6.00	S	0.77	0.20	40	11.19	11.19	11.54	-0.35
15	Tirado	19	r	2.0	14.0	58	6	6.00	S	0.78	0.20	40	12.01	12.01	11.55	0.46
16	Kolman		r	2.0		85		7.50	A	0.88	0.30	30	12.20	12.20	12.66	-0.46
17	Lunas	53	r	2.4	12.0	40	1	4.00	S	0.77	0.30	60	12.57	12.57	10.32	2.25
18	Lunas	57	r	2.4	12.0	50	1	4.00	S	0.78	0.30	60	11.59	11.59	10.62	0.97
19	Underhay	38	r	2.4	15.0	85		5.00	S	0.88	0.30	60	12.01	12.26	11.52	0.62
20	Kocyla	32	r	2.4	15.0	100		5.00	S	0.84	0.30	60	11.59	11.59	11.53	0.06
21	Rosebrugh		r	2.8		55		6.00	A	0.88	0.30	30	11.80	11.80	12.22	-0.42
22	Rosebrugh		r	3.0		60		6.00	A	0.88	0.30	30	12.10	12.10	12.38	-0.28
23	Kelly		r	3.0		66		6.00	A	0.88	0.35	30	12.20	12.20	12.37	-0.17
24	Cardano	35	n	3.0	6.0	75		6.00	S	0.68	0.35	30	11.59	12.01	12.23	-0.43
25	Plante	37	r	3.0	6.7	100		4.70	S	0.83	0.35	30	10.60	11.19	12.16	-1.27
26	Solinski	13	r	3.0		150	3	5.00	S	0.88	0.35	30	10.60	10.60	12.38	-1.78
27	Robinson	23	r	3.0	11.4	151	5	5.00	S	0.90	0.25	30	11.59	12.26	12.55	-0.63
28	Boschat	35	r	3.1	10.0	133		5.50	S	0.76	0.30	50	11.19	12.26	12.35	-0.63
29	Mayo	47	sc	3.5	13.3	54		6.00	S	0.81	0.15	20	13.04	13.35	12.78	0.42
30	Locher		r	3.5		68		6.60	A	0.88	0.30	30	12.70	12.70	12.89	-0.19
31	Rosebrugh		r	3.5		70		6.00	A	0.88	0.30	30	12.80	12.80	12.71	0.09
32	Mayo	47	sc	3.5	13.3	100		6.00	S	0.77	0.15	20	13.35	13.61	13.05	0.43
33	Bova	60	mak	3.5	14.4	130		4.00	S	0.81	0.30	50	11.59	12.01	12.27	-0.47

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
34	Bova	60	mak	3.5	14.4	130		4.50	S	0.81	0.45	30	12.01	12.26	12.55	-0.42
35	Beal	36	mak	3.5	14.4	160		5.00	S	0.74	0.30	50	12.26	12.57	12.50	-0.09
36	Miller	40	sc	3.5	14.0	160		5.00	S	0.77	0.20	20	12.57	13.04	12.89	-0.09
37	Beal	36	mak	3.5	14.4	160		6.00	S	0.74	0.30	30	12.57	13.96	12.88	0.39
38	Kolman		mak	3.5	14.4	160		7.50	A	0.78	0.30	30	13.50	13.50	13.77	-0.27
39	Kelly		r	4.0		64		6.00	A	0.88	0.35	30	12.90	12.90	12.84	0.06
40	Ewalt	57	mak	4.0	10.0	67	4	5.20	S	0.81	0.30	30	11.59	12.01	12.66	-0.86
41	Rosebrugh		r	4.0		80		6.00	A	0.88	0.30	30	12.80	12.80	13.00	-0.20
42	Rosebrugh		n	4.0		80		6.00	A	0.68	0.30	30	11.70	11.70	12.78	-1.08
43	Bodin	44	sc	4.0	10.0	143		6.50	S	0.77	0.35	60	12.26	12.57	12.85	-0.44
44	Kiernan	73	r	4.0	15.0	160		5.70	S	0.75	0.30	50	12.26	12.57	13.28	-0.87
45	Villareal	37	n	4.0	10.0	208	3	6.50	S	0.76	0.25	50	12.57	12.57	13.22	-0.65
46	Locher		r	4.3		33		6.60	A	0.88	0.30	30	12.80	12.80	12.87	-0.07
47	Locher		r	4.3		66		6.60	A	0.88	0.30	30	13.20	13.20	13.25	-0.05
48	Locher		r	4.3		138		6.60	A	0.88	0.30	30	13.70	13.70	13.48	0.22
49	Tirado	19	n	4.5	8.0	75	6	6.00	S	0.71	0.20	50	12.26	12.57	12.89	-0.48
50	Eklof	35	n	4.5	7.9	100		5.50	S	0.69	0.45	50	13.04	13.35	12.55	0.65
51	Eklof	35	n	4.5	7.9	100		6.00	S	0.69	0.35	60	13.04	13.45	12.65	0.60
52	Eklof	35	n	4.5	7.9	225		5.50	S	0.69	0.45	50	13.45	13.61	12.87	0.66
53	Himes	29	n	4.5	16.0	286		6.00	S	0.66	0.40	50	13.04	13.04	13.00	0.04
54	Schaefer	18	r	5.0	8.0	30	5	4.50	E	0.88	0.25	30	11.70	11.70	11.36	0.34
55	Schaefer	18	r	5.0	8.0	30	5	5.00	E	0.88	0.25	30	13.00	13.00	11.78	1.22
56	Mayo	47	sc	5.0	10.0	39		6.00	S	0.81	0.15	20	13.04	13.35	13.00	0.20
57	Mayo	47	sc	5.0	10.0	62		6.00	S	0.81	0.15	20	13.61	13.96	13.40	0.39
58	Calrsson	33	sc	5.0	9.0	71		4.40	S	0.79	0.35	40	12.01	12.26	12.25	-0.12
59	Carlsson	33	sc	5.0	9.0	71		4.90	S	0.79	0.35	60	11.59	12.01	12.15	-0.35
60	Tirado	19	sc	5.0	10.0	104	6	6.00	S	0.81	0.20	50	13.35	13.35	13.35	0.00
61	DePrimo	39	r	5.0	8.0	145	5	5.30	S	0.87	0.30	30	12.57	12.57	13.49	-0.92
62	Mayo	47	sc	5.0	10.0	150		6.00	S	0.81	0.15	20	13.96	14.34	13.89	0.26
63	Peter	41	r	5.0	12.0	188		5.70	S	0.90	0.35	40	13.04	13.35	13.62	-0.43
64	Pensack	37	n	5.1	5.5	152		6.30	S	0.65	0.15	50	13.61	13.61	13.60	0.01
65	Bowen		n	6.0		24		6.00	O	0.68	0.15	30	11.90	11.90	12.42	-0.52
66	Schaefer	18	n	6.0	8.0	30	5	5.00	E	0.68	0.25	30	12.40	12.40	11.82	0.58

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
67	Skiff	33	r	6.0	8.0	30	9	7.00	S	0.90	0.15	40	13.35	13.35	13.54	-0.19
68	Bowen		n	6.0		43		6.00	O	0.68	0.15	30	12.90	12.90	13.08	-0.18
69	Skiff	33	r	6.0	8.0	50	9	7.00	S	0.90	0.15	40	13.61	13.61	13.97	-0.36
70	Pinkney	22	n	6.0	6.0	65		4.00	S	0.64	0.25	40	12.01	12.26	12.00	0.14
71	Bowen		n	6.0		67		6.00	O	0.68	0.15	30	13.30	13.30	13.46	-0.16
72	Skiff	33	r	6.0	8.0	80	9	7.00	S	0.90	0.15	40	14.34	14.34	14.26	0.08
73	Meyer	32	n	6.0	5.0	87		6.00	S	0.67	0.20	20	12.01	12.26	13.63	-1.50
74	Krisciunas	35	n	6.0	5.8	98	9	6.04	S	0.72	0.15	40	13.96	14.34	13.74	0.41
75	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	13.80	13.80	13.93	-0.13
76	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	14.50	14.50	13.93	0.57
77	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	14.20	14.50	13.93	0.42
78	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	14.00	14.30	13.93	0.22
79	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	14.20	14.20	13.93	0.27
80	Krisciunas	35	n	6.0	5.8	98	9	6.40	A	0.72	0.15	30	13.70	13.70	13.93	-0.23
81	Krisciunas	35	n	6.0	5.8	98	9	6.40	S	0.72	0.15	30	13.96	13.96	13.93	0.03
82	Schaefer	18	n	6.0	8.0	100	5	4.10	E	0.68	0.25	30	12.70	12.70	12.70	0.00
83	Weisend	36	n	6.0	5.0	100	4	4.90	S	0.78	0.30	50	12.57	13.04	13.00	-0.20
84	Weisend	36	n	6.0	5.0	100	4	6.00	S	0.78	0.30	40	13.61	13.96	13.64	0.15
85	Bergeron		r	6.0	8.0	100	7	6.30	A	0.90	0.20	30	14.60	14.60	14.02	0.58
86	Bergeron		r	6.0	8.0	100	7	6.30	A	0.90	0.20	30	14.70	14.70	14.02	0.68
87	Weisend	36	n	6.0	5.0	100	4	6.40	S	0.78	0.30	60	13.61	13.96	13.49	0.30
88	Rosebrugh		r	6.0		120		6.00	A	0.88	0.30	30	14.00	14.00	13.89	0.11
89	Rosebrugh		n	6.0		120		6.00	A	0.68	0.30	30	13.50	13.50	13.67	-0.17
90	Bowen		n	6.0		120		6.00	O	0.68	0.15	30	13.70	13.70	13.82	-0.12
91	Clark	43	n	6.0	8.0	122	4	4.50	S	0.72	0.30	20	12.26	12.26	13.26	-1.00
92	Clark	43	n	6.0	8.0	122	4	5.00	S	0.68	0.30	20	13.04	13.04	13.44	-0.40
93	Vinson	41	n	6.0	8.0	133	7	5.30	S	0.69	0.35	60	13.04	13.04	13.06	-0.02
94	Skiff	33	r	6.0	8.0	140	9	7.00	S	0.90	0.15	40	14.96	14.96	14.49	0.47
95	Burkhart	43	n	6.0	8.0	150		5.10	S	0.68	0.35	30	12.57	12.57	13.51	-0.94
96	Cardano	35	n	6.0	8.5	160		6.00	S	0.69	0.35	30	13.04	13.35	13.77	-0.58
97	Kolman		n	6.0		160		7.50	A	0.68	0.30	30	14.30	14.30	14.67	-0.37
98	Skiff	33	r	6.0	8.0	165	9	7.00	S	0.90	0.15	40	15.30	15.58	14.54	0.90
99	Skiff	33	r	6.0	8.0	165	9	7.00	S	0.90	0.15	40	15.30	15.30	14.54	0.76

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
100	Wipfield	42	r	6.0	8.0	180	5	5.00	S	0.88	0.30	30	13.96	14.34	13.83	0.32
101	Weisend	36	n	6.0	5.0	180	4	6.40	S	0.74	0.30	60	13.96	14.34	13.67	0.48
102	Mayer		n	6.0		180	9	7.00	A	0.68	0.30	30	15.10	15.10	14.27	0.83
103	Thayer	61	n	6.0	5.5	200	8	4.00	S	0.68	0.25	30	12.57	12.57	13.51	-0.94
104	Vinson	41	n	6.0	8.0	200	7	5.30	S	0.69	0.35	60	13.04	13.04	13.29	-0.25
105	Hoffmann	43	r	6.0	8.0	200		6.00	S	0.84	0.25	40	13.61	13.96	14.13	-0.35
106	Hoffmann	43	r	6.0	8.0	200		6.00	S	0.84	0.25	40	13.61	13.96	14.13	-0.35
107	Hoffmann	43	r	6.0	8.0	200		6.00	S	0.84	0.25	40	12.57	13.04	14.13	-1.33
108	Cardano	35	n	6.0	8.5	200		6.00	S	0.69	0.35	30	12.57	13.04	13.84	-1.04
109	Bowen		n	6.0		200		6.00	O	0.68	0.15	30	14.00	14.00	14.02	-0.02
110	Nakamura	34	r	6.0	12.0	200		6.30	S	0.88	0.35	30	14.34	14.34	14.12	0.22
111	Mohn	40	n	6.0	8.0	202	1	6.00	S	0.67	0.30	20	12.26	13.96	13.95	-0.84
112	Cardano	35	n	6.0	8.5	220		6.00	S	0.69	0.35	30	13.35	13.61	13.86	-0.38
113	Luciano	40	r	6.0	15.0	225		5.30	S	0.84	0.30	30	13.04	13.35	13.98	-0.79
114	Hermsen	43	n	6.0	6.0	230		4.50	S	0.70	0.40	40	13.61	13.96	13.43	0.36
115	Olson	33	r	6.0	12.0	247		3.50	S	0.88	0.30	30	13.35	13.61	13.42	0.06
116	Hermsen	43	n	6.0	6.0	285		4.50	S	0.70	0.50	40	13.61	13.96	13.42	0.37
117	Skiff	33	r	6.0	8.0	295	9	7.00	S	0.90	0.15	40	15.58	15.58	14.67	0.91
118	Burkhart	43	n	6.0	8.0	300		5.10	S	0.68	0.35	30	12.57	12.57	13.83	-1.26
119	Bowen		n	6.0		300		6.00	O	0.68	0.15	30	14.00	14.00	14.13	-0.13
120	Barnum	47	n	6.0	10.0	600	4	5.00	S	0.83	0.35	30	12.57	14.66	14.07	-0.46
121	Caruso	42	r	6.5	16.0	162	6	4.70	S	0.79	0.35	50	13.04	13.35	13.40	-0.21
122	Newbill	40	r	7.0	9.0	200		5.50	S	0.90	0.30	50	14.30	14.66	14.11	0.37
123	Newbill	40	r	7.0	9.0	320		4.50	S	0.86	0.30	50	13.96	14.34	14.01	0.14
124	Newbill	40	r	7.0	9.0	320		5.50	S	0.86	0.30	50	14.66	14.96	14.28	0.53
125	Newbill	40	r	7.0	9.0	320		5.80	S	0.85	0.30	30	14.96	15.30	14.51	0.62
126	Kolman		n	7.9		160		7.50	A	0.68	0.30	30	13.50	13.50	15.17	-1.67
127	Simison	65	sc	8.0	10.0	55		3.80	S	0.73	0.25	50	12.26	12.57	11.76	0.66
128	Laurent	25	sc	8.0	10.0	62		5.00	S	0.74	0.25	30	12.57	13.04	13.08	-0.28
129	Schmidt	42	sc	8.0	10.0	77	2	4.00	S	0.77	0.30	30	12.26	12.26	12.66	-0.40
130	Talbert	45	sc	8.0	10.0	77		5.00	S	0.80	0.35	30	12.26	13.96	13.39	-0.28
131	Whitta	35	sc	8.0	10.0	77		5.00	S	0.77	0.25	40	12.26	12.57	13.29	-0.88
132	Brinkmann	27	sc	8.0	10.0	80		5.00	S	0.71	0.35	40	12.26	12.26	13.19	-0.93

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
133	LaBeau	47	sc	8.0	10.0	100	2	3.00	S	0.80	0.30	10	11.19	11.19	12.44	-1.25
134	Pryal	42	sc	8.0	10.0	100	8	6.00	S	0.79	0.30	60	13.04	13.35	13.80	-0.61
135	Baer	47	n	8.0	4.5	102		5.00	S	0.69	0.35	30	13.61	15.38	13.61	0.89
136	Kluth	42	sc	8.0	10.0	110		5.60	S	0.81	0.35	50	13.61	13.96	13.83	-0.05
137	Johnson	50	n	8.0	5.7	116		5.20	S	0.70	0.40	40	13.96	14.34	13.71	0.44
138	Plante	37	sc	8.0	10.0	117		4.50	S	0.75	0.35	30	12.01	12.26	13.46	-1.33
139	Ayer	69	sc	8.0	10.0	124		5.00	S	0.80	0.30	60	14.34	14.66	13.62	0.88
140	Schallert	66	sc	8.0	10.0	145		6.10	S	0.79	0.20	50	13.35	13.96	14.63	-0.98
141	Maher	31	n	8.0	4.5	152	6	6.00	S	0.71	0.30	30	13.96	14.34	14.31	-0.16
142	Talbert	46	sc	8.0	10.0	154		5.00	S	0.74	0.35	30	12.26	13.96	14.00	-0.89
143	Gagne	46	sc	8.0	10.0	154		5.70	S	0.72	0.25	30	13.96	14.34	14.36	-0.21
144	Vincent	39	sc	8.0	10.0	160		4.50	S	0.70	0.25	50	12.57	13.04	13.60	-0.80
145	Underhay	38	n	8.0	5.1	160		5.50	S	0.62	0.25	60	13.96	14.34	13.79	0.36
146	Miner	41	sc	8.0	10.0	160		5.50	S	0.80	0.30	30	13.04	13.04	14.31	-1.27
147	Billups	15	sc	8.0	20.0	160		5.70	S	0.79	0.30	30	13.04	13.61	14.26	-0.94
148	Billups	15	sc	8.0	20.0	160		6.00	S	0.81	0.30	30	13.61	13.61	14.38	-0.77
149	Polakis	27	n	8.0	3.8	161	7	6.50	S	0.68	0.20	20	14.96	15.30	14.56	0.57
150	Cortes	42	n	8.0	5.8	163		5.00	S	0.68	0.45	20	12.57	13.01	13.94	-1.15
151	Cortes	42	n	8.0	5.8	163		6.00	S	0.71	0.45	20	13.61	13.96	14.32	-0.54
152	Wolf	42	sc	8.0	10.0	166	3	5.00	S	0.80	0.20	20	14.34	14.96	14.27	0.38
153	Brinkmann	27	sc	8.0	10.0	167		5.00	S	0.71	0.35	40	12.57	12.57	13.84	-1.27
154	Kenney	34	n	8.0	5.0	168		5.00	S	0.70	0.30	50	12.26	12.57	13.79	-1.38
155	Jandorf	40	n	8.0	7.0	200		5.00	S	0.68	0.25	20	12.57	13.04	14.22	-1.42
156	Ewalt	57	sc	8.0	10.0	200	4	5.20	S	0.75	0.30	30	13.04	13.35	14.41	-1.22
157	Oleski	36	sc	8.0	10.0	200	4	5.50	S	0.80	0.35	40	13.61	14.34	14.27	-0.30
158	Melillo	30	sc	8.0	10.0	220		4.50	S	0.79	0.40	30	13.04	13.35	13.97	-0.78
159	Brinkmann	27	sc	8.0	10.0	220		5.00	S	0.71	0.35	40	13.96	13.96	14.01	-0.05
160	LaBeau	47	sc	8.0	10.0	222	2	3.00	S	0.82	0.30	10	12.57	12.57	13.54	-0.97
161	Vincent	39	sc	8.0	10.0	225		4.50	S	0.70	0.25	50	12.57	13.35	13.89	-0.93
162	Schallert	66	sc	8.0	10.0	227		5.00	S	0.79	0.20	50	12.26	12.57	14.46	-2.05
163	Brinkmann	27	sc	8.0	10.0	240		5.00	S	0.67	0.35	40	13.61	13.61	14.01	-0.40
164	Thiele	26	n	8.0	6.0	240		5.00	S	0.69	0.35	40	13.04	13.35	14.03	-0.84
165	Thiele	26	n	8.0	6.0	240		5.80	S	0.69	0.30	40	13.96	13.96	14.32	-0.36

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
166	Thiele	26	n	8.0	6.0	240		6.00	S	0.69	0.30	40	14.34	14.34	14.37	-0.03
167	Lee	34	sc	8.0	10.0	242		4.70	S	0.77	0.20	30	13.61	13.96	14.30	-0.52
168	Lee	34	sc	8.0	10.0	242		4.70	S	0.77	0.20	30	14.34	14.34	14.30	0.04
169	Parnell	41	n	8.0	7.5	250		4.00	S	0.69	0.35	30	13.35	13.35	13.87	-0.52
170	Wyman	47	n	8.0	7.0	250		5.10	S	0.63	0.35	30	13.04	13.35	14.22	-1.03
171	McKenna	37	sc	8.0	10.0	250		5.30	S	0.79	0.35	50	13.61	13.96	14.19	-0.41
172	Reed	48	sc	8.0	10.0	265	6	5.00	S	0.80	0.25	30	14.96	15.30	14.53	0.60
173	Campbell	44	sc	8.0	10.0	266		5.00	S	0.77	0.30	20	13.96	14.34	14.47	-0.32
174	Bodin	44	sc	8.0	10.0	286		6.50	S	0.77	0.35	60	14.66	14.96	14.36	0.45
175	Brinkmann	27	sc	8.0	10.0	330		5.00	S	0.71	0.35	40	14.34	14.34	14.21	0.13
176	Chandler	68	sc	8.0	10.0	330		5.30	S	0.78	0.35	50	14.34	14.66	14.58	-0.08
177	Rosebrugh		n	8.8		175		6.00	A	0.68	0.30	30	14.10	14.10	14.49	-0.39
178	Rosebrugh		n	9.4		188		6.00	A	0.68	0.30	30	14.10	14.10	14.64	-0.54
179	Tirado	19	sc	10.0	6.3	61	6	6.00	S	0.81	0.20	40	13.61	14.66	13.98	0.16
180	Nielsen	38	sc	10.0	10.0	96		5.50	S	0.80	0.30	50	13.96	13.96	13.95	0.01
181	Robinson	28	sc	10.0	10.0	100	3	5.00	S	0.81	0.35	30	14.34	14.66	13.86	0.64
182	Goodwyn	49	n	10.0	4.5	110		5.00	S	0.68	0.30	50	12.26	12.57	13.73	-1.32
183	Goodwyn	49	n	10.0	4.5	110		5.00	S	0.68	0.30	30	12.26	12.57	13.98	-1.57
184	Goodwyn	49	n	10.0	0.5	110		5.20	S	0.68	0.30	30	12.57	13.04	14.11	-1.31
185	McCagne	42	n	10.0	5.6	112		4.90	S	0.68	0.35	30	13.61	13.61	13.87	-0.26
186	Gainsford	53	n	10.0	6.0	120		5.00	S	0.69	0.40	40	14.34	14.34	13.92	0.42
187	Gainsford	53	n	10.0	6.0	120		5.00	S	0.69	0.50	40	14.34	14.34	13.84	0.50
188	Gainsford	53	n	10.0	6.0	120		5.00	S	0.69	0.45	40	13.96	13.96	13.88	0.08
189	Walker	31	n	10.0	5.6	140	6	6.30	S	0.69	0.25	40	14.34	14.96	14.71	-0.06
190	Wilder	50	sc	10.0	10.0	153		6.00	S	0.77	0.25	40	13.96	14.34	14.84	-0.69
191	Miner	41	n	10.0	4.5	155		5.50	S	0.70	0.30	30	13.96	13.96	14.53	-0.57
192	Robb	36	sc	10.0	10.0	166		5.00	S	0.79	0.20	30	13.96	13.96	14.48	-0.52
193	Helmut	51	sc	10.0	10.0	167		4.50	S	0.80	0.35	40	12.01	13.04	14.08	-1.56
194	Helmut	51	sc	10.0	10.0	167		5.00	S	0.80	0.50	40	12.01	13.04	14.22	-1.70
195	Helmut	51	sc	10.0	10.0	167		5.20	S	0.80	0.35	40	12.26	13.04	14.46	-1.81
196	Helmut	51	sc	10.0	10.0	167		5.60	S	0.80	0.35	40	13.04	13.61	14.65	-1.33
197	Helmut	51	sc	10.0	10.0	167		6.20	S	0.80	0.35	40	13.61	13.96	14.91	-1.13
198	Hart	51	sc	10.0	10.0	174	3	5.50	S	0.81	0.30	40	14.66	14.96	14.70	0.11

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
199	Troiani	37	n	10.0	6.0	174	9	5.80	S	0.67	0.30	60	15.30	16.06	14.27	1.41
200	Miller	41	sc	10.0	10.0	192		4.50	S	0.79	0.25	50	13.96	14.96	14.12	0.34
201	Kolman		n	10.0		200		7.50	A	0.68	0.30	30	14.60	14.60	15.68	-1.08
202	Babcock	42	n	10.0	6.1	212	6	5.00	S	0.68	0.45	30	13.96	13.96	14.38	-0.42
203	Babcock	42	n	10.0	6.1	212	6	6.00	S	0.68	0.30	30	14.66	14.96	14.87	-0.06
204	Shaw	42	n	10.0	6.0	213		4.50	S	0.65	0.45	60	12.57	13.04	13.56	-0.76
205	McCagne	42	n	10.0	5.6	237		4.90	S	0.68	0.35	30	13.96	13.96	14.51	-0.55
206	Murray	43	n	10.0	5.8	246	5	5.50	S	0.65	0.35	40	14.96	14.96	14.62	0.34
207	Walker	31	n	10.0	5.6	264	6	6.30	S	0.65	0.25	40	14.96	15.30	14.92	0.21
208	Helmut	51	sc	10.0	10.0	330		4.40	S	0.76	0.35	40	12.57	13.04	14.59	-1.79
209	Helmut	51	sc	10.0	10.0	330		5.50	S	0.76	0.35	40	13.35	13.61	14.96	-1.48
210	Helmut	51	sc	10.0	10.0	330		6.20	S	0.76	0.35	40	13.61	13.96	15.13	-1.35
211	Thayer	61	n	10.0	5.6	340	8	3.65	S	0.67	0.40	30	13.96	13.96	14.34	-0.38
212	Honkus	38	sc	10.0	10.0	357		5.50	S	0.79	0.35	30	16.31	16.31	15.00	1.31
213	Helmut	51	sc	10.0	10.0	400		5.00	S	0.76	0.35	40	13.35	13.61	14.90	-1.42
214	Shaw	42	n	10.0	6.0	406		4.50	S	0.67	0.45	60	12.57	13.04	14.04	-1.24
215	Helmut	51	sc	10.0	10.0	417		4.50	S	0.80	0.35	40	12.01	13.04	14.82	-2.30
216	Helmut	51	sc	10.0	10.0	417		5.60	S	0.80	0.35	40	13.04	13.61	15.12	-1.80
217	Pitcairn	31	n	10.1	4.5	156		6.20	S	0.70	0.25	30	13.96	14.34	14.82	-0.67
218	Feyen	40	n	10.1	4.5	165	4	4.40	S	0.70	0.30	60	12.57	13.04	13.57	-0.77
219	Kelly	30	n	10.1	4.5	190		5.50	S	0.67	0.45	50	13.96	13.96	14.17	-0.21
220	Manning	62	n	10.3	7.3	150		5.00	S	0.59	0.45	40	13.96	13.96	14.09	-0.13
221	Manning	62	n	10.3	7.3	300		5.00	S	0.59	0.45	40	14.34	14.66	14.58	-0.08
222	Kemble	66	sc	11.0	10.0	167	7	5.00	S	0.82	0.30	40	15.58	15.58	14.64	0.94
223	Kemble	66	sc	11.0	10.0	267	7	5.00	S	0.82	0.30	40	15.58	15.58	15.06	0.52
224	diCicco		c	11.0		330	9	7.00	A	0.68	0.30	30	15.90	15.90	15.58	0.32
225	Jules	47	sc	11.0	10.0	340		5.00	S	0.82	0.30	40	13.96	14.66	15.06	-0.75
226	Kemble	66	sc	11.0	10.0	418	7	4.00	S	0.76	0.40	40	13.96	14.34	14.81	-0.66
227	Kemble	66	sc	11.0	10.0	418	7	4.50	S	0.76	0.40	60	16.06	16.06	14.60	1.46
228	Kemble	66	sc	11.0	10.0	418	7	4.50	S	0.76	0.30	40	14.96	15.58	15.12	0.15
229	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.40	40	13.96	14.34	15.17	-1.02
230	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.35	40	15.58	15.58	15.23	0.35
231	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.30	40	15.58	15.58	15.29	0.29

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
232	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.30	40	15.58	15.58	15.29	0.29
233	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.35	50	15.58	15.58	15.10	0.48
234	Kemble	66	sc	11.0	10.0	418	7	5.00	S	0.76	0.30	40	14.34	14.66	15.29	-0.79
235	Kemble	66	sc	11.0	10.0	418	7	5.50	S	0.76	0.35	40	16.06	16.06	15.38	0.68
236	Kemble	66	sc	11.0	10.0	418	7	5.90	S	0.76	0.30	40	15.58	16.06	15.54	0.28
237	Kemble	66	sc	11.0	10.0	418	7	6.00	S	0.76	0.30	40	15.30	16.06	15.56	0.12
238	Kemble	66	sc	11.0	10.0	418	7	6.00	S	0.76	0.30	50	16.06	16.06	15.46	0.60
239	Wettlaufer	40	r	12.0	16.6	125		5.00	S	0.69	0.45	40	14.96	15.30	14.06	1.07
240	Anderton	45	n	12.0	7.6	130		4.50	S	0.68	0.45	40	12.26	12.57	13.80	-1.39
241	Clark	65	c	12.0	22.0	220		5.20	S	0.61	0.40	20	13.04	13.35	14.98	-1.79
242	Wettlaufer	40	r	12.0	16.6	555		5.00	S	0.69	0.45	40	15.30	15.58	14.95	0.49
243	Kolman		n	12.1		240		7.50	A	0.68	0.30	30	15.10	15.10	16.09	-0.99
244	Kolman		n	12.1		310		7.50	A	0.68	0.30	30	15.30	15.30	16.18	-0.88
245	Himes	29	n	12.5	6.0	60		6.00	S	0.66	0.40	50	12.26	12.57	13.82	-1.41
246	Schaefer	18	n	12.5	8.0	100	5	4.00	E	0.68	0.25	30	13.20	13.20	13.29	-0.09
247	Normandin	41	c	12.5	16.0	104		6.00	S	0.57	0.30	30	12.57	13.35	14.65	-1.69
248	Lavigne	25	n	12.5	6.0	118		6.40	S	0.64	0.35	10	14.34	14.66	15.00	-0.50
249	Himes	29	n	12.5	6.0	122		6.00	S	0.68	0.40	50	13.96	13.96	14.49	-0.53
250	Gable	41	n	12.5	6.0	146	5	4.50	S	0.69	0.30	30	13.96	14.34	14.19	-0.04
251	Lipnisky	39	n	12.5	5.0	164		4.50	S	0.71	0.30	30	12.57	13.04	14.33	-1.53
252	Cortes	42	n	12.5	5.8	174		6.00	S	0.68	0.45	20	14.66	14.96	15.08	-0.27
253	Glass	60	n	12.5	5.8	174		6.00	S	0.62	0.45	20	14.66	14.96	15.15	-0.34
254	Morales	42	n	12.5	7.0	185	9	5.40	S	0.71	0.20	30	15.58	15.58	15.03	0.55
255	Bortle	45	n	12.5	5.6	238	9	6.30	S	0.68	0.30	30	15.30	15.58	15.43	0.01
256	Parnell	41	n	12.5	6.0	268		4.50	S	0.69	0.30	40	13.96	14.34	14.70	-0.55
257	Himes	29	n	12.5	6.0	286		5.00	S	0.64	0.40	30	14.34	14.66	14.84	-0.34
258	Himes	29	n	12.5	6.0	286		6.00	S	0.66	0.40	50	14.34	14.34	14.95	-0.61
259	Novak	27	c	12.5	9.0	317		5.60	S	0.70	0.30	20	14.96	15.30	15.30	-0.17
260	Bortle		n	12.5		375	9	7.20	A	0.68	0.30	30	16.30	16.30	16.04	0.26
261	Parnell	41	n	12.5	6.0	376		4.00	S	0.69	0.35	30	14.34	14.66	14.81	-0.31
262	Moore	30	n	13.0	4.5	180		5.50	S	0.67	0.40	40	13.35	13.61	14.74	-1.26
263	Coe	39	n	13.0	5.6	230		6.20	S	0.77	0.20	20	14.96	15.58	15.65	-0.38
264	Polakis	27	n	13.0	4.5	312	7	4.50	S	0.60	0.20	20	14.96	14.96	14.94	0.02

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
265	Polakis	27	n	13.0	4.5	312	7	6.50	S	0.60	0.20	20	16.06	16.31	15.55	0.64
266	DePrimo	39	n	13.1	4.5	136	5	5.20	S	0.68	0.35	30	14.66	14.66	14.55	0.11
267	Davis	32	n	13.1	4.5	142		6.10	S	0.67	0.35	20	14.96	15.30	15.08	0.05
268	York	37	n	13.1	4.5	150	7	6.50	S	0.66	0.15	50	15.30	15.58	15.29	0.15
269	Taylor	30	n	13.1	4.5	164		5.40	S	0.64	0.35	30	13.96	13.96	14.74	-0.78
270	DePrimo	39	n	13.1	4.5	214	5	5.20	S	0.68	0.35	30	14.96	14.96	14.94	0.02
271	DePrimo	39	n	13.1	4.5	214	7	5.30	S	0.64	0.30	30	14.66	14.66	14.98	-0.32
272	Lucas	47	n	13.1	4.5	272	9	6.00	S	0.64	0.30	60	15.30	15.58	15.06	0.38
273	DePrimo	39	n	13.1	4.5	323	5	5.20	S	0.67	0.35	30	14.96	14.96	15.20	-0.24
274	Normandin	41	sc	14.0	11.0	109		6.00	S	0.71	0.30	40	14.34	14.96	14.89	-0.24
275	Katowik	40	sc	14.0	11.0	300		5.00	S	0.75	0.35	30	14.66	14.96	15.27	-0.46
276	Radville	31	n	14.5	4.5	290	3	5.00	S	0.81	0.30	30	13.96	14.34	15.35	-1.20
277	Caruso	42	c	16.0	18.0	133	6	5.10	S	0.62	0.40	30	14.66	14.96	14.62	0.19
278	Kolman		n	16.0		160		7.50	A	0.68	0.30	30	15.30	15.30	16.32	-1.02
279	Stevens	34	n	16.0	4.9	200		6.00	S	0.67	0.25	50	15.58	16.06	15.41	0.41
280	Kolman		n	16.0		320		7.50	A	0.68	0.30	30	15.50	15.50	16.70	-1.20
281	Poyner	30	n	16.0	5.0	336		5.50	S	0.68	0.35	50	15.30	15.58	15.38	0.06
282	Renner	40	c	16.0	14.0	360	7	5.00	S	0.67	0.30	30	14.96	15.30	15.55	-0.42
283	Cortes	42	n	16.0	4.5	381		5.50	S	0.65	0.30	40	15.30	15.58	15.68	-0.24
284	Bodin	44	n	17.5	4.5	160		6.50	S	0.65	0.40	60	14.96	15.30	15.25	-0.12
285	Bodin	44	n	17.5	4.5	160		6.50	S	0.65	0.25	60	14.96	15.58	15.51	-0.24
286	Pool	31	n	17.5	4.5	167		5.50	S	0.70	0.20	30	15.30	15.58	15.36	0.08
287	Bodin	44	n	17.5	4.5	222		6.50	S	0.65	0.30	60	15.30	15.58	15.62	-0.18
288	Bodin	44	n	17.5	4.5	222		6.50	S	0.65	0.40	60	15.30	16.06	15.43	0.25
289	Bodin	44	n	17.5	4.5	222		6.50	S	0.70	0.25	60	15.30	15.58	15.77	-0.33
290	Seal	46	n	17.5	4.5	240		7.20	S	0.68	0.15	60	16.62	17.05	16.42	0.42
291	Schallert	66	n	17.5	5.0	253		5.00	S	0.68	0.25	50	13.61	13.96	15.41	-1.63
292	Schaeffer	32	n	17.5	4.5	282		4.80	S	0.66	0.30	30	15.58	16.06	15.35	0.47
293	Fitch	38	n	17.5	4.5	286		6.00	S	0.67	0.40	30	15.58	15.58	15.83	-0.25
294	Bodin	44	n	17.5	4.5	286		6.50	S	0.65	0.30	60	15.30	15.58	15.74	-0.30
295	Bodin	44	n	17.5	4.5	286		6.50	S	0.65	0.40	60	15.30	16.06	15.55	0.13
296	Bodin	44	n	17.5	4.5	286		6.50	S	0.70	0.25	60	15.30	15.58	15.91	-0.47
297	Schaeffer	32	n	17.5	4.5	420		4.80	S	0.66	0.30	30	15.30	15.58	15.66	-0.22

TABLE 1 (Continued)

#	OBSERVER	AGE	TEL	APER	f /	MAG	E	MZ	SEQ	T	KV	Z	MLIM	MLIM?	MPRED	ERROR
298	Oleski	36	n	18.0	4.4	200	4	5.00	S	0.84	0.30	40	15.30	15.58	15.22	0.22
299	Thompson	39	n	18.0	4.4	420		6.40	S	0.80	0.40	50	15.58	16.06	16.09	-0.27
300	Cragg		n	18.0		540	9	7.00	A	0.68	0.20	30	17.30	17.30	16.59	0.71
301	Schaefer	18	r	20.0	18.0	225	5	3.50	E	0.77	0.25	30	13.50	13.50	14.47	-0.97
302	Schaefer	18	r	20.0	18.0	225	5	4.00	E	0.77	0.25	30	14.00	14.00	14.84	-0.84
303	Schaefer	18	r	20.0	18.0	225	5	4.50	E	0.77	0.25	30	14.40	14.40	15.18	-0.78
304	Bortle	45	n	20.0	5.0	241	9	6.30	S	0.71	0.35	30	16.31	16.31	16.22	0.09
305	Kriege	35	n	20.0	5.0	411		4.80	S	0.71	0.30	30	15.30	15.58	15.91	-0.47
306	Verdenet		n	21.0		630	9	7.00	A	0.68	0.30	30	17.50	17.50	16.81	0.69
307	Cavadore	20	n	24.0	3.6	175		6.10	S	0.70	0.15	30	16.31	16.62	16.16	0.31
308	Morris		n	24.0		720	9	7.00	A	0.68	0.20	30	17.70	17.70	17.05	0.65
309	Harvey	48	n	29.0	4.6	400		5.00	S	0.68	0.30	30	16.31	16.62	16.52	-0.06
310	Bowen		n	60.0		260		6.00	O	0.68	0.15	30	16.80	16.80	17.53	-0.73
311	Bowen		n	60.0		500		6.00	O	0.68	0.15	30	17.60	17.60	18.16	-0.56
312	Bowen		n	60.0		750		6.00	O	0.68	0.15	30	18.00	18.00	18.26	-0.26
313	Bowen		n	60.0		1500		6.00	O	0.68	0.15	30	18.00	18.00	18.26	-0.26
314	Warner		n	82.0		1000	9	7.00	O	0.66	0.15	30	20.50	20.50	19.23	1.27

arrival and detection is randomly distributed in time, some time intervals will have sufficient photons for detection while other time intervals will not. Blackwell shows that the difference from 10% to 50% and from 50% to 90% detection probability is roughly half a magnitude. In the laboratory it is straightforward to measure the probability of detection. However, at the telescope, it is not well-defined what probability level is assigned to each detection by the various observers. I assume that the "faintest star seen for sure" corresponds to a 90% confidence level, while the "faintest star suspected" corresponds to a 10% probability. The experimental results (eq. (2)) are based on a 50% probability of detection. So the error of the model should be the average of the 10% and 90% observational limits minus the model prediction (see the last column of Table 1). Further trouble arises since different observers may examine the field for different lengths of time. So, for example, an observer looking for five seconds may glimpse a star once and hence not conclude that the star is visible, but the same observer looking for 15 seconds will glimpse the star three times and may record a confident detection. Blackwell shows that the difference between a 6-second observation and a 60-second observation corresponds to roughly half a magnitude.

The error in the model prediction is given in the last column for each observation. I have tested to see if the error is correlated with any of the input values and have found no significant correlation. In addition, I have checked for correlations with f-ratio, sex, magnitude sequence, latitude, and temperature with null results. Only the observer's experience, e, has a significant correlation with the error in the model prediction. It is in the sense that an experienced observer will see fainter than an inexperienced observer. This can be quantified as

$$m(e) = m + 0.16\,(e - 6) \;, \tag{20}$$

where m is as given from equation (16). The cause of this correction may be due to the experienced observer's better observing methods, such as knowing the most sensitive position for averted vision and better care to dark adapt. Another cause may be that an experienced observer may be more confident of a faint detection. For example, a star which is visible only 10% of the time may be a marginal detection for an average observer, but an experienced observer may know that this is a confident detection.

Various unmodeled conditions may affect the observed value of m. For example, hyperventilation is a well-known trick gaining several tenths of a magnitude,

$$m(\text{hyperventilation}) \approx m + 0.3 \;. \tag{21}$$

This gain in sensitivity is presumably related to extra oxygen in the bloodstream. Other unmodeled effects may well exist.

A histogram of the model errors is presented in Figure 2. The histogram is roughly a bell-shaped curve with a HWHM of 0.75 magnitude and a center of −0.24 magnitude. The large width of the histogram is distressing because it shows that the limit of accuracy for my model is much larger than I would have hoped or expected. The scatter cannot be caused by incorrect functional dependencies on the input parameters, since there is no correlation between any parameters and the model errors. One possible source of scatter is observational error, when the observer quotes values not appropriate for the observation. My primary suspicion along these lines is that the observer did not evaluate m_z with the same care that m was evaluated. Another difficulty is that the confidence levels for m were not well-defined, so that conservative observers might have a bright limiting magnitude. Finally, when equation (20) is included in the model, the HWHM is reduced to 0.5 magnitude (see Fig. 3).

The model presented in Section 2 can be tested with the observations presented in Table 1. The lack of any significant correlations with any of the input parameters demonstrates that the model does not have any biases in the functional form for each parameter. The near-zero center of the histogram in Figure 2 demonstrates that the model does not have any significant normalization biases. Hence, I conclude that the model from Section 2 is an accurate and unbiased representation of the observations. There are unmodeled effects, such as hyperventilation and experience, for which empirical corrections can be applied. The cause of the 0.5-magnitude uncertainty in the model prediction (after the correction for experience) is not known.

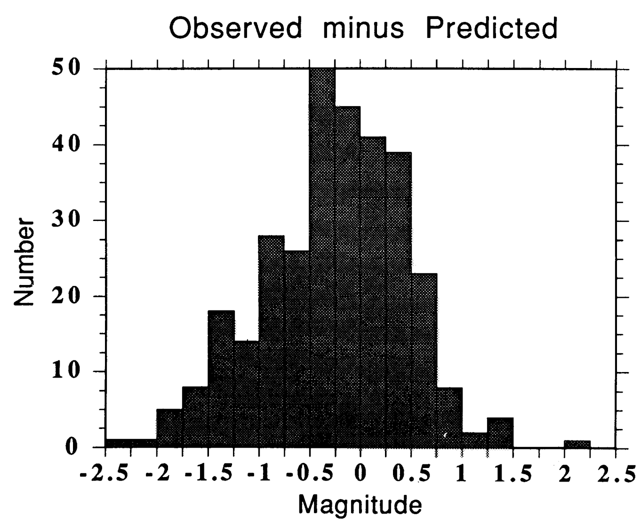

FIG. 2–Histogram of model errors. This shows the distribution of model errors; that is, the observed minus predicted limiting magnitude, for all 314 observations. A positive value for the error means that the observer saw fainter than I would have predicted. The errors are distributed roughly as a Gaussian-shaped curve with a HWHM of 0.75 magnitude and a center of −0.24 magnitude.

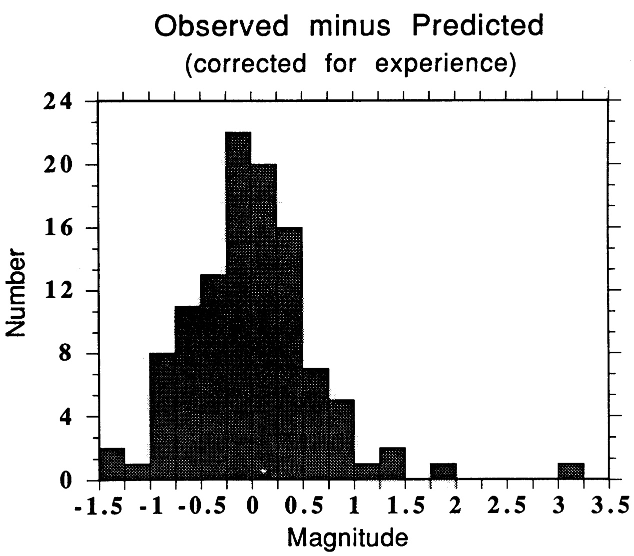

FIG. 3–Histogram of model errors after correction for experience. The model errors are correlated with the observer's experience, so that a very experienced observer may see over a magnitude fainter than a beginner. Equation (20) is an empirical representation of this correlation. When the experience effect is removed from the data, the size of the scatter in the errors is substantially reduced. The histogram of the errors after correction for experience is Gaussian shaped with a HWHM of 0.5 magnitude and a center of 0.0 magnitude.

4. Visual Recovery of Halley's Comet

O'Meara (1985) claims to have visually recovered Halley's comet with a 24" telescope when it was essentially a stellar image of visual magnitude 19.6. Some pundits (see Green 1985) have expressed scepticism that such a faint limiting magnitude would be possible. However, the internal evidence of O'Meara's claim is by itself convincing that not only was Halley's comet seen but that even fainter normal stars were seen. Specifically, he was able to detect unknown stars of roughly 20th magnitude that were later verified from the Palomar Sky Survey. In addition, another observer saw a 19.1-magnitude star with the same instrumentation so that an observer of O'Meara's exceptional acuity (see below) could plausibly have seen much fainter. Finally, he was able to correctly identify the comet, detect its motion, and estimate its correct magnitude despite the position being unknown.

O'Meara's observation had the advantage of many special conditions that improved his limiting magnitude. First, he observed from the top of Mauna Kea, which is arguably the best observing site in the world. Second, the telescope optics had been recently cleaned. Third, he used a high magnification of 549× which is near the theoretical optimum value. Fourth, he was breathing bottled pure oxygen. Fifth, O'Meara has a great visual acuity.

The great sensitivity of O'Meara's eyes is a tale that I have heard several times around the country at amateur gatherings. The high resolution of his eyes has been experimentally verified by Green (1985). On the night of his recovery of Halley's comet, O'Meara observed an unknown field with his unaided eye. Later comparison with the SAO Catalog showed that he consistently saw stars of magnitude 8.4. In 1978 I informally compared my limiting magnitude against O'Meara's (using the Harvard 9" Clark refractor and AAVSO charts) and found him going two magnitudes fainter than I. In 1988 I performed a controlled experiment where I randomly chose a star field and had him sketch all stars visible within it. Later I found that all stars in his sketch corresponded to real stars and that he had seen a star of magnitude 8.2.

We can estimate the F_s value for O'Meara on the night of the Halley observation. The night-sky conditions at the summit were likely to have been close to $B_S = 65$ mμL and $k_v = 0.11$ magnitude per air mass (Krisciunas *et al.* 1987 and private communication). For m_z equaling 8.4, F_s must equal 0.22 by equation (18).

The other correction factors can be estimated as follows: $F_b = 1.41$, $F_e = 1.13$, $F_t = 1.32$, $F_p = 1$, $F_a = 0.00012$, $F_r = 1$, $F_{SC} = 1$, $F_m = 301000$, and $F_c = 0.5$. For a sky brightness of 65 mμL the background brightness as perceived through the telescope will be 1.9 mμL. From equation (2) the faintest point-source brightness that can be detected is then 2.1×10^{-10} footcandles. When corrected to astronomical magnitudes (eqs. (14) and (16)), this corresponds to a V magnitude of 19.02.

The theoretical conclusion that O'Meara could have seen a 19.02-magnitude star has not taken into account his experience as an observer and his breathing of pure oxygen gas. From equations (20) and (21) a further improvement of roughly 0.8 magnitude is expected. So, my final prediction of the limiting magnitude of O'Meara's observation is roughly 19.8. And he might have gotten occasional glimpses of stars even half a magnitude fainter.

In summary, the internal details of O'Meara's observations are sufficient to prove his claim for having recovered Halley's comet, and the theory of telescopic limiting magnitudes shows that an observer of his acuity could easily have seen fainter than needed.

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